Kalman filter simulation and characterization of BDS satellite clock

Bin Wang*, Junping Chen*† and Binghao Wang‡
Email: {binw, junping}@shao.ac.cn wangbinghao7@126.com
*Shanghai Astronomical Observatory, CAS, Shanghai, China
† School of Astronomy and Space Science, UCAS, Beijing, China
‡ Zhengzhou Institute of Surveying and Mapping, Zhengzhou, China

***Abstract*—In this paper, a Kalman filter algorithm for simulation and characterization of BDS (beidou satellite system) satellite clock in orbit is presented. The algorithm is designed to produce near optimal estimation of characteristic parameters for BDS satellite clock. Besides normal clock noises, white frequency modulation, random walk frequency modulation and random run frequency modulation, the flicker phase modulation and flicker frequency modulation are also considered as the linear combination of several Gauss Markov processes. Additional states are used to model the periodic variations of BDS satellite clock, and the first type model of clock periodic states is used in the paper. It is shown that simulation results fit the real oscillator behavior well.**

Keywords—BDS; clock simulation; clock characteristic parmeter estimation; kalman filter; flicker noise

I. INTRODUCTION

An accurate and correct characterization of satellite clock is of great importance for clock prediction, and the prerequisite and basis for the production of time scale in GNSS (global navigation satellite system). Significant effort has been reported on the analysis of the clock stability and systematic variations of GNSS satellite clocks. Periodic variations are found in BDS (beidou satellite system) satellite clocks [1]. Through spectral analysis, the presence of 1 and 2 cpr (cycle per revolution) periodicities is evident. BDS satellite clock fundamental harmonic period is different from that of satellite orbit. It is inferred that clock periodic variations are related to solar illumination. Besides harmonics, in time intervals smaller than 1e3s, flicker phase modulation (FLPM) can be found, and in time intervals larger than 2e5s, flicker frequency modulation (FLFM) is remarkable for BDS satellite clock [2].

Kalman filter is extensively used in real-time applications of clocks and oscillators. Two-state model was suggested by Fran to describe the behavior of cesium clock [3]. The inclusion of a linear clock drift term brought about three state clock models, particularly necessary to model the drift of masers and rubidium clocks [4]. Kalman filter algorithms usually model the clock noise as a linear combination of white frequency modulation (WHFM), random walk frequency modulation (RWFM) and random run frequency modulation (RRFM). However, flicker noise and periodic variations are common in the GNSS satellite clock. Flicker noise processes can be modelled approximately as a linear combination of Gauss Markov processes [5]. There are three different models of clock periodic states can be used in Kalman filter [6]. The first model that adds the periodic states to only the phase state without any stochastic correlation between the stochastics driving the harmonics and the stochastics driving the base model, is simple and fast. It is preferred in the use of simulation and characterization of GNSS satellite clock.

In this paper, to simulate and characterize the BDS satellite clock in orbit, the satellite clock model that account for the periodic signals and flicker noise processes (FLPM and FLFM) is presented. After that Kalman filter algorithm to characterize BDS satellite clocks from real measurements is proposed.

This work is support by the NSFC (No.11673050), the 863 (No. 2014AA123102) projects and also the open project program of the key lab of space navigation and positioning technology (No. KFKT-201705), Shanghai.

II. BDS SATELLITE CLOCK MODEL

A. Clock base model

Basic clock model $x_{base}(t)$, is typically represented as:

$$x_{base}(t) = x_0 + y_0(t-t_0) + \frac{1}{2}z_0(t-t_0)^2 \tag{1}$$

where x_0, y_0 and z_0 is the initial values of clock-offset, frequency offset and linear frequency drift, respectively at epoch t_0, and t-t_0 can be denoted by τ. In general, the basic clock model is a purely deterministic of clock behavior.

B. Harmonic signals

The harmonic signals, denoted by $x_{har}(t)$, can be expressed as

$$x_{har}(t) = \sum_{i=1}^{n_{har}}\left[c_i\cos(2\pi f_i t) + s_i\sin(2\pi f_i t)\right] \tag{2}$$

where n_{har} is the number of harmonics, c_i and s_i is harmonic coefficients, f_i is the frequency of i-th harmonic, and $f_i = if_0$, f_0 is the basic frequency. In this paper, it is assumed that the basic frequency is known.

