Analysis of Sagnac Correction for Time Transfer in Optical Fibers

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Abstract. Over the past few years, atomic clocks have been improving and are now reaching stabilities and accuracies of a few parts in 10^{18} in fractional frequency. Fiber-based time and frequency transfer techniques have demonstrated excellent performance in the comparison of state-of-art optical atomic clocks over thousands of kilometers. For highly accurate time and frequency transfers, relativistic effects which affect the signal propagation in optical fibers, need to be taken into account. The most important is the Sagnac correction because of the computation complexity and also the non-reciprocity in the two-way time transfer. Sagnac correction is an important source of uncertainty in fiber-based time and frequency transfer. Besides, not all important parameters are known with sufficient precision when we compute the Sagnac correction such as the large position error of fiber nodes and also the sparse fiber nodes. It is necessary to evaluate the Sagnac correction due to imperfect knowledge of parameters. In our work, several simulation fiber links in China are analyzed as specific examples to evaluate the influences of imperfect knowledge on the accuracy of the time transfer. The results show that in order to ensure one picosecond precision of time transfer using optical fiber, position accuracy of nodes should be higher than 500 m when the information of enough number of nodes can be obtained.

Keywords: Time transfer · Sagnac correction · Optical fibers

1 Introduction

Precise time and frequency are the most demanding physical quantities in the development of science and technology. Optical clocks, due to their unprecedented precision [1] and accuracy [2], are already being used in the experiment of physical theories [3], and will play an important role in the redefinition of the Intentional System of Units and the development of global timescales [4]. Traditional satellite-based techniques used for long distance comparison of frequency standards, are no longer capable of supporting the required accuracy of optical clocks, currently at the 10^{-18} level [2]. However, fiber-based time and frequency transfer techniques have demonstrated excellent performance in the comparison of state-of-art optical atomic clocks over thousands of kilometers [5], and such fiber network would be one of the key technologies for the application of new generation of atomic clocks in many fields, such as geodesy, radio astronomy, spectroscopy, and so on.

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Requirements on accuracy and stability of time and frequency transfer push the evaluation of the phenomena affecting the signal propagation in fiber link, and also the uncertainty of correction models to a new level. Uncertainty of time transfer mainly arises from the employed time modems and the fiber link. High-precision calibration can be performed to reduce the influences of delays in time transfer modems [6]. While fiber link is subject to various factors, such as temperature variations, vibrations, acoustic noise, relativistic effects, and so on. Among them, relativistic effects can be precisely modeled [7], but difficult to correct in some cases due to poor knowledge of fiber link information [8].

Model of relativistic corrections with an uncertainty of 1 ps has been proposed by Geršl et al. [7]. The time corrections for one way time transfer using fiber link are composed of three terms, Newtonian term, Sagnac correction and also the Shapiro correction. Newtonian and Shapiro corrections can be compensated by the two-way time transfer, while not for the Sagnac correction. Therefore, Sagnac correction is an important source of uncertainty in the long haul optical time transfer.

Sagnac correction depends on the area of equatorial projection of the surface swept by the earth radial vector moving along the optical fiber. Difficulty in the computation of Sagnac correction mainly results from the limited knowledge of the route of optical fiber. The real coordinates of fiber route are often not known exactly or confidential for internal use of optical network operator only. Therefore, several approximated methods have been proposed [7–12]. These methods can be classified into four types of methods. The first two methods are sphere approximation [11] and ellipsoid approximation [9] of the earth when only the coordinates of the end points are known. The third method is used in the case that some nodes are exactly known an also the distance between two adjacent nodes [7]. The fourth method is used in the case of benchmark real-like optical paths [8]. As for the confidence interval, there are also four types of methods. First method uses the sector area of the parallels passing through the ends of the fiber link [9]. Second method uses the sector area of the parallels with the fixed length of fiber link [10]. Third method uses the maximum and minimum triangle area constrained by the fixed length [7], while fourth method uses the maximum semicircle area constrained by the fixed length [12].

