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Comparison of BDS Station Clock Short-term Prediction Models and their Applications in Precise Orbit Determination† ⋆

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Abstract BDS (BeiDou Navigation Satellite System) ground tracking stations are equipped with high accuracy atomic clocks, and they are synchronized with the BDS time scale (BDT) via the Precise Orbit Determination (POD) processing. During the periods of satellite maneuver and post-maneuver, station clocks are kept fixed as known values in the POD processing. To improve the real-time POD capability, station clocks need to be predicted. In this paper, the performance of three clock prediction models is evaluated, including quadratic polynomial model (QP), periodical term model (PM), and grey model (GM). The precision of clock fitting and prediction, as well as the performance of the prediction models in POD are compared. Data of six stations are used for test, and the results show that: the mean fitting accuracy of quadratic polynomial model, periodical term model, and grey model is 0.14 ns, 0.05 ns, 0.27 ns, respectively; the 1 h and 2 h prediction precision of the three models is 1.17 ns, 0.88 ns, 1.28 ns, and 2.72 ns, 2.09 ns, 2.53 ns, respectively. Applying the 1 h and

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2 h predicted station clocks in the POD, the 3D orbit accuracy reaches the best using the periodical term model, while the radial accuracy of satellite orbit is rather close for the three models with the difference within 3 cm.

Key words clock error prediction—spectrum analysis—periodical term model—orbit determination

1. INTRODUCTION

For time-keeping and synchronization purposes, the master control stations, command and control stations, and monitoring stations of the GNSS (Global Navigation Satellite System) Operational Control Segment (OCS) are equipped with high precision atomic clocks, including hydrogen, rubidium, and cesium types[1]. These clocks have high prediction precisions, and could contribute to the orbit determination process during the periods of satellite post-maneuver.

BDS GEO (Geosynchronous Orbit) and IGSO (Inclined Geosynchronous Satellite Orbit) satellites experience frequent maneuvers to maintain their nominal positions,, and the satellite is set as unhealthy status during this period, which informs users that the service of this satellite is unavailable. During the maneuver, the orbits are determined based on the reverse-point-positioning approach, and the real-time orbits with precision of tens of meters are provided for the Radio Determination Satellite Service (RDSS). During the periods of post-maneuver, where no pulse forces are pushing satellites, and satellites gradually move following a nominal dynamic law. More precise satellite orbits are required by the OCS to ensure meter-level Radio Navigation Satellite Services (RNSS) during this period. In the orbit determination process for satellite experiencing maneuvers, the post-processed station clocks are normally kept as known to reduce the number of parameters and to improve the orbit precision[2−4]. Due to the latency of the time synchronization processing, the station clocks of the latest 1-2 hours are not available for orbit determination process. Consequently, the unavailable period of the satellite is prolonged. In this paper, we propose an effective approach to solve this problem.

Instead of relying on post-processed station clocks, we introduce clock predictions in orbit determination. Three typical clock fitting and prediction models, include quadratic polynomial (QP) model, gray model (GM), and the periodical model (PM)[5−9], are used and tested for this new orbit determination approach. Among the three models, the QP model models the clock physical characteristics, the periodical model estimates the clock periodic characteristics, and GM is not sensitive to the amount of data. In Section 2, we introduce the orbit determination model and clock prediction models, Section 3 evaluates the clock fitting and prediction performance of the 3 different models, Section 4 analyzes and compares orbit accuracies based on the three clock prediction models.

2. BDS ORBIT DETERMINATION AND CLOCK PREDICTION MODELS

2.1 Orbit Determination and Clock Prediction Models

Under normal circumstances, multi-satellite POD (MPOD) is implemented in the BDS OCS to provide precise orbits[10]. During the periods of satellite post-maneuver, MPOD excludes the maneuver satellite, and calculates the parameters such as the station clock, the satellite orbit, and clock of the healthy satellites. Subsequently, the calculated station clocks of MPOD are taken as known input values into the single-satellite POD (SPOD).

