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球状星团的天体测量

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摘 要

球状星团是银河系中最年老的天体之一，是储存着银河系早期演化珍贵信息的"化石"．球状星团的天体测量，主要包括球状星团天区内恒星相对自行的测定，并由这些相对自行数据采用适当方法定出星团的绝对自行，或者直接测定绝对自行．利用这些自行数据，或者进一步与测光和视向速度数据结合，可以开展与球状星团的距离、运动、动力学状况、质量、年龄、演化等等以及银河系的结构和演化等有关的一系列重要的研究．在本文中对本世纪 70 年代中期以来在球状星团相对自行测定和成员概率估计、内部运动检测、绝对自行测定和空间运动研究这三方面取得的成果和进展以及存在的问题作了评述．

关 键 词 球状星团：一般 — 天体测量 — 方法：数据分析

Astrometry of Globular Clusters

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Abstract

The globular clusters are of the oldest objects in the Milky Way and "fossils" stored with valuable messages of the formation and evolution of the Galaxy. In astrometry of a globular cluster, relative proper motions of stars in region of the cluster are determined and absolute proper motion of the cluster can be derived from relative motions by an appropriate process, or determined directly. These motion data, or further, combined with radial velocity and photometric data, enable the important information of distance, space motion, dynamics, mass, age, evolution, etc. of the globular cluster and of structure and evolution of the Galaxy

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to be investigated. The results, progress, and open questions in respects of the determination of relative proper motions, the estimation of membership probabilities, the detection of internal motions, the determination of absolute proper motions, and the investigation of space motions of the globular clusters since the middle of 1970's are reviewed.

Key words globular cluster: general— astrometry—methods: data analysis

1 Introduction

The globular clusters are of the oldest objects in the Milky Way and "fossils" stored with valuable messages of the formation and evolution of the Galaxy. They are particularly useful in

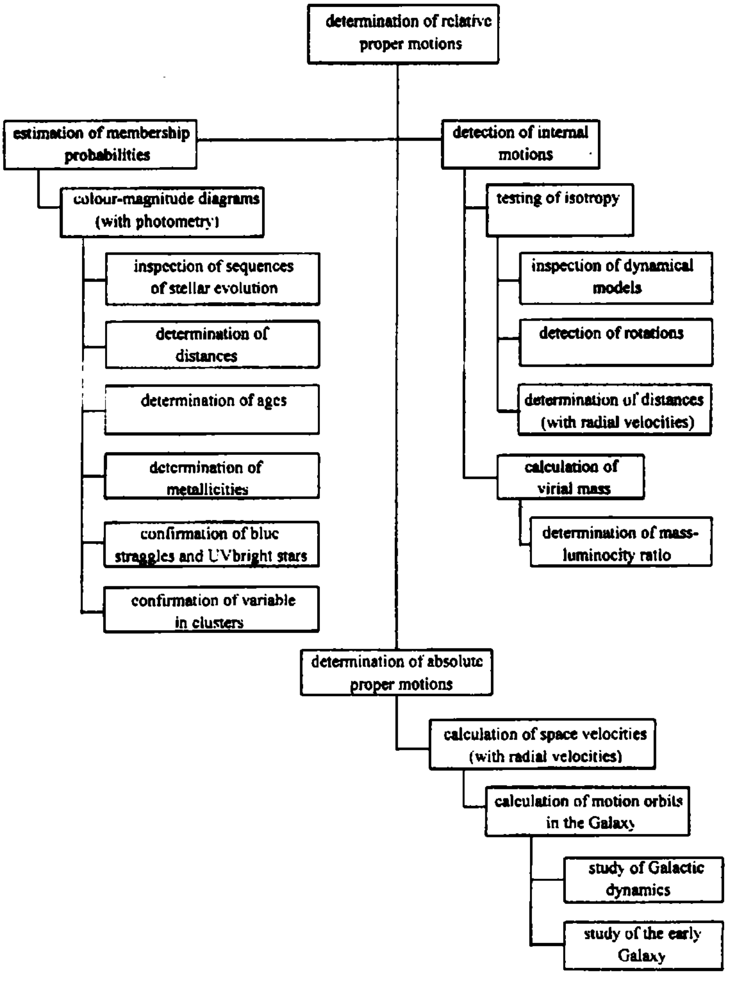

Fig. 1 The connection of astrometry of globular clusters with some relevant studies