In our work we do some analysis of Sagnac correction on china fiber link. Due to the very limited knowledge of the optical fiber that only the coordinates of some nodes are known, we have to find some real-like transmission lines for analysis. It's known that fiber cables on land are typically installed in ducts for utility distribution, such as gas and electricity, often running across metropolitan areas and along highways, routes planning function of map API (Application Programming Interface) is used to generate a real-like optical-fiber link. Based on the generated optical-fiber link routes, influences of imperfect information (especially the position error and also the sparse degree of fiber nodes) on Sagnac correction are analyzed.

2 Sagnac Correction in the Fiber-Based Time Transfer

Systematic relativistic description of signal propagation in optical fibers can be found in [7], and the time correction for one way time transfer over optical fiber is also derived with an uncertainty of 1 ps:

$$\Delta t_{\pm} = \frac{1}{c}\int_0^L n \, \mathrm{d}l \pm \frac{1}{c^2}\int_0^L \mathbf{v} \cdot \mathbf{s}_l \, \mathrm{d}l \pm \frac{1}{c^3}\int_0^L n\left(w + v^2/2\right) \, \mathrm{d}l \tag{1}$$

where c is the light speed, n is an effective refractive index, $\mathbf{v}$ is velocity vector of a fiber point, $\mathbf{s}_l$ is the tangent vector field with parameter l, w is earth potential, $v^2 = \mathbf{v} \cdot \mathbf{v}$. The first c^{-1} term is Newtonian term and the third c^{-3} term is Shapiro correction. These two terms are independent of direction, and can be compensated in two-way time transfer. The main part of the second term (c^{-2}) in above formula is the Sagnac correction and comes from the earth rotation. Sagnac correction can also be expressed as:

$$\Delta t_{\pm sagnac} = \frac{2\Omega A_{\pm}}{c^2} \tag{2}$$

where Ω is the nominal mean value of earth's angular velocity, numerically 7.2921115e-5 rad/s [13], and contribution of earth tides on the earth's angular velocity is omitted. $A_{\pm}$ corresponds to the equatorial projection area of the surface swept by the geocentric radial vector moving along the optical fiber. Besides, Sagnac correction depends on the direction of signal propagation, when the signal propagates along the direction of earth rotation, the sign is positive, otherwise, the sign is negative. Above equation assumes that $A_{\pm}$ is directed area dependent of the relative position of the two ends of optical fiber or segments of optical fiber. It will be shown that this assumption can be used to significantly simplify the numerical calculation process of Sagnac correction for a long-haul optical fiber.

3 Comparison of Sagnac Correction Methods

In order to obtain exact Sagnac correction, Sagnac area should be computed as precise as possible. However, the real coordinates of optical fiber are often not known exactly or confidential for internal use of optical network operator due to security reasons. Therefore, several approximation methods of Sagnac correction have been proposed, and also the confidence interval computation. In the following, comparison of these methods is performed.

Four methods can be found for Sagnac correction computation. First method (Method I) approximates the earth as an ideal sphere, when the ends of fiber link are known, the arc of great circle is used to approximate the real fiber link [11], as shown in Fig. 1. When computing the Sagnac correction, the following procedures can be used. Firstly, normal vector of plane OAB can be obtained through the cross product of radial vectors $\boldsymbol{OA}$ and $\boldsymbol{OB}$, then we have the equation of route L_{AB} on the great circle with combination of OAB plane equation and the sphere equation. After the cancellation of z variable, we can get the equation of $L_{A'B'}$ on the xoy plane, and projected area can be obtained through the integral.