The SPOD adopts the traditional dynamical orbit determination model, where a variety of forces such as the non-spherical gravitation perturbation of the earth, the solar pressure, the Earth's albedo radiation pressure perturbation, etc. are modeled. In the SPOD, the pseudo-range observations are used, and the observation equations between station r and satellite s are abbreviated as follows[4,5]:

$$\rho_{\mathrm{r}}^{\mathrm{s}} = \sqrt{(x^{\mathrm{s}} - x_{\mathrm{r}})^2 + (y^{\mathrm{s}} - y_{\mathrm{r}})^2 + (z^{\mathrm{s}} - z_{\mathrm{r}})^2} + c\mathrm{d}t_{\mathrm{r}} - c\mathrm{d}t^{\mathrm{s}} + \delta + \varepsilon\,, \tag{1}$$

where indices r and s refer to the station and satellite, respectively; $\rho_{\mathrm{r}}^{\mathrm{s}}$ denotes the pseudo-range observation, $(x^{\mathrm{s}}, y^{\mathrm{s}}, z^{\mathrm{s}})$ and $(x_{\mathrm{r}}, y_{\mathrm{r}}, z_{\mathrm{r}})$ denote the satellite position and the station location, respectively; $\mathrm{d}t_{\mathrm{r}}$ is the station clock, calculated by MPOD; $\mathrm{d}t^{\mathrm{s}}$ is the satellite clock, obtained by the Two-Way Satellite Time Frequency Transfer (TWSTFT) techniques; δ includes the tropospheric correction, ionospheric correction, and relativistic correction, which can be corrected by the models; and ε denotes the multipath effects and other kinds of noise.

Figure 1 is a schematic diagram showing the time span of the observation data and station clocks of the SPOD. In the figure, $t0$ denotes the starting epoch of the observation data for one SPOD batch, $t1$ denotes the last epoch of the MPOD station clock, t is the current time when the SPOD is evoked. The data time span used by the conventional SPOD depends on the end time $t1$, where both station clock and observation data are available. Following the conventional approach, the latest observation data can't be included even the observation data is real-timely available at the current time t. In this case, the satellite's orbits from $t1$ to t have to be predicted according to the broadcast ephemeris of last SPOD batch, thus orbit precision suffers, and system service performance is affected.

In Figure 1, the station clock can be predicted from the epoch $t1$ to current time t using proper prediction models, and the predicted clocks can be used in the SPOD. The SPOD with station clock prediction is defined as a new SPOD in the following text. Common models for clock prediction include QP and GM. For BDS, in the station clock possesses periodic characteristic due to the strong correlation between the satellite orbit and clock, and the periodic term model (PM) can be used[11−13].

The QP model reflects the basic physical characteristics of its clock rate and clock acceleration, the formula is as follows:

$$T_i = a_0 + a_1(t_i - t_0) + a_2(t_i - t_0)^2\,, \tag{2}$$

where t_0 is the reference time of the station clock, t_i denotes the observation epoch, T_i is the clock prediction; a_0, a_1, and a_2 denote the initial clock error, clock rate, and clock acceleration, respectively.

The PM model is as follows:

$$T_i = a_0 + a_1\Delta t + a_2\Delta t^2 + \sum_{p=1}^{m}[A_p \sin(\omega_p\Delta t) + B_p \sin(\omega_p\Delta t)]\,, \tag{3}$$

where $\Delta t = t_i - t_0$, $p = 1, 2 \cdots m$, and m is the number of frequencies determined by the power spectrum, A_p and B_p are the amplitudes, and ω_p denotes the frequency. The solution procedure is: (1) Use the quadratic polynomial to remove the trend terms to obtain the residuals time series, (2) determine the frequency terms ω_p using Fourier transform based on the residuals, and (3) The coefficients in the formula (3) are solved based on the least square method.

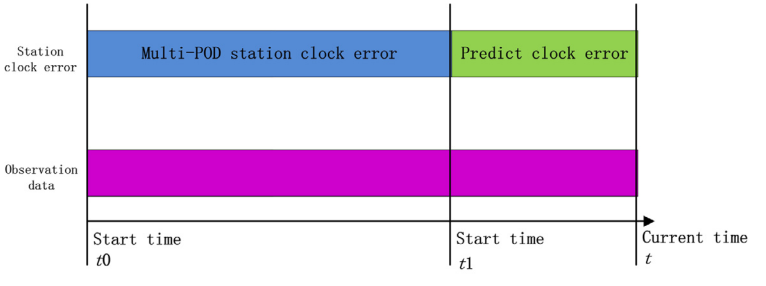

Fig. 1 Time span of the MPOD station clock observation data

For the GM model, the original clock can be expressed as $\boldsymbol{X}^{(0)} = \left[x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right]$, with n being the number of clock data, and a new clock time series $\boldsymbol{X}^{(1)} = \left[x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\right]$ can be generated with $x^{(1)}(k) = \sum_{i=1}^{k} x^{(1)}(i)$, $(k = 1, \cdots, n)$.