the investigation of stellar evolution because much more stars are included in a globular cluster than in an open cluster and the inspection of theoretical sequences of stellar evolution on their colour-magnitude diagrams (CMDs) is thus possessed of much higher statistical significance. They are playing an important role in the test of dynamical theory of stellar system in view of that they are analogous with galaxies to high extent on numerous stars and they are much closer to us than extragalaxies so that we can directly measure their internal motions to verify some conclusions derived from dynamical theory. In astrometry of a globular cluster, relative proper motion of stars in region of the cluster is determined and the absolute proper motion of the cluster can be derived from the relative motions by an appropriate process, or determined directly. These motion data, or further, combined with radial velocity and photometric data, enable the important information of distance, space motion, dynamics, mass, age, evolution, etc. of the globular cluster and of structure and evolution of the Galaxy to be investigated. The connection of astrometry of globular clusters with some relevant studies is shown in Fig. 1.

The results, progress, and open questions in respects of the determination of relative proper motions and estimation of membership probabilities, the detection of internal motions, and the determination of absolute proper motions and investigation of space motions of the globular clusters since the middle of 1970's are reviewed here.

2 Determination of Relative Proper Motions and Estimation of Membership Probabilities

In 1931, Barnard[1], a famous astronomer who firstly devoted himself to determine relative proper motions of globular clusters, found detectable relative motions of two stars in the region of the globular cluster M92 from the measurements taken with the micrometer of the 1 m refractor at Yerkes Observatory between 1898 and 1922. Since the end of the 19th century, plates of some globular clusters were taken at Yerkes Observatory, etc. Afterwards, these plates became early epoch observational material in photographic astrometry of globular clusters. Round about 1910, astrometric plates of several globular clusters, which are now of great value, were taken at Shanghai Astronomical Observatory, too.

The earliest photographic determination of relative proper motions for globular clusters was carried out by Brown (1951)[2] on plates of the cluster M15 at Yerkes Observatory. The standard errors of these proper motions were about $\pm 0''.2$ century^{-1} or more. After some years, the precision of his result[3] for the cluster M13 reached about $\pm 0''.095$ century^{-1}. In 1960's, the relative proper motions of stars in the regions of the clusters M13, M3, and M5 were determined on plates of Pulkovo Observatory by Kadla[4] and Zhukov[5,6] with the precision of $\pm 0''.2$ – $0''.4$ century^{-1}. These early data of proper motions were only usable in very rough determination of membership and quite unable to detect the internal motions in these clusters.

Since 1976, Cudworth of Yerkes Observatory had taken advantage of the automatic measuring engine at Lick Observatory (called AME for short in Table 1) to measure the plates of globular clusters. Later, he and his collaborators scanned their plates on the PDS microdensitometers at University of California, Berkeley (B–PDS in Table 1) and at MADRAF (Midwest Astronomical Data Reduction and Analysis Facility) in Madison,WI (M–PDS in Table 1). The number of plates in their studies was increased substantially and epoch spans were greater and greater with the passage of time. In their reductions, a theoretically more rigorous algorithm with the central overlapping principle was used. All these improved the precision of the determination of relative proper motions in magnitudes to about $\pm0''.02$ century^{-1}. A summary of the determination of relative proper motions for some globular clusters published by astronomers of Yerkes Observatory since 1976[7–20] is listed in Table 1.

The determination of the membership for stars in the region of a globular cluster is less important than that of an open cluster because of more crowding of member stars, smaller angle radius, and less contamination by field stars, particularly in high galactic latitude regions. Nevertheless, it must be made at least for the clusters in the low galactic latitude zone. Compare two CMDs of the globular cluster M4 given by Cudworth & Rees[18] in Fig. 2, where panel (*a*) is one of all stars measured in the regions of M4 and panel (*b*) of members with proper motion membership probabilities greater than 0.3. Obviously, some details in the latter panel are clearer and more reliable than in the former.