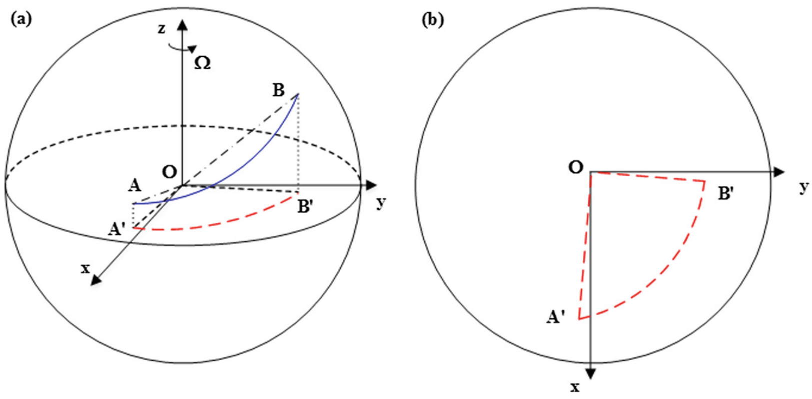

Fig. 1. Sagnac correction computation in the case of sphere approximation of the earth. The fiber link is shown in the left subplot (a), projected Sagnac area on the equatorial plane is shown in (b).

Second method (Method II) approximates the earth as an ellipsoid, when the ends of fiber link are known, the meridians and the parallels are used to approximate the real fiber link [9, 10], as shown in Fig. 2. Even though the real optical fiber may runs beyond the region between the meridians and parallels through its two ends, the projected area of the circular sectors (B′OC′ and A′OD′) closed by the projections of meridians and parallels on the equatorial and the earth radial vector would be a good approximation of the Sagnac area. With the known coordinates of two ends, parallels radius can be easily computed, then we can obtain the area of B′OC′ and A′OD′. Sagnac correction can be considered as the mean value of two corrections corresponding to the area of B′OC′ and A′OD′. Besides, uncertainty of the estimation can also be obtained using the difference between the area of B′OC′ and A′OD′.

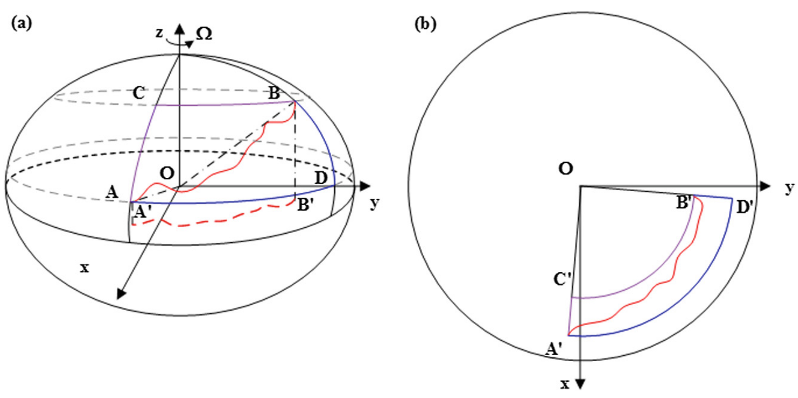

Fig. 2. Sagnac correction computation in the case of ellipsoid approximation of the earth. The fiber link is shown in the left subplot (a), projected Sagnac area on the equatorial plane is shown in (b). The violet and blue lines correspond to the considered projection of hypothetical fiber link.

If the position of some nodes and also the lengths between two adjacent nodes of the optical fiber can be exactly known, then another method (Method III) can be used. In this case, actually two methods have been proposed for the computation of projected Sagnac area. One is the computation of circular sectors area with the fixed length of fiber route segment [10], as shown in the subplot (a) of Fig. 3. The other is the computation of triangular area with exact positions, while the fixed length of fiber route segment is used as a constraint condition for the computation of estimation uncertainty [7], as shown in the subplot (b) of Fig. 3.

The fourth method (Method IV) is used in the case of benchmark real-like optical paths [8], where position of more points are exactly known than that of aforementioned case. Triangle area in the equatorial plane with exact positions of fiber route segments are computed, and the length of fiber is used as a constraint condition to find the maximum semicircle for the computation of estimator uncertainty [12].

