The Service Improvement of BDS Positioning Based on Advanced Equivalent Satellite Clock Calculation

Yangfei Hou, Junping Chen, Bin Wang and Jiexian Wang

Abstract Positioning precision of navigation satellite system can be measured by two indicators: the dilution of precision (DOP) and the user equivalent range error (UERE). As the DOP values are only related to the spatial distribution of navigation satellites, the reduction of UERE is the main approach to improve the positioning precision. Equivalent satellite clock (ESC) has been used by Beidou satellite system (BDS) to reduce the UERE and to improve the user's positioning accuracy. In this contribution, both the pseudo-range and carrier-phase measurements of BDS are used to compute the ESC respectively, and the corresponding navigation positioning performance are also compared. It is shown that the UERE improvement based on phase observables is 50.1%, while 32.1% based on pseudo-range observables. Kinematic positioning experiments of 4 MEGX stations are performed respectively under the standard PNT service and wide area differential service (WADS). It is shown that horizontal, vertical and three-dimensional positioning results of WADS are better than that of the standard PNT service.

Keywords Equivalent satellite clock · User equivalent range error
Navigation performance improvement

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1 Introduction

Users' positioning precision is mainly related to the precision of navigation signals and the spatial distribution of navigation satellites, which can be described as $Pos_{error} = UERE \times DOP$. The DOP is the indicator of navigation satellites distribution, and the UERE is related to the users' range errors and users' equipment errors [1–3]. Beidou satellite system (BDS) is designed to provide two kinds of navigation service, including the standard navigation service (to all users) and the wide area differential service (only to authorized users). For authorized users, navigation positioning precision can be improved by using the equivalent satellite clock (ESC) parameter to reduce the UERE. Currently, the model used to calculate the ESC values is based on pseudo-range observables [4, 5]. In this contribution, the advanced algorithm based on phase observables is proposed and experiments to calculate more precise ESC values are performed in order to improve the UERE.

2 Equivalent Satellite Clock Computation Model

2.1 Equivalent Satellite Clock

As the radial orbit error of BDS satellites present small projection differences in China area, it is difficult to fully separate the radial orbit errors with satellite clock errors. Therefore, it is better to merge them, called the equivalent satellite clock. Ionosphere-free observables for one station and one satellite can be expressed as:

$$P_i = \rho(x^{sat}) + c \cdot (dt_{rec} - dt^{sat}) + (b_{ifb} - b^{tgd}) + m \cdot ZTD + \varepsilon \tag{1}$$

P_i is the range observables, ρ is the geometric range between the satellite and receiver, dt_{rec}, dt^{sat} are respectively the receiver and satellite clock offset, b_{ifb}, b^{tgd} are respectively the receiver and satellite code hardware delays, m and ZTD are the troposphere mapping function and corresponding zenith troposphere delays, ε represents many kinds of errors including multipath effects. The equivalent satellite clock can be expressed as:

$$ESC = \delta_{clk} + \delta_{orb} \tag{2}$$

δ_{clk} is the satellite clock error, while δ_{orb} is the projection error of satellite orbit. By taking all the other corrections out of the observation equation, the previous equation can be rewritten as:

$$p_i^j = \delta_{rec,i} - \delta_{ESC}^j + \varepsilon_i^j \tag{3}$$

p_i^j is the observation residuals between satellite j and station i, $\delta_{rec,j}$ is the receiver clock error, δ_{ESC}^j is the equivalent satellite clock. Through aforementioned equations, we can construct the following equation, which can be solved using the least squares adjustment:

$$\begin{bmatrix} p_1^1 \\ p_1^2 \\ \vdots \\ p_n^1 \\ p_n^2 \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & -1 & 0 & \cdots \\ 1 & 0 & \cdots & 0 & -1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & 0 & 1 & -1 & 0 & \cdots \\ \cdots & 0 & 1 & 0 & -1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \delta_{rec,1} \\ \delta_{rec,2} \\ \vdots \\ \delta_{ESC}^1 \\ \delta_{ESC}^2 \\ \vdots \end{bmatrix} \tag{4}$$

Due to lack of the time reference, the constructed normal equation is rank defect, which means the normal equation is singular. Therefore, we should firstly define one station clock as the reference, then iteratively estimate all the other clock errors using the least square adjustment, finally we can obtain a set of equivalent satellite clock.

