Mitigation of Orbit Integration Errors for Eclipsing Satellites

Bingbing Duan, Junping Chen, Jiexian Wang, Yize Zhang, Sainan Yang, Jiejun Zhang and Qingchen Zhang

Abstract Numerical integration assumes that accelerations at each node are continuous and smooth. However, when satellite enters into shadow, perturbation caused by solar radiation pressure will jump. Therefore, mathematical theory of numerical integrator cannot match the real motion of satellite, which will bring about integration errors. In order to deal with the problem, some experiments are done for different constellations. The result shows that 99 % of error occurs in along-track direction and will accumulate when crossing more shadow boundaries. For different integrators, errors are different. Runge-Kutta4 integrator is sensitive to step size, especially for eclipsing satellites, and is not competent for long arc integration. Adams integrator relies on former steps, needs a fixed step size, and will induce more integration errors when crossing shadow boundaries. Runge-Kutta9 integrator brings less error during eclipsing season than Runge-Kutta4 and Adams integrators, and can change step size flexibly. To mitigate integration errors during eclipses, this contribution introduces an improved method based on Runge-Kutta9 integrator. We use dichotomy to detect the exact epoch of penumbra boundary, change the step size, and restart the integration. Result shows that after boundary detection, accuracy for 1-day arc improves 65.8 %, 2-day arc improves 55.5 %, 3-day arc improves 33.2 %. This method is suitable for both extended filter and least square method.

Keywords Eclipse · Penumbra · Integrator · Runge-Kutta · Adams

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1 Introduction

The International GNSS Service (IGS) is a volunteer organization of more than 200 individual agencies and institutions that maintain a global network of monitoring stations as well as products. Since the IGS started its operations on June 21, 1992, orbits of an unprecedented accuracy are available today for all active GPS satellites with a delay of about 13 days. As the advent of new GNSS, the IGS is fully committed to expand to a true multi-GNSS service. In 2012, the IGS has initiated the Multi-GNSS Experiment (MGEX) to provide Multi-GNSS measurements and products. Accuracy of BeiDou GEO satellite is around 1 meter, BeiDou IGSO satellite is about 30 cm, and BeiDou MEO satellite owns the best trajectory of about 10–15 cm. Galileo orbits provided by TUM show us a precision of 10 cm [1–3]. In some cases, when satellites are in eclipse, it is not easy to get the exact attitude of satellites so that solar radiation pressure model, antenna phase center correction, and phase wind-up cannot be well modeled [4]. In addition, if integration crosses shadow boundary, there will be a perturbation in acceleration caused by solar radiation pressure. Acceleration inside one step size will not be smooth and continue anymore, which can bring about integration errors. In order to solve this problem, dichotomy is used to detect the exact epoch of penumbra boundary and the restart the integration.

2 Character of Integrators

Gauss-Jackson, Runge-Kutta, and Adams integrator are popular among orbit determination, which have already been fully tested in accuracy, efficiency, etc. [5–9]. Laurichesse [10] used Runge-Kutta4 integrator in extended Kalman filter for the purpose of real-time orbit determination. A step size of 60 s was adopted and would change to 10 s when eclipse happens [10]. Li [11] tested Runge-Kutta12 integrator in orbit determination and concluded that high order Runge-Kutta integrator is more precise but also more time consuming. Adams integrator has a stable accuracy but relies on former steps. Li [12] put forward using collocation method to numerical integration and obtained some advantages: a big step size is allowed, positions and velocities of satellite can be obtained directly without any interpolation.

In this paper, Runge-Kutta4, Runge-Kutta9, and Adams integrator are tested based on 2-body problem. As we know, there will be an entirely analytical solution if only the 2-body problem is considered, and at the same time, if we use the same initial orbit excluding all the other force models we shall get a consistent trajectory by numerical integration.

Table 1 presents accuracy of different integrators using different step sizes. It reveals that Runge-Kutta4 integrator is sensitive to step size, a small step size is needed for precise usage. Runge-Kutta9 has an attractive accuracy and is nearly independent of step size. Adams integrator is both efficient and precise but relies on former steps.

Table 1 Accuracy test with different step size (24 h arc)

s	RK4 (m)	RK9 (m)	Adams (m)
10	3.5534E−05	7.1175E−06	1.6901E−06
60	4.9609E−02	1.9425E−06	1.1480E−06
150	2.1973E+00	1.4003E−06	1.0985E−06
300	4.2070E+01	4.3363E−05	2.1257E−06

3 Integration Error of Eclipsing Satellites and Method for Improvement

Models for solar radiation pressure are normally developed for full sunlight, and when in umbra models will be closed. However, there will be about 60 s between completely full sunlight and complete darkness, which is called penumbra. Even if we use Runge-Kutta integrator that is independent of former steps, there will still be a possibility that one separated step cross the shadow boundary, as shown in Fig. 1.