C. Gauss-Markov noise

Non-stationary flicker noise (over a finite time period) can be approximated as a linear combination of several independent Gauss-Markov processes [5]:

$$x_{Flicker}(t) = \sum_{i=1}^{n_{flicker}} x_{GM_i}(t) \tag{3}$$

For BDS satellite clock, four independent Gauss-Markov processes are used to approximate the FLPM and FLFM respectively in time intervals smaller than 1e3s and larger than 2e5s.

BDS satellite clock model is composed of above three components, and can be expressed as

$$x(t) = x_{base}(t) + x_{har}(t) + x_{flpm}(t) + x_{flfm}(t) \tag{4}$$

III. KALMAN FILTER IMPLEMENTATION

In order to implement BDS satellite clock model with the Kalman filter algorithm, several elements should be defined. In this section, state vector, state-transition matrix, process noise covariance matrix, measurement matrix is introduced. Besides, process noise parameters estimation and Kalman filter initialization are also discussed.

A. State vector

At epoch k, state vector of BDS satellite clock Kalman filter can be defined as

$$\mathbf{X}_k = \begin{bmatrix} \mathbf{x}_{base} & \mathbf{x}_{har} & \mathbf{x}_{flpm} & \mathbf{x}_{flfm} \end{bmatrix}^T \tag{5}$$

where $\mathbf{x}_{base} = [x_k, y_k, z_k]$ is used to describe the basic clock model, $\mathbf{x}_{har} = [c_{1,k}, s_{1,k}, \ldots, c_{n1,k}, s_{n1,k}]$ is used to describe n1 harmonics, $\mathbf{x}_{flpm} = [GM_{1,k}, GM_{2,k}, \ldots, GM_{n2,k}]$ is n2 Gauss-Markov components used to approximate the FLPM in the time interval smaller than 1e3s, and $\mathbf{x}_{flfm} = [GM_{n2+1,k}, GM_{n2+2,k}, \ldots, GM_{n2+n3,k}]$ is n3 Gauss-Markov components used to approximate the FLFM in the time interval larger than 2e5s.

B. State transition matrix

State-transition matrix of BDS satellite clock model Kalman filter can be expressed as

$$\mathbf{\Phi}_k = \left[\begin{array}{ccc|c|c|c} 1 & \tau & \tau^2/2 & & & \mathbf{\Phi}_{GMf1} \\ & 1 & \tau & & & \mathbf{0}_{1,n3} \\ & & 1 & & & \mathbf{0}_{1,n3} \\ \hline & & & \mathbf{I}_{2n1} & & \\ \hline & & & & \mathbf{\Phi}_{GMp} & \\ \hline & & & & & \mathbf{\Phi}_{GMf2} \end{array}\right] \tag{6}$$

where $\mathbf{I}_{2n1}$ is a $2n1\times2n1$ identity matrix used for the harmonics, $\mathbf{\Phi}_{GMp}$ is a $n2\times n2$ matrix used for the approximation of FLPM, $\mathbf{\Phi}_{GMf1}$ is a $1\times n3$ vector used to describe the influences of FLFM on clock offset, $\mathbf{\Phi}_{GMf2}$ is a $n3\times n3$ matrix used for the approximation of FLFM, the expression of $\mathbf{\Phi}_{GMp}$, $\mathbf{\Phi}_{GMf1}$ and $\mathbf{\Phi}_{GMf2}$ can be found in [5].

C. Process noise covariance matrix

Process noise covariance matrix is given as

$$\mathbf{Q}_k = \left[\begin{array}{ccc|c|c|c} Q_{11} & Q_{base,12} & Q_{base,13} & & & Q_{GMf1} \\ Q_{base,21} & Q_{base,22} & Q_{base,23} & & & \mathbf{0}_{1,n3} \\ Q_{base,31} & Q_{base,32} & Q_{base,33} & & & \mathbf{0}_{1,n3} \\ \hline & & & q_{har}\mathbf{I}_{2n1} & & \\ \hline & & & & Q_{GMp} & \\ \hline Q_{GMf1}^T & \mathbf{0}_{n3,1} & \mathbf{0}_{n3,1} & & & Q_{GMf2} \end{array}\right] \tag{7}$$

where Q_{base} is the process noise covariance matrix of the basic clock model:

$$Q_{base} = \begin{bmatrix} \sigma_1^2\tau + \frac{1}{3}\sigma_2^2\tau^3 + \frac{1}{20}\sigma_3^2\tau^5 & \frac{1}{2}\sigma_2^2\tau^2 + \frac{1}{8}\sigma_3^2\tau^4 & \frac{1}{6}\sigma_3^2\tau^3 \\ \frac{1}{2}\sigma_2^2\tau^2 + \frac{1}{8}\sigma_3^2\tau^4 & \sigma_2^2\tau + \frac{1}{3}\sigma_3^2\tau^3 & \frac{1}{2}\sigma_3^2\tau^2 \\ \frac{1}{6}\sigma_3^2\tau^3 & \frac{1}{2}\sigma_3^2\tau^2 & \sigma_3^2\tau \end{bmatrix} \tag{8}$$

where $\sigma_i^2\ (i=1,2,3)$ is the noise spectral density of WHFM, RWFM and RRFM respectively, $Q_{base,ij}$ is the (i, j)th element of Q_{base} matrix. It is noted that

$$Q_{11} = Q_{base,11} + \tau^3 \sum_{i=1}^{n_3} \sigma_{GMf_i}^2 a_{11}\left(R_{f_i}\tau\right) \tag{9}$$

where $\sigma_{GMf_i}^2$ is two-sided spectral density of the ith Gauss-Markov processes associated with the FLFM, a_{11} is defined in [5]. q_{har} is the variance of process noise associated with harmonics. Q_{GMp} is used for the approximation of FLPM, Q_{GMf1} and Q_{GMf2} are used for FLFM.

D. Measurement matrix

BDS satellite clock Kalman filter is designed to use IGS (international GNSS service) MGEX (multi-GNSS experiment) satellite clock data. The measurement matrix, describes the relationship between state-vector and the measurements, can be defined as

$$\mathbf{H}_k = \left[\begin{array}{ccc|c|c|c} 1 & 0 & 0 & H_{har} & \mathbf{1}_{1,n2} & \mathbf{0}_{1,n3} \end{array}\right] \tag{10}$$

where H_{har} is $1\times2n1$ vector, used to relate the harmonics with clock offset:

$$H_{har} = \left[\cos(\omega_1 t) \quad \sin(\omega_1 t) \quad \cdots \quad \cos(\omega_{n1} t) \quad \sin(\omega_{n1} t) \right] \quad (11)$$

where $\omega_i = 2\pi f_i$ is the angular velocity of i-th harmonic.

E. Process noise parameters estimation

In order to obtain optimal estimation results, it is necessary to carefully configure Kalman filter parameters for each BDS satellite clock. Determination of process noise parameter associated with each of state vector is of vital importance. These parameters are usually estimated using a sufficient set of past data and some judging criteria should be defined [7]:

1) prediciton error deviation matches with RMS prediciton errors obtained from the Kalman filter predicitons;

2) time domain variances estimated from simulation data match with that estimated from real measurements;

3) kalman filter residuals obey white noise processes.

Simulating data using Kalman filter can be found in [8]. Simulation vector **X** evolve between epoch k-1 and k:

$$\mathbf{X}_k = \mathbf{\Phi}(\tau)\mathbf{X}_{k-1} + \mathbf{L}_Q(\tau)\mathbf{e}(\tau) \quad (12)$$

where $\boldsymbol{L}_Q$ is lower triangle matrix, can be obtained from the cholesky decomposition of **Q**:

$$\mathbf{Q} = \mathbf{L}_Q(\tau)\mathbf{L}_Q^T(\tau) \quad (13)$$

$\mathbf{e}_Q$ is Gauss white noise, and process noise covariance matrix $\mathbf{P}_k$ evolves as:

$$\mathbf{P}_k = \mathbf{\Phi}(\tau)\mathbf{P}_{k-1}\mathbf{\Phi}^T(\tau) + \mathbf{Q}(\tau) \quad (14)$$

F. Kalman filter initialisation

State vector and their error covariance matrix should be set to realistic estimates. The non-diagonal elements of error covariance matrix can be set to zero. Three measurements can be used to initialize the state vector of basic clock model [9]. In addition, initialization process can be completed using the empirical value.

IV. RESULTS AND DISCCUSION

Noise spectral densities of BDS satellite clock are estimated using the Kalman filter simulation algorithm. BDS constellation are comprised by GEO (geostationary earth orbit), IGSO (inclined geosynchronous satellite orbit) and MEO (middle earth orbit) satellites. Clock on each satellite orbit type are simulated and the results are discussed.