The first-order differential equation of $\boldsymbol{X}^{(1)}$ is as follows:

$$\frac{\mathrm{d}\boldsymbol{X}^{(1)}}{\mathrm{d}t} + a\boldsymbol{X}^{(1)} = b\,. \tag{4}$$

Equation (4) is the self-differential equation of GM, where a is the development gray number, and b is the control gray number. The regression analysis is used to find the best solution:

$$\hat{x}^{(1)}(k) = \left[\hat{x}^{(1)}(1) - \frac{b}{a}\right]\mathrm{e}^{-a(k-1)} + \frac{b}{a}\,, \tag{5}$$

where $\hat{x}^{(1)}$ denotes the estimated value of regression analysis, e is the natural logarithm base, and the least square analysis can be used to derive the parameters a and b.

All the above-mentioned models can be used for station clock prediction and the prediction can be used as the inputs in the SPOD.

2.2 Precision assessment of SPOD and Station Clock Prediction

In the station clock fitting and prediction, the Root Mean Square (RMS) is normally used for precision evaluation. The formula is:

$$\mathrm{RMS}_q = \sqrt{\frac{\sum_{j=1}^{l}(Y_j - y_j)^2}{l}}\,, \tag{6}$$

where RMS_q denotes the RMS statistics of clock fitting or predicted error, Y_j denotes the clock fitting or predicted value, y_j is the original value of the MPOD station clock at the corresponding epoch time j, and l is the number of epochs of each batch q.

The orbit precision of BDS MPOD is about an amount higher than that of SPOD, and it is used as a reference for orbit precision evaluation of SPOD. The orbital radial deviation has the greatest impact on users navigation, the orbit evaluation includes orbital radial precision and 3-dimensional (3D) position precision, calculated as follows:

$$\mathrm{RMS}_{R,q} = \sqrt{\frac{\sum_{j=1}^{l}(R_j - r_j)^2}{l}}\,, \tag{7}$$

$$\mathrm{RMS}_{P,q} = \sqrt{\frac{\sum_{j=1}^{l}(P_j - p_j)^2}{l}}\,, \tag{8}$$

where indices R and P refer to the radial and 3D position, respectively; $\mathrm{RMS}_{R,q}$ and $\mathrm{RMS}_{P,q}$ denote the RMS statistics of orbital radial and 3D precision, respectively; R_j and r_j are the radial orbital components of the SPOD and MPOD at epoch q, respectively; P_j and p_j denote the 3D satellite orbits of the SPOD and MPOD, respectively.

3. DATA ANALYSIS

3.1 Results of Clock Fitting and Prediction

The MPOD station clock with an interval of 1 min of six BDS monitoring stations on the 332 day of 2018 is used for the clock fitting and prediction analysis, and QP, PM, and GM models are compared. During this period, no satellite maneuver is determined. Precise orbit and station clock can be obtained based on MPOD, and they are used as reference in the comparisons.

The experiments are designed according to the data processing specifications of BDS OCS, where the data analysis was performed in 1 h batch and each batch contains 2 hours data. From 02:00 to 21:00 there are 20 batches. The RMSs of the clock fitting and the 1 h / 2 h predictions for each station are shown in Figures 2–4.

Figure 2 shows the RMS time series of the precision of each station based on the 20 batches, it clearly demonstrates the PM has the highest precision, and is the most stable, where the fitting precision of the six stations is better than 0.15 ns. The GM and the QP models show significant fluctuations for different batches, the fitting precision of the QP model is better than 1 ns for all six stations, and the fitting precision of the GM is worse than the other two models.

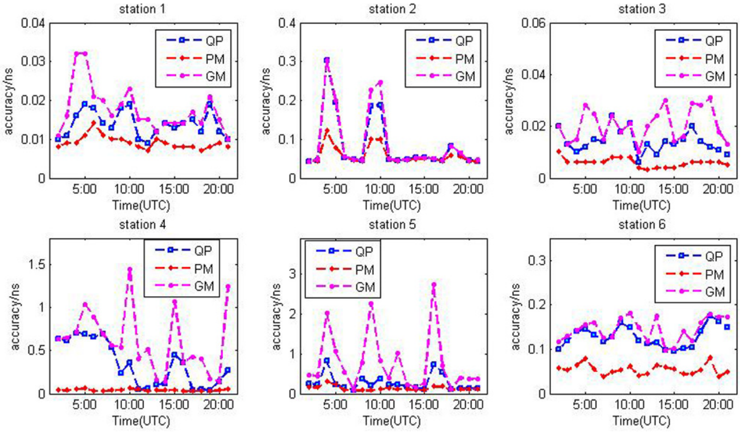

Fig. 2 The RMS of station clock error fitting

Different stations show fitting precision diversities, where for stations 1 and 3 the fitting precision is better than 0.05 ns for all the three models, while for stations 2 and 6 the fitting precision is within 0.3 ns. The fitting precision for stations 4 and 5, however, is significantly worse than that of the other stations, which indicates that their worse stability of the atomic clocks.