Method II, Method III and Method IV are compared by Czubla, Krehlik [10], it is shown that highest precision and accuracy can be obtained with Method IV, and accuracy of Method II and Method III is at the same level, while Method II has better precision than Method III. Slapak and Vojtech [8] compare all these four methods with the published information of several fiber links, and find that more detailed information of nodes is known, more precise Sagnac correction will be. In the case that when we have enough knowledge of the optical fibre path, the Method IV can achieve at least two times lower uncertainty.

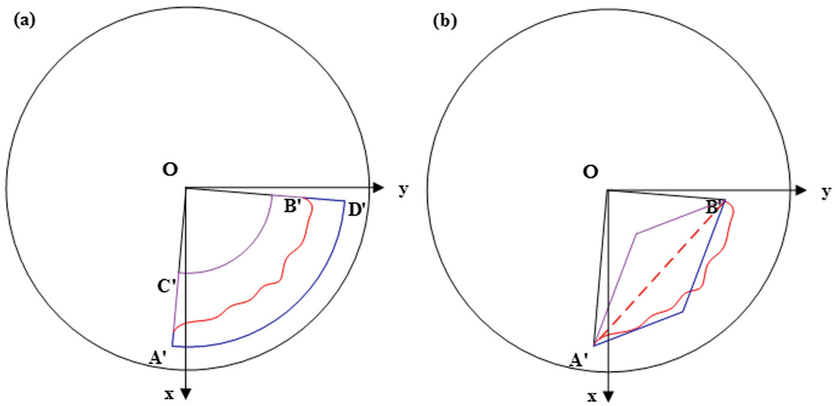

Fig. 3. Sagnac correction computation in the case of exact position and fixed length of fiber route segment. Circular sectors area with the fixed length of fiber route segment is shown in the left subplot (a), while triangle area with the fixed length of fiber route segment is shown in subplot (b).

4 Results and Discussion

Due to the confidential data of the fiber routes, we have very limited knowledge that only the coordinates of some nodes are known. In order to analyze the Sagnac correction of china fiber link, we have to find real-like transmission lines for analysis. It's known that fiber cables on land are typically installed in ducts for utility distribution often running across metropolitan areas and along highways. Routes planning function of baidu map API (http://lbsyun.baidu.com/) is used to generate a real-like optical-fiber link. Based on the generated optical-fiber link routes, Sagnac correction and also their uncertainty from imperfect information (especially the fiber and station coordinates) are analyzed.

4.1 Software Validation

In order to analyze the Sagnac correction of China optical fiber used for precise time transfer, a software package has been developed. The automated calculation algorithm of Sagnac correction (Method IV) published by Šlapák et al. [12] is used, and some improvements have been made. Concept of directed triangle area is used, and procedure of loop detection is removed. As for the confidence interval of Sagnac correction estimated value, length of fiber is constrained to find the maximum semicircle area enclosed by a given ratio of chord length and arc length.

Published results of several benchmarking paths [8] are used as the reference to evaluate the software package. Six benchmarking paths are used for the validation, and the information about them can be found in the website (https://photonics.cesnet.cz/en/sagnac-benchmark). Comparison results are shown in Table 1. It can be seen that difference of Sagnac correction value is at tens of fs level. The large difference of uncertainty for some path (Bethel–Pakhachi) may result from the combination effect of long length and small points.

Table 1. Comparison of Sagnac corrections on benchmarking paths between the published results and our software package results.