A. Data source

Two types of satellite clock data can be obtained for BDS, ODTS (orbit determination and time synchronization) and TWTT (two-way time transfer) satellite clock data [2]. However, ODTS satellite clock data is easy to get, and IGS MGEX provides public services of satellite clock and satellite orbit. In this paper, BDS satellite clock data provided by WHU (wuhan university) analysis center are used. The analysis period is the entire week of GPS week 1945. Sampling interval of BDS satellite clock data is 5min.

B. Kalman filter simulaiton results and discussion

In order to evaluate the BDS satellite clock simulation result, we should choose reasonable judging criteria. Hereinafter, difference between time domain variances respectively estimated from simulation data and real measurements is selected to evaluate the simulation or parameter estimation results. In view of the facts that overlapping allan deviation of BDS satellite clock at one day interval is about 1e-14. Therefore, it is defined that if the difference is smaller than 1e-14, then the simulation is valid.

Comparison of overlapping allan deviation estimated from Kalman filter simulation data and real measurements for GEO satellite is shown in Fig. 1. Frequency stability analysis result of simulation data is plotted with yellow line, while that of the measurements is plotted with green line, and the difference between them is plotted using blue-dot line.

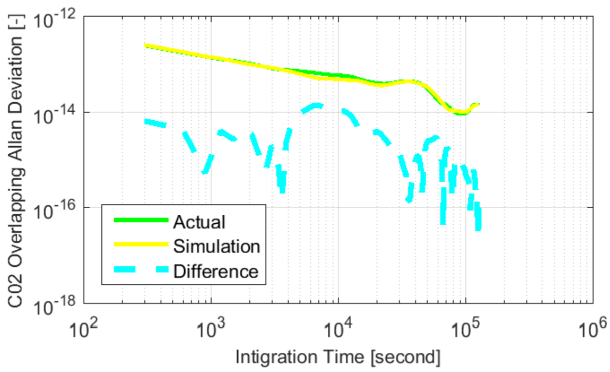

Fig. 1. Comparison of frequency stability between simulation data and real measurements of C02 (GEO) satellite clock.

From Fig. 1, it can be seen that for most of time period, Kalman filter simulation results can approximate the stochastic characterization of real measurements, except for the time interval of 4e3s-1e4s. In time interval of 4e4s-8e4s, periodic variations of GEO satellite clock are notable.

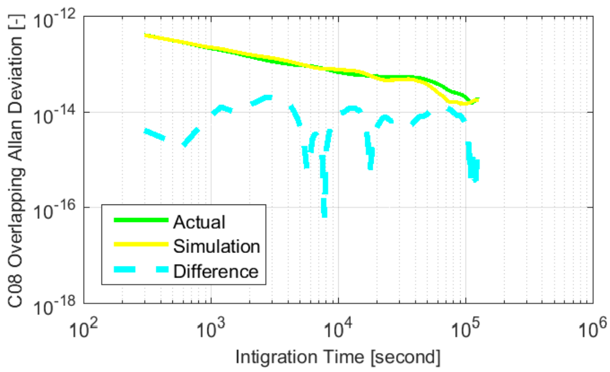

Fig. 2. Comparison of frequency stability between simulation data and real data of C08 (IGSO) satellite clock.

Simulation result of IGSO (C08) satellite clock is shown in Fig. 2. Long influences duration of IGSO satellite clock periodic variations can be found. In the time interval of 4e4s-8e4s, simulation result of IGSO satellite clock is worse than that of GEO satellite clock. But in the time interval of 1e3s-4e3s, simulation result of IGSO satellite clock is not good.

Simulation result of MEO (C12) satellite clock is shown in Fig. 3. It is found that Kalman filter simulation performance of MEO satellite is better than that of GEO/IGSO satellite clock.

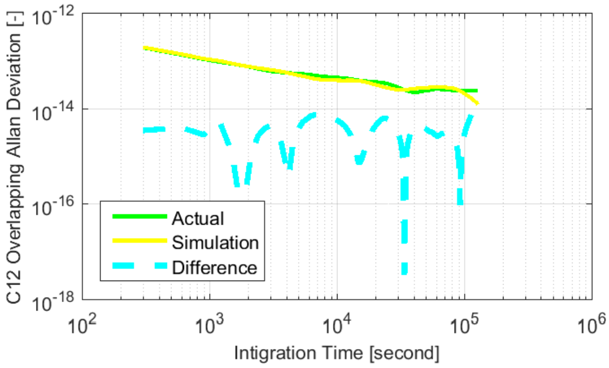

Fig. 3. Comparison of frequency stability between simulation data and real data of C12 (MEO) satellite clock.