Figure 3 shows the 1 h clock prediction RMS for each station. All the three models have better prediction precision for stations 1, 2, 3, and 6, with most of the prediction errors within 0.5 ns. Most of the differences between the three models are within 0.1 ns. For Station 4 and Station 5, however, the clock prediction error is quite different in different batches, and the maximum error of the QP and GM model is of 8–12 ns.

The precision of the fitted RMS and the predicted RMS over 1 h and 2 h is averaged over 20 batches for each station and each model as shown in Table 1. Fig. 4 illustrates the 2 h clock prediction RMS for each station. The prediction precision of stations 1, 2, and 3 is within 0.8 ns most of the time, and the precision of the three models of the same batch is consistent in most of the time. The prediction precision using the QP model possesses jumps in individual batches. For stations 4, 5, and 6, the precision of the QP and GM models is poorer in some batches, and for stations 4 and 5, the prediction errors were even more than 15 ns in individual batches. According to Table 1, the PM model is the best among

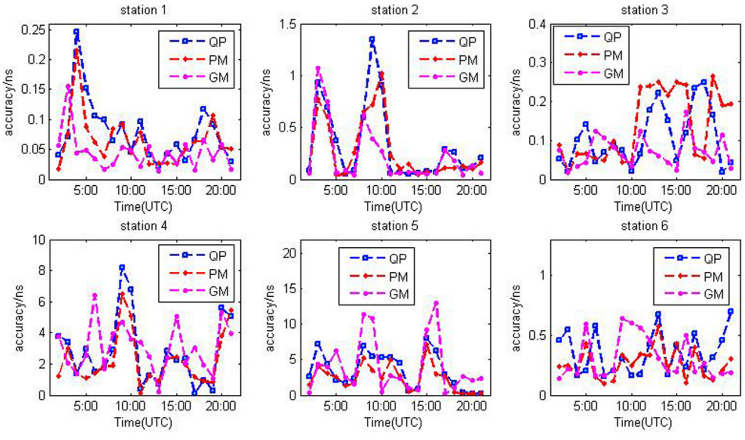

Fig. 3 The RMS of 1 h clock prediction for each station

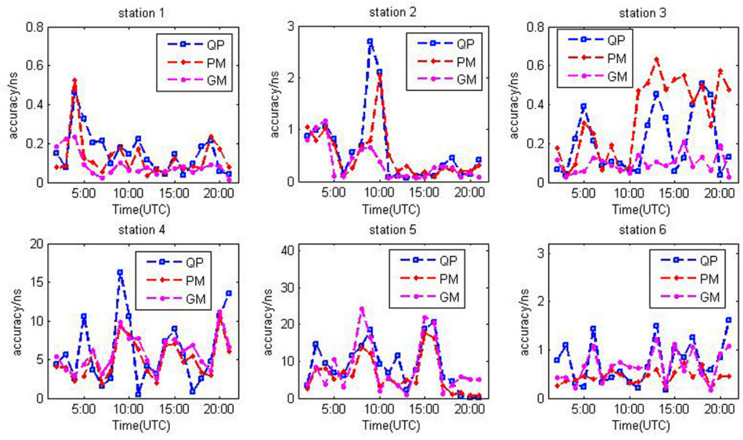

Fig. 4 The RMS of 2 h clock prediction for each station

the three in terms of fitting precision, which is better than 0.15 ns for all stations. The fitting RMS is better than 0.4 ns and 0.8 ns for the QP and GM model, respectively. The average fitting precision is 0.14 ns, 0.05 ns, and 0.27 ns for the QP, PM, and GM models, respectively, which shows PM has the highest fitting precision.