Table 1 Determinations of relative proper motions for some globular clusters (Yerkes Observatory, 1976–1993)

cluster	authors	publishing year	number of plates	max. epoch differ. (years)	number of stars	limit magnitude	sky field (arcsec from center)	measuring machine	average precision of p. m. ($''$/100yr)	reference
M15	Cudworth	1976	10	74	210	15.5	80–840	AME	0.03	[7]
M92	Cudworth	1976	10	75	219	15.5	80–600	AME	0.03	[8]
	Rees	1992	22	88	365	16.0	17–800	M–PDS	0.01	[9]
M13	Cudworth & Monet	1979	22	77	443	15.5	0–835	B–PDS	0. 013	[10]
M3	Cudworth	1979	9	75	266	16.0	90–960	B–PDS	0.020	[11]
M5	Cudworth	1979	8	77	317	15.5	120–600	B–PDS	0.02	[12]
	Rees	1993	17	88	515	15.6	33–790	M–PDS	0.01	[13]
M71	Cudworth	1985	21	79	359	16.8	0–255	M–PDS	0.01	[14]
M22	Cudworth	1986	16	84	672	15.5	50–390	M–PDS	0.02	[15]
M2	Cudworth & Rauscher	1987	14	85	301	16.0	40–840	M–PDS	0.015	[16]
NGC 6712	Cudworth	1988	29	30	617	18.3	0–165	M–PDS	0.02	[17]
M4	Cudworth & Rees	1990	38	90	530	16.4	40–870	M–PDS	0.02	[18]
M28	Rees & Cudworth	1991	20	92	254	16.3	60–325	M–PDS	0.02	[19]
M107	Cudworth et al.	1992	18	25	365	16.8	20–720	M–PDS	0.03	[20]

Moreover, the determination of membership is necessary even for globular clusters in the high galactic latitude zone to confirm some objects scattered in CMD such as blue straggles and UV bright stars. Also the judgment of membership for some variables must have the aid of data of proper motions. For examples, membership for the four new variables with very low amplitudes discovered in the region of the cluster M4 by Yao[21] was confirmed on their proper motion membership probabilities by Cudworth & Rees[18]. Because of their peculiar locations in the CMD it became a very interesting problem in stellar physics.

The estimation of membership probabilities has been an appropriate extension of determining relative proper motions for some globular clusters at Yerkes Observatory since 1976. Owing to very high precision of their proper motion data, the derived probabilities yield an excellent segregation of cluster members from field stars, with very few stars having intermediate probabilities.

3 Detection of Internal Motions

The velocities of internal motions for stars in a globular cluster are in magnitude of 10 km·s^{-1}. Only if the precision of relative proper motions reaches about 0''.02 century^{-1} or better, the detection of such motions may be possible because the distances of globular clusters away from us are thousands of parsecs or more.

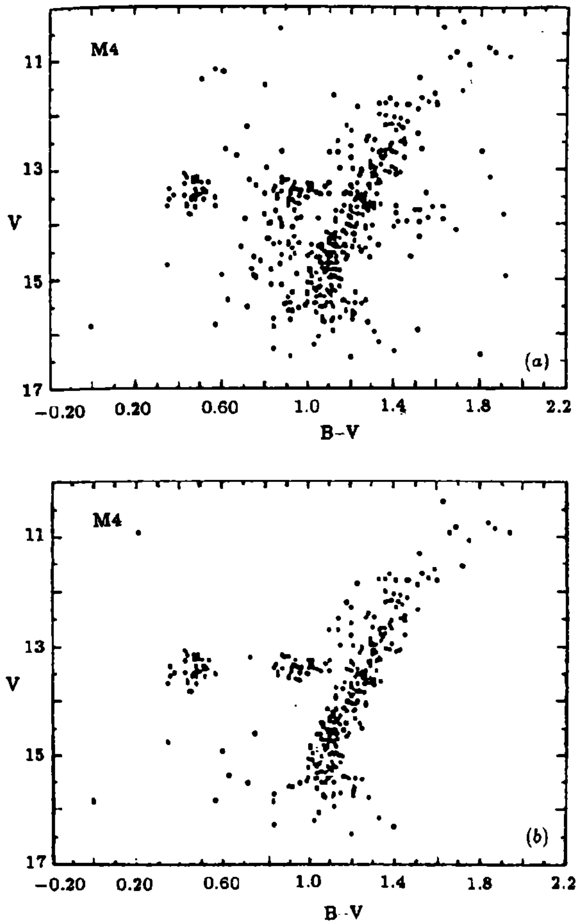

Fig.2 The CMD of stars in the region of the globular cluster M4[18]