Model parameters used in the BDS satellite clock Kalman filter are shown in Table I. It can be seen that for BDS satellite clock, spectral densities of WHFM is about 1e-23, while 1e-35 and 1e-47 for RWFM and RRFM respectively.

TABLE I. MOLDEL PARAMETERS USED IN THE BDS SATELLITE CLOCK KALMAN FILTER

PRN	**Spectral Densities**		
	WHFM (sec^2/sec)	***RWFM*** (sec^2/sec^3)	***RRFM*** (sec^2/sec^5)
C02	2e-23	9e-35	1e-47
C08	5e-23	1e-35	1e-47
C12	1e-23	1e-35	1e-47

V. CONCLUSIONS AND FUTURE WORK

Kalman filter algorithm for simulation and characterization of BDS satellite clock in orbit is presented. The algorithm is designed to produce near optimal estimation of characteristic parameters for BDS satellite clock. Several types of noise are considered in the algorithm, including WHPM, WHFM, RWFM, FLFM and also RRFM. Flicker noise is approximated by the linear combination of several Gauss-Markov processes. Besides, clock periodic variations are also considered and the simple model is used. Periodic states are only added to the phase state without any stochastic correlation between the stochastics driving the harmonics and the stochastics driving the base model.

For most of time period, Kalman filter simulation results can approximate the stochastic characterization of real measurements, except for the time interval of 4e3s-1e4s and time interval of 4e4s-8e4s. In the time interval of 4e4s-8e4s, the periodic variations of GEO are more remarkable than that of IGSO/MEO satellite clock. Simulation results show that for BDS satellite clock, spectral densities of WHFM is about 1e-23, while 1e-35 and 1e-47 for RWFM and RRFM respectively.

In this paper, only one week data is used to estimate the model parameters of BDS satellite clock Kalman filter, and the data may be not enough. In the future, more historical data would be used. And the processes of initialization and configuration of Kalman filter parameters may be automated with the help of machine learning.

ACKNOWLEDGMENT

We would like to thank Wang Wei for valuable discussions, and also the IGS MGEX campaign for providing multi-GNSS data and products.

REFERENCES

[1] Bin Wang, Yidong Lou, Jingnan Liu, Qile Zhao and Xing Su, "Analysis of BDS satellite clocks in orbit," GPS Solutions. vol. 20, pp. 783-794, 2016.

[2] Bin Wang and Junping Chen, "Analysis of BDS satellite clock in orbit with ODTS and TWTT satelltie clock data," Preoceedings of China Satellite Navigation Conference (CSNC) 2018, in press.

[3] Fran B. Varnum, "Kalman filtering with a two-state clock model." Proceedings of the 15th Annual PTTI Meeting 1983. Washington, D.C., USA.

[4] P.V. Tryon and R.H. Jones, "Estimation of parameters in models for cesium beam atomic clocks," Journal of Research of the National Bureau of Standards. vol. 88, pp. 3–16, 1983.

[5] J.A. Davis, C.A. Greenhall, and P.W. Stacey, "A Kalman filter clock algorithm for use in the presence of flicker frequency modulation noise," Metrologia, vol. 42, pp. 1-10, 2005.

[6] M.J. Coleman, and Ken Senior, "A comparison of GPS clock models for the next generation GPS system timescale," Proceedings of the 43rd Annual Precise Time and Time Interval Systems and Applications Meeting. Long Beach, CA, 2011.

[7] Bhattarai S., "Satellite clock time offset prediction in global navigation satellite systems," Department of Civil, Environmental and Geomatic Engineering, University College London, 2014.

[8] Davis, John, "Use of Kalman filters in time and frequency analysis," Proceedings of the 25th European Frequency and Time Forum, 2011.

[9] Stein S R, Filler R L, "Kalman filter analysis for real time applications of clocks and oscillators," Proceedings of the 42nd Frequency Control Symposium, Baltimore, MD, USA, 1988.