In terms of prediction precision, the prediction precision suffers from the predicted time, and the predicted precision of the same model varies from station to station, e.g., the clocks fitting and prediction of stations 4 and 5 are significantly worse than the other stations. The optimal model for stations 1, 2, and 3 is the GM, followed by the PM model. The optimal model for stations 4, 5, and 6 is the PM, which are obviously better the other two models. With the QP, PM, and GM models, the average precision of 1 h station clock prediction for the six stations is 1.17 ns, 0.88 ns, and 1.28 ns, respectively, and the 2 h prediction precision is 2.72 ns, 2.09 ns, and 2.53 ns, respectively. Based on the average precision of clock predictions, the PM is the best model because the MPOD station clocks absorb orbital errors showing period fluctuations[14], and they can be modeled in the PM model.

Table 1 The average precision of station clock error fitting and prediction (unit: ns)

Station ID	Fitting			Prediction for 1 h			Prediction for 2 h		
	QP	PM	GM	QP	PM	GM	QP	PM	GM
station 1	0.01	0.01	0.02	0.08	0.06	0.05	0.15	0.12	0.09
station 2	0.08	0.06	0.09	0.32	0.27	0.22	0.60	0.49	0.34
station 3	0.01	0.01	0.02	0.11	0.14	0.07	0.21	0.33	0.10
station 4	0.34	0.04	0.61	2.73	2.21	3.06	5.99	4.98	5.85
station 5	0.27	0.13	0.76	3.45	2.37	3.96	8.65	6.17	8.15
station 6	0.13	0.05	0.15	0.35	0.26	0.31	0.73	0.43	0.66
MEAN	0.14	0.05	0.27	1.17	0.88	1.28	2.72	2.09	2.53

3.2 Results of SPOD

The predicted station clocks based on the above models are applied to the SPOD, and the average orbit precision of the 20 batches under each model is calculated for 4 GEO (Geosynchronous Orbit) satellites (C01-C04) and 5 IGSO (Inclined Geosynchronous Satellite Orbit) satellites (C06-C10). To better evaluate the performance of each clock prediction model, the orbits obtained from the conventional SPOD are analyzed and compared with the precise MPOD orbits. Figure 5 shows the radial precision and 3D position precision for traditional SPOD and new SPOD with 1 h station clock prediction under 4 different scenarios.

The radial and 3D precision of the SPOD is between 0.54–0.81 m and 2.0–7.5 m respectively in the scenario with 1 h predicted station clock introduced into the new SPOD, while the radial and 3D precision of the conventional SPOD is between 0.66–0.85 m and 2.2–8.1 m, respectively. The new SPOD performs better in general than the traditional SPOD. The radial precision differences are mostly in the order of millimeters, and the 3D position differences are within 20 cm among the new SPOD using the three models.

Figure 6 displays the radial and 3D position precision for the traditional SPOD and new SPOD with 2 h station clock predictions. The radial orbit precision of the traditional POD is within 0.71–1.02 m, and the 3D precision is between 2.5–8.5 m. The improvement of the new SPOD is obvious, especially in the radial component for GEO satellites. The biggest improvement comes from the new SPOD based on the PM model, while the 3D differences are no more than 30 cm for the new SPOD under different prediction models.

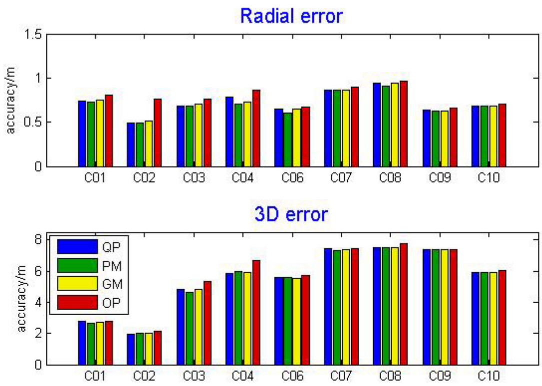

Fig. 5 The comparison of the orbit precision of traditional SPOD (OP) and new SPOD with 1 h station clock prediction based on different models

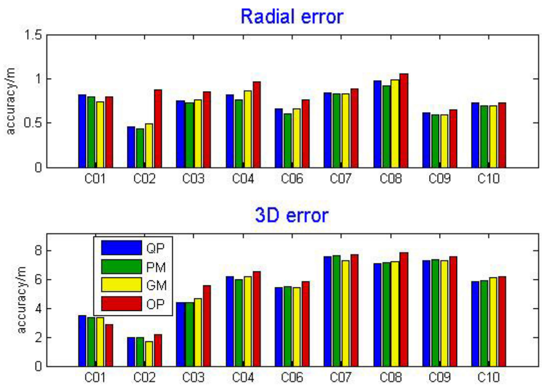

Fig. 6 The comparison of the orbit precision of traditional SPOD (OP) and new SPOD with 2 h station clock prediction based on different models

Table 2 gives the statistical values of SPOD precision for each satellite. In Table 2, the 1 h station clock predictions are used for SPOD, and the average radial precision of the orbits of the QP, PM, and GM models is 0.676 m, 0.656 m, 0.673 m, respectively, and the average 3D position precision is 5.570 m, 5.501 m, 5.529 m, respectively. The precision of the new SPOD based on each model is improved respectively by 10.40%, 13.08%, 10.75% in radial component, and 5.62%, 6.80%, 6.31% in 3D positions compared to the conventional SPOD.