(*a*) all stars regardless of membership probabilities

(*b*) stars with proper motion membership probabilities $p \geq 0.3$

The earlier studies of the internal motions in some globular clusters were based on data of radial velocities and therefore were limited in brighter and fewer stars of the clusters. A study of the cluster M3 published by Gunn & Griffin[22] in 1979 showed that the dispersions of internal motion velocities derived from radial velocities agreed with that of a theoretical model very well up to 15 core radii from the cluster center ($r = 375''$. This result is given in Fig. 3, where the abscissa is the logarithms of distances from the cluster center in arc-seconds, the ordinate is the dispersions of velocities in km·s^{-1}, the circles are observational values, the vertical bars indicate the ranges of observational errors, the curve is the theoretical model adopted, and r_c is the core radius.

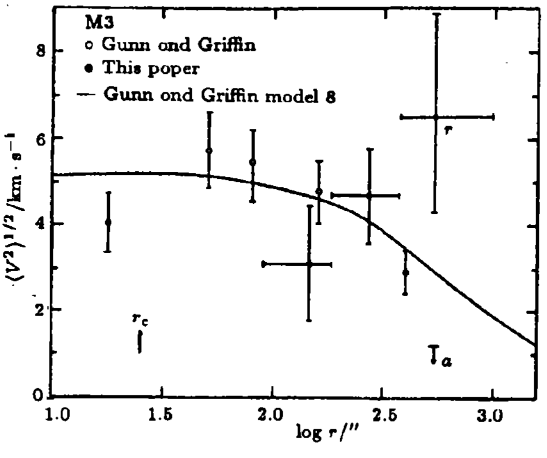

Fig.3 The dispersions of internal motion velocities versus the distances from the cluster center in the globular cluster M3[11]

The improvement of the precision of relative proper motion determinations represented by the work at Yerkes Observatory enabled one to investigate the internal motions in some globular clusters from relative proper motions. In order to detect the internal motions from relative proper motions, the proper motions of stars in a cluster are usually resolved into the radial and azimuthal components with respect to the cluster center. It is called the isotropy if the dispersion of radial components σ_r and the dispersion of azimuthal components σ_a are equal. Otherwise it is called the anisotropy. If σ_r is much greater than σ_a, it indicates that the majority of stars in the cluster are moving on orbits with great eccentricities; otherwise, it indicates that the majority of stars in the cluster are moving on approximately circular orbits. The different states of internal motions show the different dynamic properties in the clusters.

The result of an internal motion analysis from the relative proper motions of the cluster M3 by Cudworth[11] is also given in Fig. 3, where the dots are the observations, and the horizontal bars indicate the sizes of radial intervals. It is obvious that the errors of velocity dispersions from the proper motions are very large and the data points agree very marginally with the theoretical model in the error ranges. As the prediction of the model, evident anisotropy is shown in the interval of $r > 375''$ farthest away from the center (where the points r and a indicate the radial and azimuthal components, respectively). These conclusions were uncertain because of very large observational errors. Furthermore, the data of proper motions in the cluster core were not available. A great effort in improving relative proper motion measurements, therefore, is still desired to obtain meaningful results for cluster dynamics.

Assuming the dispersions of radial velocities and proper motions identical, the distance of a globular cluster can be derived from the observations of these dispersions. Such a distance determination based on astrometry should provide very important evidence of the reliability for distance determination on photometry or spectroscopy. The distance of 9.6 ± 2.6 kpc for the cluster M3 found by Cudworth[11] on this method is coincident in the error range with 9.2 kpc found by Sandage[23] on the mean sequence fitting and 10.4 kpc found by Gunn and Griffin[22] on the isochrone fitting though the precision is low.

In some cases, the rotation of a globular cluster may be detected on analysis of the internal motions. In particular, if the rotation axis of the cluster is parallel to the line of sight, the rotation of the cluster cannot be detected from relative proper motions or radial velocities either. On the other hand, if the axis is perpendicular to the line of sight, the rotation can be detected from the resolution of relative proper motions into two directions parallel to and perpendicular to the axis, respectively, and a comparison between two component dispersions.