2.2 *Variation of the ESC*

In Eq. (4), the ESC values are estimated only using pseudo-range observables, which are heavily affected by measurement noise. To improve the estimation precision of the ESC, a two-step smoothing method can be applied to suppress the pseudo-range measurement noise. The first step is to calculate the variation of the epoch-wise ESC values using phase-differenced observables, and the second step is to smooth the pseudo-range observables using the variation of epoch-wise ESC values [6]. The ionosphere-free phase measurement equation is as follows:

$$L = \rho(x^{sat}) + c \cdot (dt_{rec} - dt^{sat}) + (b_{ifb} - b^{tgd}) + N + m \cdot ZTD + \varepsilon \tag{5}$$

ε is the error of phase observable, b_{ifb}, b^{tgd} are receiver and satellite hardware delays respectively; other parameters have the same meaning as the Eq. (1) except for the phase ambiguity parameter N. The phase ambiguity can't be fixed in real time, which hinder the ESC calculation when using phase observables, however the epoch-wise variation of the ESC can be obtained simply:

$$\Delta L(t_{i-1}, t_i) = \Delta\rho(\mathrm{x}_{i-1}^{sat}, \mathrm{x}_i^{sat}) + \mathrm{c} \cdot (\Delta dt_{rec} - \Delta dt^{sat}) + \Delta m \cdot ZTD + \Delta\varepsilon \tag{6}$$

From Eq. (6), It can be seen that the phase ambiguity problem donot exist any more, and hardware delays are canceled, and the troposphere errors are only related to the mapping function differences. Until now, the high-precision phase observables can be processed using similar least squaresadjustment as pseudo-range observables, and the epoch-wise ESC values can be acquired.

2.3 Smoothing

Estimation of the epoch-wise ESC variation in Sect. 2.2 can be used to smooth the estimated ESC absolute values in Sect. 2.1. With the absolute ESC values calculated by pseudo-range observables, we can form a new set of measurement equations as:

$$\hat{X}_i - X_{l,i} = V_{l,i} \tag{7}$$

On the other hand, the estimation of the epoch-wise ESC variation can be represented as:

$$(\hat{X}_i - \hat{X}_{i-1}) - (X_{\varphi,i} - X_{\varphi,i-1}) = V_{\Delta\varphi,i} \tag{8}$$

Through Eqs. (7) and (8), we can get the normal equation as:

$$E^T P_l E \hat{X} = E^T P_l E X_l \tag{9}$$

$$C^T \cdot P_\varphi \cdot C \cdot \hat{X} = C^T \cdot P_\varphi \cdot \Delta \mathrm{x}_\varphi \tag{10}$$

C is the corresponding coefficients matrix:

$$C = \begin{pmatrix} -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{pmatrix}_{n \times n} \tag{11}$$

P_l and P_φ are weight matrix for pseudo-range and phase observables respectively. Through aforementioned smoothing procedure, we can acquire a set of better-quality ESC values by using both pseudo-range and phase observables.

2.4 Improved Equivalent Satellite Clock Estimation Algorithm

Although the two-step algorithm is quite effective in ESC estimation, the absolute ESC values derived in above sections can still be improved. When there are no cycle slips in the carrier phase, the integer ambiguity keeps unique. With this property in mind, we can directly calculate the ESC using phase observables, and it is much more convenient than the above two-step procedure. The improved ESC estimation algorithm based on phase observables can be described using the following steps. It is assumed that the reference epoch of ESC is t_i.