When the predicted 2 h station clocks are used for the SPOD, the mean radial precision of the satellite orbits of the QP, PM and GM models is 0.725 m, 0.693 m, and 0.723 m, respectively, and the mean 3D position precision is 5.767 m, 5.672 m, and 5.679 m, respec-

Table 2 The accuracy for each prediction model used in POD (unit: m)

The SPOD precision based on 1 h station clock predictions (unit: m)								
Satellite ID	Radial				3D			
	QP	PM	GM	OP	QP	PM	GM	OP
C01	0.748	0.678	0.718	0.783	2.755	2.580	2.610	3.028
C02	0.580	0.545	0.576	0.733	2.115	1.913	2.037	2.209
C03	0.635	0.621	0.614	0.743	5.128	4.983	5.100	5.383
C04	0.691	0.675	0.688	0.791	6.309	6.175	6.277	6.569
C06	0.621	0.619	0.639	0.696	5.830	5.842	5.776	5.985
C07	0.813	0.811	0.804	0.857	7.300	7.305	7.214	7.848
C08	0.726	0.686	0.720	0.763	7.406	7.416	7.450	7.663
C09	0.597	0.597	0.622	0.667	7.256	7.259	7.261	8.170
C10	0.674	0.671	0.680	0.758	6.031	6.032	6.039	6.262
MEAN	0.676	0.656	0.673	0.755	5.570	5.501	5.529	5.902
The SPOD precision based on 2 h station clock predictions (unit: m)								
Satellite ID	Radial				3D			
	QP	PM	GM	OP	QP	PM	GM	OP
C01	0.818	0.722	0.777	0.890	3.397	3.297	3.170	3.622
C02	0.647	0.572	0.616	0.817	2.320	2.206	2.158	2.523
C03	0.694	0.682	0.705	0.844	5.207	4.943	4.959	5.518
C04	0.764	0.733	0.784	0.898	6.341	6.400	6.242	6.704
C06	0.652	0.605	0.618	0.727	5.900	5.727	5.866	6.350
C07	0.859	0.843	0.865	1.021	7.185	7.391	7.469	7.993
C08	0.739	0.740	0.775	0.864	7.585	7.446	7.522	7.962
C09	0.627	0.629	0.647	0.719	7.63	7.571	7.598	8.561
C10	0.723	0.708	0.722	0.779	6.339	6.063	6.124	6.449
MEAN	0.725	0.693	0.723	0.840	5.767	5.672	5.679	6.187

tively. The best results are obtained by the new SPOD with PM model. Compared with the conventional SPOD, the radial precision of the new SPOD based on the three models is improved by 13.71%, 17.53%, and 13.89%, respectively, and 3D position improvement is 6.78%, 8.33% and 8.21%, respectively.

4. CONCLUSION

This paper evaluates the performance of BDS station clocks using 3 typical fitting and prediction models, include quadratic polynomial model, gray model, and the periodical model. The 1 h and 2 h predicted station clocks are applied to the precise single satellite orbit determination according to the data processing specification of the BDS OCS, the conclusion is as follows:

(1) The station clock of the MPOD solution contains orbital errors, and the periodic term clock fitting model has the highest fitting precision, where fitting precision is within 0.15 ns for all stations, and the average fitting precision reaches 0.05 ns.

(2) The station clock varies from station to station. The average precision of 1 h prediction is 1.17 ns, 0.88 ns, and 1.28 ns for the QP, PM, and GM model, respectively. The average precision of 2 h prediction is 2.72 ns, 2.09 ns, and 2.53 ns, respectively.

(3) New SPOD based on the PM model shows the highest precision, where the orbit precision improvement using the 1 h and 2 h station clock predictions is respectively of 13.08%, and 17.53% in radial component, 6.80% and 8.33% in 3D positions, compared to the conventional SPOD. The results of the new SPOD using predicted station clocks show marginal difference among different models, where radial orbit precision is less than 2-3 cm on average.

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