In the strict sense, the effects of the rotation of a cluster should be eliminated from the dispersions of radial velocities and proper motions before the distance is derived from them. A successful attempt was given by Rees[9] in a new study of proper motions of the cluster M92. The position angle of rotation axis $165° \pm 15°$ and the ratio of dispersions parallel to and perpendicular to this axis 1.41 were obtained. This angle is coincident with that of the major axis found from the oblate shape of the image of the cluster M92 on the copies of Palomar Sky Survey plates by visual inspection. After the rotation effects are removed, an astrometric estimate of the distance 8.3 ± 1.6 kpc derived for the cluster M92 is well agree with the distance values derived with other methods.

4 Determination of Absolute Proper Motions and Investigation of Space Motions

Even before the earliest determination of relative proper motions of stars in the regions of globular clusters was published, a program of absolute proper motions of the clusters had been started. In 1927, an absolute proper motion of the cluster M13 was published by van Maanen[24]. In 1948, another determination of the absolute proper motion of the cluster M13 referenced to the extragalactic objects was performed by Gamaley. The difference between these two measurements was very large and even nearly equal to the measurements themselves. This problem is the primary trouble in the determination of the absolute proper motions of globular clusters so far.

There are three methods to determine absolute proper motions: the use of a classical reference system, the use of a field star reference frame with the assumptions about the solar motion and the rotation of the Galaxy, and the use of an extragalactic reference frame. A comparison of the absolute proper motions determined with these methods for the clusters M15 and M3 was given by Geffert *et al.*[26] and showed that the difference between the various measurements was large up to $0''.6$ century^{-1} with an average of $\pm 0''.2$ century^{-1}, which was the same magnitude as the absolute proper motions of these two clusters.

Without a doubt, relating to extragalactic objects is the reliable method to measure. No assumption is necessary for this method. The first attempt to determine the absolute proper motion of a globular cluster directly referred to a great number of background galaxies was performed by Scholz *et al.*[27] for the cluster M3 on the plates taken with the Schmidt telescope at Tautenburg Observatory. In this study, five pairs of plates with the epoch intervals from 20 to 40 years were reduced with respect to about 2,000 reference galaxies. A local inertial reference frame was well defined on these galaxies with an accuracy of $\pm 0''.05$ century^{-1}. However, a disadvantage from the Schmidt plates is low precision of the proper motions for individual stars, which are merely from 0″.2 to 0″.3 century^{-1} in the work of Scholz *et al.*[27] Of course, for the study of the space motion of a cluster, the whole cluster is regarded as an object and low precision of the proper motions for individual stars may not be a serious problem. This work has been extended to other clusters such as M12, M92, and M15 by Scholz *et al.*[38,40,42]

For the determination of the absolute proper motion of a globular cluster referred to field stars, a crucial problem is the assumptions about the solar motion and the rotation of the Galaxy. Some recent improvements may be helpful to raise the accuracy of the determination of absolute proper motions, but it is difficult to completely eliminate the systematic differences to the actual inertial system. By the use of a list of the stellar mean parallaxes derived by van Altena (1988) on a new model of the Milky Way, the solar motion derived by Hanson (1989) on the Northern Proper Motions (NPM) of Lick Observatory, and the Oort constants given by Kerr & Lynden-Bell (1986), the absolute proper motions of 14 globular clusters were calculated by Cudworth & Hanson[28] on the relative proper motions determined at Yerkes Observatory. Because of difficulty in estimating the systematic differences of them, these absolute motions are less significant in comparison with those of Scholz et al. though the former is with higher precision.

The absolute proper motion of a globular cluster determined merely by the use of a classical reference system is less significant, too, because such a system is not the rigorous inertial system. The programs of the NPM of Lick Observatory and the SPM (Southern Proper Motions) of Yale University Observatory in determining stellar absolute proper motions related to extragalaxies, however, will make this method more significant. In particular, the NPM program has been nearly completed and the NPM Catalogue has been available besides the hidden zone about the Galactic plane. The absolute motion of a cluster may be determined directly referred to the NPM Catalogue and such work is going on at Yerkes Observatory. On the other hand, the systematic differences of the classical systems represented by such as AGK3, PPM (the Positions and Proper Motions), etc. to the extragalactic reference system in the local regions of the sky can be determined on the NPM Catalogue and the absolute motions determined on the classical systems can be corrected.