1. Using long-time phase observables, we can fix the phase ambiguity N_i^j in the period of $[t_i - \Delta_t, t_i)$;
2. At epoch t_i, firstly, we perform carrier phase cycle-slip detection. Then we perform polynominal fitting of the phase measurements $\tilde{\varphi}_i$ in the period of $[t_i - \Delta_t, t_i)$ [7]:

$$\begin{aligned} \tilde{\varphi}_i &= a_0 + a_1(t_i - t_0) + a_2(t_i - t_0)^2 + \cdots + a_n(t_i - t_0)^n \\ &\quad (i = 1, 2, \ldots, m;\ \ m > n + 1) \end{aligned} \tag{12}$$

Through fitting, coefficients $a_0, a_1, \ldots, a_n$, and corresponding root mean squares error $\sigma = \sqrt{\frac{[V_i V_i]}{m-(n+1)}}$ can be obtained, where V_i are the fitting residuals. With the coefficients estimation, we can extrapolate the phase measurements in the following epoch, and make a comparison between the extrapolated value and real measurement. If the difference is less than 3σ, the total period can be regard as continuous without any cycle slips. In experience, three order polynominals should be used.

3. If there is no cycle slip in the period $[t_i - \Delta_t, t_i)$, the carrier phase ambiguity can be estimated, and then the measurement equations can be simplified as

$$\begin{bmatrix} l_1^1 \\ l_1^2 \\ \vdots \\ l_n^1 \\ l_n^2 \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & -1 & 0 & \cdots \\ 1 & 0 & \cdots & 0 & -1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & 0 & 1 & -1 & 0 & \cdots \\ \cdots & 0 & 1 & 0 & -1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \delta_{rec,1} \\ \delta_{rec,2} \\ \vdots \\ \delta_{ESC}^1 \\ \delta_{ESC}^2 \\ \vdots \end{bmatrix} \tag{13}$$

   Using Eq. (13), all the ESC can be estimated directly.
4. Excluding the first measurement in aforementioned period, we add the next carrier phase measurement into the measurement value series, and reprocess

using the above steps. When the difference between extrapolated value and real measurement is larger than 3σ, the ESC estimation procedure should be stopped till the next carrier phase ambiguity solved.

3 User Equivalent Range Error

The UERE are the errors related to the satellite orbits errors, satellite clock errors, atmosphere model errors, receiver clock errors, and so on. It includes the navigation signal range errors (URE) and the users' equipment errors (UEE) [8].

The UERE can be expressed as:

$$UERE^j = \bar{P}^j - \rho^j - \delta^j_{sat} - \delta^j_{\text{model}} \tag{14}$$

$\bar{P}$ is the range observation, ρ is the geometric range between receiver and satellite, δ_{model} includes troposphere, ionosphere, relativity, earth displacement and receiver clock correction. By subtracting the ESC, we can get a new variable: the user difference range error (UDRE) [9]:

$$UDRE^j = \bar{P}^j - \rho^j - \delta^j_{sat} - ESC - \delta^j_{\text{model}} \tag{15}$$

4 Experiments

The experiment period is from 23/10/2017 to 25/10/2017. In the experiment period, ESCs are calculated using the two-step algorithm and the improved estimation algorithm. Three types of the UERE/UDRE corrections are compared with each other. Mode 1 is pseudo-range corrections based on two-step algorithm, mode 2 is pseudo-range corrections based on improved ESC estimation algorithm, and mode 3 is phase corrections based on improved ESC estimation algorithm.

4.1 User Ranging Error Statistics

4.1.1 Statistical Results of the UERE, UDRE

In Fig. 1, we present the RMS of UERE and UDRE of C14 (MEO) satellite for 3 kinds of ESC correction on 24/10.