Endpoints	Length [km]	Points	Published Sagnac correction [ps]	Our Sagnac correction [ps]
Berlin–Poznan	260	51	756.561 ± 0.415	756.586 ± 0.406
Berlin–Ustı nad Labem	265	89	145.421 ± 0.317	145.428 ± 0.104
Bethel–Pakhachi	1943	31	−4063.525 ± 63.392	−4063.589 ± 61.752
Dusseldorf	30	46	−11.910 ± 0.010	−11.910 ± 0.003
Karlskoga–Gustavberg	260	99	582.180 ± 0.243	582.197 ± 0.235
Sydney–Adelaide	1397	137	−5006.103 ± 10.037	−5006.545 ± 9.933

4.2 Sagnac Correction Analysis of Optical-Fiber Time Transfer

Eight cities, (Beijing, Wuhan, Guangzhou, Sanya, Nanjing, Shanghai, Xian, Chengdu), are used as the input parameter for routes planning of baidu map API, and seven optical fiber paths are output as the simulation results (Fig. 4).

Fig. 4. Simulation results of China optical fiber paths using baidu map API.

Sagnac corrections are computed based on the simulated seven paths, and the results are shown in Table 2. It can be seen that uncertainty of Sagnac corrections is at fs level. In the following, these results would be considered as the reference. The influences of optical-fiber nodes position error on Sagnac correction computation are analyzed from two aspects, three-dimensional position error and degree of sparsity of fiber nodes.

Table 2. The results overview of Sagnac corrections on seven simulated optical fiber paths

Endpoints	Length [km]	Points	Sagnac correction [ps]
Beijing–Wuhan	1184.5	4489	−673.807 ± 0.002
Wuhan–Guangzhou	996.6	4199	−433.567 ± 0.000
Guangzhou–Sanya	878.0	3510	−1917.522 ± 0.004
Wuhan–Nanjing	543.7	2171	1845.816 ± 0.000
Nanjing–Shanghai	296.8	1571	1108.437 ± 0.000
Wuhan–Xian	747.2	3939	−2203.347 ± 0.000
Xian–Chengdu	714.7	4515	−2023.673 ± 0.000

In order to analyze the influences of optical-fiber nodes position error on Sagnac correction, random position error of 10 m, 20 m, 50 m, 100 m, 200 m, 500 m, and 1 km, respectively on north, east, and up direction is added into the known position of fiber nodes. Then the Sagnac correction is computed, and compared with reference value. We repeat 5000 times of above processing, and obtain the statistics. Influences of position error on Sagnac correction for all these seven paths are almost the same. Analysis results of Beijing–Wuhan path are shown in Table 3. It can be seen that in order to ensure 1 ps precision of time transfer, position error of nodes should be smaller than 500 m. This result is consistent with the analysis results of Gersl et al. [7].

Table 3. Influences of optical-fiber nodes position error on Sagnac correction for Beijing–Wuhan path

Endpoints	Length [km]	Points	Sagnac correction [ps]
Beijing–Wuhan	1184.5	4489	−673.807 ± 0.002
Position error [m]	Mean [ps]	STD[ps]	RMS [ps]
5	0.000	0.017	0.017
10	0.000	0.035	0.035
20	0.000	0.069	0.069
50	0.000	0.173	0.173
100	0.000	0.345	0.346
200	0.000	0.691	0.692
500	−0.001	1.726	1.729
1000	−0.001	3.453	3.458

In order to analyze the influences of fiber nodes sparsity on Sagnac correction, several degrees, such as 5%, 10%, 20%, 30%, until 70%, are used. We also repeat 5000 times. The influences of nodes sparsity are little, because even 70% nodes are deleted, one node per 1 km is still guaranteed for all these seven simulation paths.

5 Conclusion

In order to analyze the Sagnac correction of china optical fiber links routes, planning function of baidu map API (http://lbsyun.baidu.com/) is used to generate a real-like optical-fiber link. Based on the generated optical-fiber link routes, magnitude of Sagnac correction and their uncertainty from imperfect information (especially the fiber and station coordinates) are analyzed. It is found that in order to ensure one picosecond precision of time transfer using optical fiber, position error of nodes should be smaller than 500 m.

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