The work of determining the absolute proper motions of the globular clusters on a combination of the AGK3 Catalogue and Lick proper motions was started in 1981 by Brosche *et al.* In view of the NPM Catalogue unavailable at that time, they used directly some NPM plates of Lick Observatory to determine the absolute proper motions of the reference stars related to extragalaxies. In the first step, they reduced measurements to the AGK3 system. The proper motions on the AGK3 were then replaced by those from Lick plates to construct a catalogue with the positions as closed to the AGK3 system as possible and the proper motions based on the extragalactic reference system. Finally, the absolute motion of the cluster was derived on this catalogue. The absolute motions of the clusters NGC5466 and NGC4147 were obtained with this method by Brosche *et al.*[29] A similar method was used by Geffert *et al.*[26] to determine a correction of the absolute proper motion of the cluster M15 derived by Le Compion on the PPM system and correct it onto the extragalactic system but they used the duplications of Schmidt plates of Palomar Observatory to determine the absolute proper motions of the PPM stars in the region of the cluster M15 related to extragalactic objects.

Table 2 Absolute proper motions and space motions of 26 globular clusters[30]

cluster	μ_α (″/100yr)	μ_δ (″/100yr)	type	reference	distance (kpc)	radial velocity (km·s^{-1})	Π (km·s^{-1})	Θ (km·s^{-1})	Z (km·s^{-1})
47 Tuc	+0.55	−0.16	1	[31]	4.6	−18.8	+27 ± 52	+191 ± 17	+33 ± 31
NGC288	+0.50	−0.68	1	[32]	8.4	−45.9	+8 ± 40	−96 ± 51	+52 ± 00
NGC362	+0.35	−0.26	1	[33]	8.6	+223.0	+11 ± 44	−37 ± 2	−73 ± 24
Pal3	+0.033	+0.030	1	[34]	90.8	+22.0	−311 ± 24	+156 ± 86	+165 ± 86
NGC4147	0.27	+0.09	1	[29]	18.8	+183.0	+167 ± 57	+244 ± 26	160 ± 26
M53	0.03	−0.40	1	[35]	18.5	−80.0	−151 ± 192	−100 ± 105	−124 ± 28
NGC5139	−0.22	−0.73	3	[36]	4.9	+232.2	−89 ± 37	−82 ± 3	−87 ± 37
M3	−0.31	−0.23	1	[27]	10.1	−146.6	+63 ± 10	+32 ± 26	−106 ± 4
NGC5466	−0.54	+0.06	1	[37]	15.8	+107.1	+261 ± 32	−125 ± 137	+225 ± 30
Pal5	−0.244	−0.087	1	[34]	23.0	−56.0	−124 ± 19	+5 ± 32	+64 ± 18
M5	+0.67	−0.78	1	[38]	7.6	+54.3	−290 ± 28	+232 ± 21	−202 ± 29
M4	−1.16	−1.57	2	[28]	2.0	+70.7	−56 ± 4	+44 ± 19	+9 ± 7
M107	−0.07	−0.31	2	[28]	6.4	−34.2	+24 ± 32	+150 ± 26	−43 ± 26
M13	−0.09	+0.55	2	[28]	7.2	−246.4	+290 ± 22	−66 ± 70	−114 ± 27
M12	+0.31	−0.75	1	[38]	5.6	−42.6	−24 ± 6	+152 ± 24	−168 ± 24
M10	−1.40	−0.63	3	[39]	4.3	+75.5	−80 ± 11	+8 ± 66	+197 ± 54
M92	−0.44	+0.11	1	[40]	7.5	−120.5	+106 ± 1	−6 ± 22	+70 ± 24
NGC6397	+0.33	−1.52	2	[28]	2.2	+18.8	+21 ± 8	+135 ± 10	−103 ± 11
M28	+0.03	−0.34	2	[28]	5.9	+15.5	+26 ± 5	+163 ± 24	45 ± 18
M22	+0.86	−0.51	2	[28]	3.0	−148.8	+171 ± 2	+185 ± 24	−113 ± 24
NGC6712	+0.42	−0.20	2	[28]	6.8	−107.5	+216 ± 7	+22 ± 11	−133 ± 20
M56	+0.21	+0.54	3	[39]	9.8	−135.9	+333 ± 9	−178 ± 132	+15 ± 109
M71	−0.23	−0.51	2	[28]	3.9	−23.1	+7 ± 8	+188 ± 16	−2 ± 15
NGC6934	−0.11	+0.14	3	[41]	14.9	−412.2	−25 ± 35	−239 ± 44	+254 ± 58
M15	−0.01	+0.02	1	[42]	10.5	−108.5	+120 ± 21	+102 ± 18	+66 ± 16
M2	+0.55	−0.42	2	[28]	11.9	−3.1	+68 ± 41	−92 ± 72	−306 ± 72