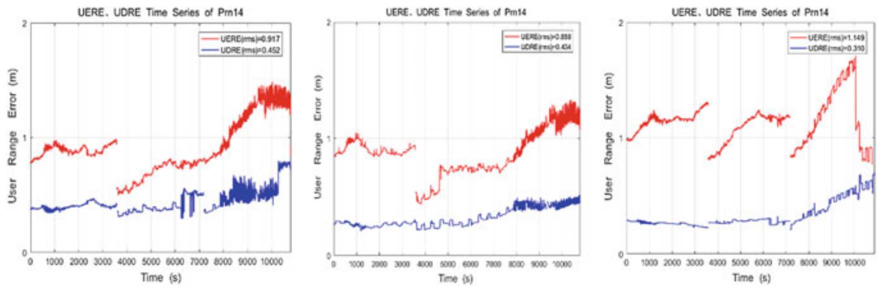

Fig. 1 UERE and UDRE of three kinds of ESC correction for C14 satellite

In the above figures, the red dots represent the UERE of the BDS standard service users, the blue dots represent the UDRE of the BDS authorized service users. From those figures, we can find that the values of UDRE are less than corresponding UERE corrections, which imply the effective use of ESC corrections. Meanwhile, the UDRE of mode 3 is much smoother than that of mode 1 and mode 2, which indicates that the ESC estimation calculated by improved ESC estimation algorithm is much more stable than the two-step algorithm; the differences between UERE and UDRE are similar in mode 1 and mode 2, which indicates that the ESC estimated by improved algorithm may be not compatible with pseudo-range observables.

Table 1 RMS statistics of all UDRE satellites in 3 days (unit: m)

PRN	October 23 UDRE (RMS)			October 24 UDRE (RMS)			October 25 UDRE (RMS)		
	1	2	3	1	2	3	1	2	3
1	0.334	0.307	0.215	0.419	0.407	0.172	0.374	0.393	0.154
2	0.338	0.354	0.236	0.442	0.472	0.256	0.406	0.380	0.158
3	0.375	0.387	0.232	0.420	0.472	0.180	0.343	0.361	0.212
4	0.473	0.455	0.215	0.785	0.472	0.287	0.346	0.359	0.264
5	0.431	0.382	0.288	0.436	0.482	0.294	0.529	0.520	0.294
6	0.447	0.391	0.319	0.479	0.511	0.256	0.922	0.857	0.368
7	0.631	0.608	0.381	0.455	0.505	0.293	0.587	0.653	0.264
8	0.447	0.440	0.331	0.501	0.436	0.371	0.521	0.473	0.235
9	0.415	0.383	0.315	0.559	0.614	0.356	0.487	0.414	0.158
10	0.373	0.432	0.240	0.581	0.577	0.399	0.362	0.350	0.141
11	0.322	0.281	0.208	0.712	0.727	0.575	0.649	0.697	0.367
12	0.461	0.478	0.258	0.495	0.469	0.301	0.584	0.575	0.265
13	0.525	0.519	0.295	0.528	0.554	0.293	0.529	0.574	0.264
14	0.497	0.494	0.370	0.660	0.658	0.378	0.514	0.546	0.328

4.1.2 RMS of the UDRE

The RMS of UDRE is shown in Table 1 and Fig. 2 for three days and three modes.

From Table 1 and Fig. 2, we can find that mode 3 is superior to mode 1 and 2 with much smaller RMS of UDRE. The RMS of UDRE for mode 1 and mode 2 are similar, which indicates that phase observables are superior to pseudo-range observables in UDRE calculation.

4.1.3 Comparison

To quantify the precision improved in mode 3, we present the proportion reduction for mode 1 and mode 3 in Fig. 3:

Form Fig. 3, we can find that the average of proportion reduction is 32.1% for mode 1, and 50.1% for mode 3, which verifies the high-precision property of the improved ESC estimation.

4.2 Positioning Results Statistics

In order to study the effect of ESC on the positioning results, four MEGX stations DUND, DAE2, GMSD and JFNG were selected. And the results of dual-frequency positioning under standard service and enhanced service are compared.