When integrator integrates from epoch n to $n+1$ or from epoch $m-1$ to m, acceleration within this period will not continue and smooth anymore. In order to mitigate integration errors during eclipsing season, a strategy for detecting the boundary of penumbra is put forward. Afterwards, integration is divided into three main parts: full sunlight, penumbra, and umbra. As in Fig. 1, the shadow factor at n epoch is $v = 1$, while $n+1$ epoch is $v = 0$. The boundary is inside the duration, which we can use dichotomy to detect the exact boundary position by doing several iterations. Whenever the shadow factor $v > 0.99$ we can treat it as a boundary epoch. So, boundary epochs b_1, b_2, b_3 and b_4 can be obtained. After that, we can change the step size at epoch n and integrate to epoch b_1, then do the same thing at epoch b_2, $m-1$, b_3 and b_4.

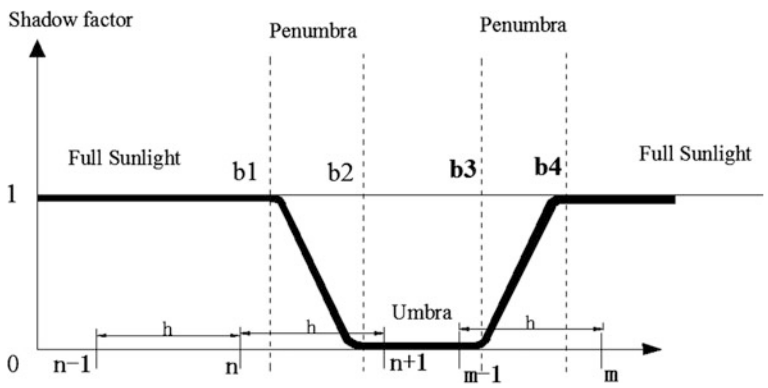

Fig. 1 Integration cross penumbra

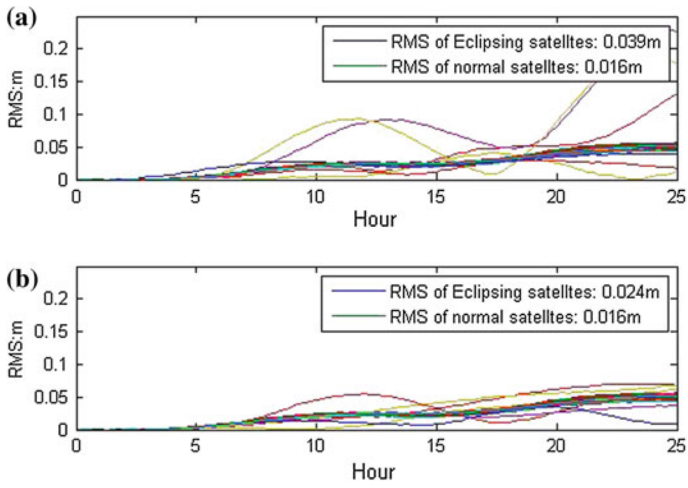

Fig. 2 Comparison of Runge-kutta4 integrator with step size 45 and 60 s

After restarting the integration, we need a standard to judge the precision. However, if the solar radiation pressure model is added, it will be complicated to do analytical solution. But we have already concluded from Table 1 that Runge-Kutta9 and Adams integrator are not sensitive to step size. If we use a very close step size, the results should be the same for the normal satellites, and the difference of eclipsing satellites could be an expression of integration precision.

Figure 2a shows a comparison of Runge-Kutta4 integrator using different step size 45 s and 60 s. It reveals that eclipsing satellites are very sensitive to step size and even normal satellites experience a linear growing difference. Figure 2b shows the same situation but after boundary checking, there is an improvement of 38.5 % for eclipsing satellites, but still not good enough. So, for long time running and precise using, we would not recommend Runge-Kutta4 integrator.

Adams integrator based on former steps is popular among post-processing of orbit determination. When crossing shadow boundaries, some integration errors occur. In order to find out how large the errors can be, we test data from 2015-10-01 to 2015-10-03 for GPS, BeiDou, and Galileo constellations.

Figure 3 shows comparison of Adams integrator with step size 45 s and 60 s for different constellations. The difference is obvious, GPS Block IIF experiences the largest difference, which can reach 1.2 m after 3 days. Table 2 is the time range of penumbra period and times of eclipse.

It is found that difference in Fig. 3 of each satellite increases with the number of eclipse, which means that the more shadow boundaries the integrator cross, the lager difference it will reach. Because that Adams integrator cannot change step size flexibly we do not check the boundary of penumbra and restart the integration.