After the absolute proper motion of a globular cluster is obtained, if its radial velocity and distance are also known, the components of its space motion velocity in the system with respect to the Galactic standard of rest, Π, Θ, and Z, can then be easily found, only with adopting an appropriate assumption about the solar motion. Here the components Π and Θ are parallel to the Galactic plane, with the direction of Θ identical to that of the rotation of the Galaxy at the cluster and the direction of Π perpendicular to Θ and away from the Galactic center, and the component Z is perpendicular to the Galactic plane towards the north Galactic pole. Up to now, the absolute proper motions of 26 globular clusters have been determined. The absolute proper motions and space motion velocities of these 26 globular clusters are given in Table 2, where the data are taken from a paper of Dauphole et *al.*[30] to be published. In their choice of the absolute proper motion values the following principles were obeyed. In the first place, the proper motions on extragalactic adjustment were taken (called Type 1 in Table 2). In the next, the data of Cudworth & Hanson[28] were adopted (Type 2). For a few clusters lacked in the data of two types above mentioned, the data from other sources were used (Type 3). The types and sources of the proper motion data are listed in Table 2. The errors of the velocity components in Table 2 are typically about $\pm 40\mathrm{km}\cdot\mathrm{s}^{-1}$. They come mainly from the errors of proper motions and distances. The uncertainty of extragalactic adjustments is another important source of error but the error from this source is difficult to evaluate quantitatively and not included in the estimation of errors in Table 2.

From the space motions in Table 2, the orbits of the clusters in the Galaxy were derived via numerical integration and orbital eccentricities, total energy, the apogalactic and perigalactic distances and the maximum distance from the Galactic plane and their relation to the metallicity of the clusters have been studied by Dauphole *et al.*[30] They found that 10 of these 26 globular clusters are on retrograde orbits and globular clusters with metallicity larger than [Fe/H]= −1.4 appear to have smaller maximum distances from the Galactic plane and apogalactic distances than the metal–poorer clusters. Their apogalactic distances indicate a metallicity gradient in the halo globular clusters. However, only 26 clusters are very few statistically and the uncertainties of kinematic parameters such as apogalactic distances from errors of space motion velocities are very large. Thus, the results above mentioned are to be confirmed.

5 Conclusion

Among those above mentioned, we would include the following:

(1) For fine investigations of CMDs of globular clusters, particularly in the low galactic altitude regions, cleaning of CMDs by rejection of field stars on estimation of proper motion membership probabilities is necessary. In addition, the confirmation of some peculiar members such as variables, blue straggles, UV bright stars, etc. must also be based on proper motions. Thus, the high-precision determination of relative proper motions is important in the studies of globular clusters.

(2) For detection of internal motions of stars in a globular cluster, particularly in order to provide reliable observational constraint for the dynamic evolution theory of globular clusters and to determine distances and rotations of globular clusters astrometrically with high precision, the precision of relative proper motion measurements should be further improved. For this purpose, in addition to the use of existing plates as many as possible, as the most effective method, the time spans of plate materil should be further extended. Thus, even ten or twenty years later, the high-precision determination of relative proper motions in globular clusters will still be a quite interesting subject.

(3) In order to reliably determine space motion velocities of globular clusters in the system with respect to the galactic standard of rest and further in order to provide observational constraint for the construction and evolution theory of the Milky Way, the determination of absolute proper motions of the clusters must be based on the extragalactic reference system. The most reliable method of determining absolute proper motions is undoubtedly the use of Schmidt plates

because there are a great number of extragalactic objects on such plates. The NPM and future SPM Catalogues provide convenience and higher reliability for transforming the relative proper motions and the proper motions related to classical reference system to the absolute proper motions referred to extragalactic objects. The work on this area is in the first steps and more meaningful results will be obtained with further studies.

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