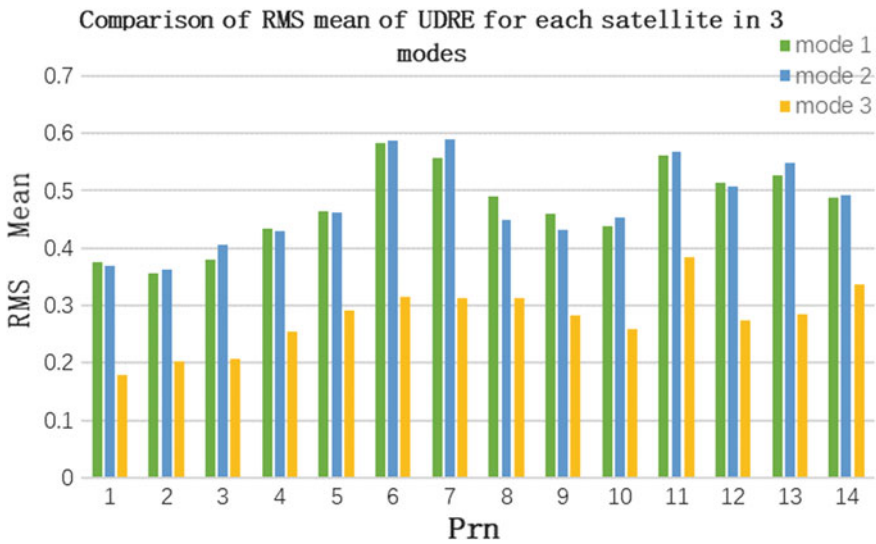

Fig. 2 RMS of UDRE for three kinds of mode for all BDS satellites

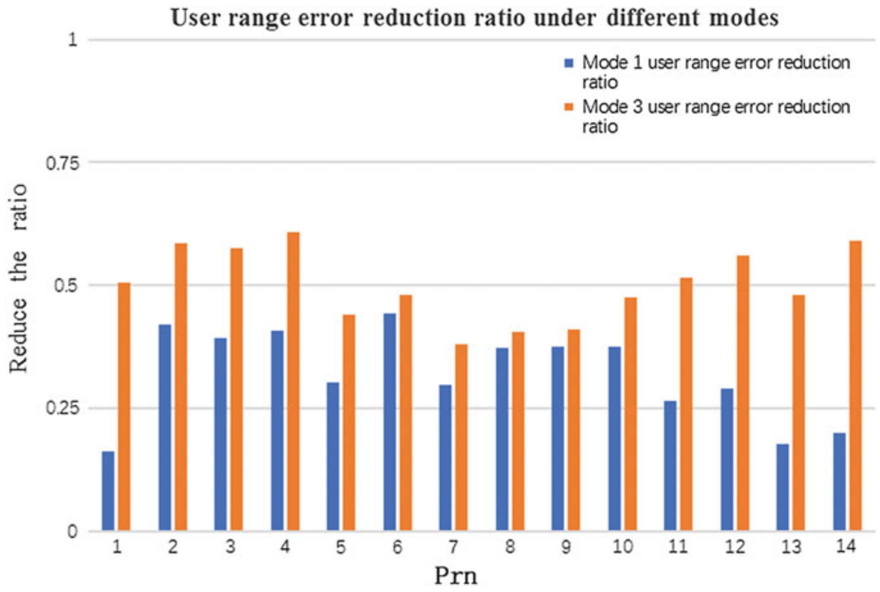

Fig. 3 Proportion reduction of satellite user range error for Mode 1, Mode 3

4.2.1 Positioning Results of Standard Service and Enhanced Service

Taking the positioning results of GMSD station on October 24 as an example, statistics are made on the kinematic positioning results of the users under the standard and enhanced service, respectively. The results are as shown in Fig. 4.

The left two pictures in Fig. 4 are the positioning results of pseudo-range and carrier phase measurements under the standard service of GMSD station respectively. The two pictures on the right are the positioning results of pseudo-range and carrier phase measurements under enhanced service of GMSD station. In Fig. 4, RMS of north, east and up positioning error shows that after the ESC correction, enhanced positioning results are significantly better than that of standard positioning results.

4.2.2 Positioning to Enhance the Proportion of Statistical Results

Horizontal, vertical and three-dimension positioning results of four MEGX stations are obtained. And the proportion improvement of enhanced navigation relative to the standard navigation is shown in Table 2.