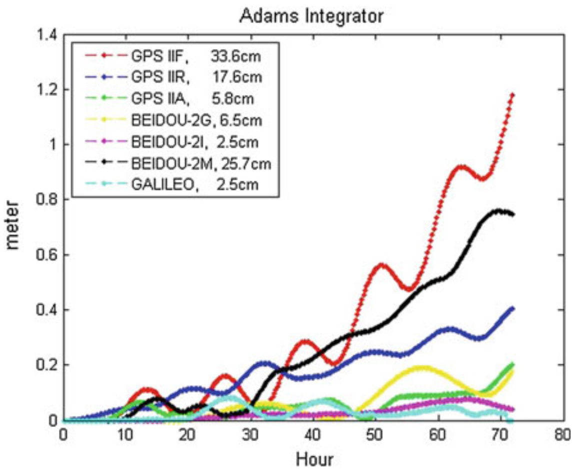

Fig. 3 Comparison of Adams integrator with step size 45 and 60 s

Table 2 Eclipsing information

	Mean penumbra duration (s)	Eclipsing times
GPS IIF	66	6
GPS IIR	62	6
GPS IIA	64	6
BEIDOU-2G	145	3
BEIDOU-2I	230	3
BEIDOU-2M	68	6
GALILEO	220	4

Runge-Kutta9 integrator can achieve the same precision as Adams integrator, but will spend more time. However, Runge-Kutta9 integrator has an advantage that step size can be modified flexibly. We use the boundary checking method that described above and restarts the integration. In order to compare with Adams integrator, the same data is tested for Runge-Kutta9 integrator.

Figure 4 shows the difference of GPS, BEIDOU, and GALILEO constellations after integrating for 3 days. BEIDOU-2M reveals the largest difference, which can reach to 45 cm. Compare Fig. 4 with Fig. 3, we can get that error in Runge-Kutta9 integrator is less than that in Adams. Figure 5 illustrates errors after restart integration. The largest difference drops down to 25 cm, details of improvement are shown as Table 3.

As shown in Table 3, after boundary detection and restarting the integration, accuracy for 1-day arc improves 65.8 %, 2-day arc improves 55.5 %, 3-day arc improves 33.2 %. In addition, 99 % errors occur in along-track direction.

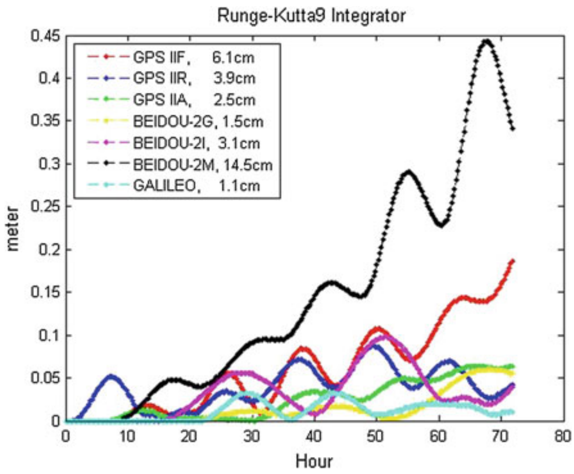

Fig. 4 Comparison of Runge-kutta9 integrator with step size 45 and 60 s

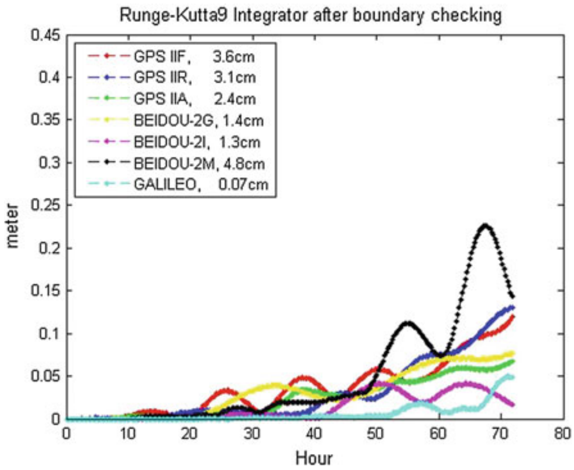

Fig. 5 Comparison of Runge-kutta9 integrator with step size 45 and 60 s after boundary checking

Table 3 Improvement after boundary detection (%)

Day	IIF	IIR	IIA	GEO	IGSO	MEO	IOV	Mean
3	39.9	18.3	2.8	5.6	57.5	66.6	41.9	33.2
2	40.9	74.6	5.3	9.8	80.7	85.2	91.6	55.5
1	44.2	83.1	55.1	21.5	90.4	90.9	75.6	65.8

4 Conclusion

This paper analyzes integration errors caused by crossing shadow boundary. A strategy based on shadow factor is developed to mitigate integration error. Conclusions are as following:

1. Runge-Kutta4 integrator is sensitive to step size especially for eclipsing satellites. The precision decreases after 1 day integration.
2. Adams integrator is precise and stable but relies on former steps. When crossing shadow boundaries, it will bring about errors. As tested in this paper, integration error increases with the number of eclipse. GPS Block IIF satellites can lead to 1.2 m error after integrating for 3 days.
3. Runge-Kutta9 integrator is precise and flexible. It is independent of former steps, and will not cause so much integration error as Adams integrator. Besides, it can modify step size at any epoch. After boundary detection, it will be easy to restart integration.
4. 99 % errors occur in along-track direction. For Runge-Kutta9 integrator, after boundary detection and restarting the integration, accuracy for 1-day arc improves 65.8 %, 2-day arc improves 55.5 %, 3-day arc improves 33.2 %.

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