As can be seen from Table 2, the horizontal, vertical and three-dimension positioning results of the four MEGX stations have been improved after the ESC correction adding to the measurements. This proves that the ESC can be used to improve the positioning accuracy of users required better navigation services. It is

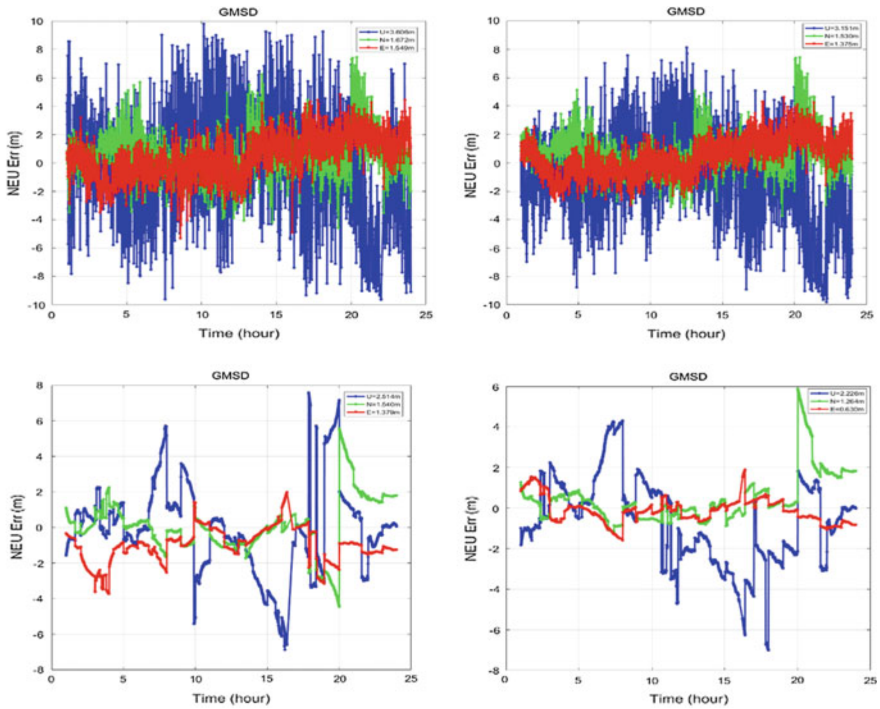

Fig. 4 Positioning results of GMSD under standard service and enhanced service respectively

Table 2 Proportion improvement of enhanced navigation relative to the standard navigation

Station	Pseudo-range enhancement navigation boost ratio (%)			Carrier phase enhancement navigation boost ratio		
	Horizontal	Vertical	3D	Horizontal	Vertical	3D
DUND	10.54	3.32	5.83	12.79	7.86	10.62
DAE2	13.43	7.98	6.13	4.58	23.0	18.46
GMSD	9.75	12.62	11.79	31.68	11.46	19.01
JFNG	8.45	8.80	8.84	23.53	22.72	21.09

noted that the average improvement percentage of the positioning result is smaller than the proportion reduction of the user's ranging error. This is because the BDS reference station is selected to calculate the user ranging error, and the quality of the observations of the MEGX stations used in the positioning is relatively poor, and the DOP of satellites for MEGX station is not too good.

5 Conclusion

The broadcast equivalent satellite clock corrections are very useful to improve navigational positioning precision in the BDS. In this study, we present the old-fashion two-step pseudo-range-based algorithm to calculate the ESC corrections, and propose a new advanced phase-based algorithm to improve the calculation of ESC corrections. Through 3 days experiment Beidou observables, the results show that the RMS of the UDRE corrections gets significant reduction, with reduction ratio reaching to 50.1%; The results of dynamic positioning of four MEGX observatories under basic service and enhanced service are calculated, it is shown that horizontal, vertical and three-dimensional positioning results of WADS are better than that of the standard PNT service.

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