Modeling and initial assessment of the inter-frequency clock bias for COMPASS GEO satellites

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Abstract

The COMPASS system is a project established by China to develop an independent global satellite navigation system, which has five GEO (Geostationary Orbit) satellites and thirty Non-GEO satellites. An apparent inter-frequency clock bias (IFCB) for COMPASS GEO satellites is investigated using the real data. The bias also is modeled by the different models. Based on the 15 months (DOY 121, 2011–214, 2012) single-day-estimated results, the periodic variation of IFCBs of the COMPASS GEO satellite is studied using a harmonic analysis. The notable periods of 12 h and 8 h are noted. The harmonics-based models with different periods and different orders and quadratic function based model are used to describe the IFCB. The performances show that the 4-order harmonics-based model with the periods of 24, 12, 8 and 6 h is most optimal than others for describing the IFCB of COMPASS GEO satellite. Its amplitudes and phases estimated from a least square fit are used to study the features of the IFCB. The results show that the current amplitudes and phases do not present special features. Although the irregular amplitudes and phases of the model are disadvantageous for the long-term prediction of IFCB, it is obvious that the modeling IFCB can simplify its service and a few of coefficients can replace the IFCB series. The performance of the model in short-term prediction IFCB is tested using the ten-day data (DOY 215-224, 2012).

Keywords: Triple-frequency signals; Precise Point Positioning; Inter-frequency clock bias; COMPASS system

1. Introduction

The Compass system is a project made by China to develop an independent global satellite navigation system (GNSS) of five GEO (Geostationary Orbit) satellites and thirty Non-GEO satellites. The system is based on the previously deployed BeiDou-1 navigation demonstration system. The follow-up BeiDou-2 system will be completed through two phases. In the first phase, 5 satellites in Geostationary Orbit (GEO), 5 in Inclined Geosynchronous Orbit (IGSO) and 4 in Medium Earth Orbit (MEO) are comprised. It was fully completed at the end of 2012. The constellation will be completed by the end of 2020 and it consists of 5 GEO, 3 IGSO and 27 MEO satellites (Shi et al. 2013).

Current GNSS and augmentations, such as the modernized GPS, the European Galileo system and Japanese Quasi-Zenith Satellite System (QZSS) provide triple-frequency signals. COMPASS is the first system provides triple-frequency signals for all operational satellites. An apparent inconsistency between three frequency carrier phases in the GPS systems and COMPASS system (Montenbruck et al. 2011, 2012a, b) was noticed. It is understood to be
caused a thermally dependent inter-frequency bias (IFB). The IFB of QZSS also was investigated and the result shows that it has no the variation characterization (Hauschild et al. 2012). The IFB variation leads to the satellite clocks derived from L1/L2 (B1/B2) carrier-phase observations cannot be used for L1/L5 (B1/B3) based Precise Point Positioning (PPP) without careful consideration of these biases. To enable a consistent use of L1/L2 (B1/B2) clock products in L1/L5 (B1/B3) based positioning and contribute to a better clock predictability at timescales of several hours, the inter-frequency clock biases (IFCB) of Block IIF satellites are estimated and modeled (Montenbruck et al. 2011, 2012a; Li et al. 2012a,b). In IFCB modeling, a high-order harmonics-based model is used (Montenbruck et al. 2012a; Li et al. 2012b). Based on the modeled results, the features of the IFCB are discussed.

With the developments of all GNSS systems, the IFCB will be provided as a routine service. It is well known that the service of satellite clock can be simplified by modeling the IFCB, because a few of estimated coefficients can replace the IFCB series. In this contribution, the 15 months (DOY 121, 2011-214, 2012) data is processed to study the features of IFCB of COMPASS GEO satellites. Based on the estimated results and the features, the empirical models are discussed in detail. In the following, Section 2 introduces estimation approaches and empirical model of IFCBs. Section 3 presents the data analysis and discusses the results. Finally, Section 4 summarizes the main findings.

2. Estimation and modeling

The estimated and modeled results all can be used to express the IFCB. Obviously, the advantage of the model is that it can replace IFCB series from estimation strategy and reduce storage space. It is necessary for modeling IFCB to study its feature based on the estimated results so that the estimation approach also is introduced in this section.

2.1. Estimation

The undifferenced (Montenbruck et al. 2012b) and epoch-differenced (ED) approaches (Li et al. 2010a,b, 2012c) are used to estimate IFCB. The processing times show that the ED approach is much more efficient, especially for real-time application (Li et al. 2012b). The two approaches use the differenced ionosphere-free measurement (DIF). The DIF measurement is defined as:

\[
DIF(B_1, B_2, B_3) = IF(B_1, B_2) - IF(B_1, B_3) = \delta + amb \tag{1}
\]

where \(IF(B_1, B_2)\) is the ionosphere-free combination formed with B1 and B2; \(IF(B_1, B_3)\) is the ionosphere-free combination formed with B1 and B2; \(amb\) is the ambiguity of DIF measurement; The term \(\delta\) is the IFCB. In undifferenced approach, the IFCB could be estimated based on (1) (Montenbruck et al. 2011, 2012a). It is very important to realize the contribution of the receiver to the IFCB, because it dominates the selection of the reference. When it cannot be neglected, the reference must be selected as the estimation of satellite clock (Bock et al., 2009; Ge et al. 2012; Li et al. 2010a, b) in IFCB estimation. The approaches stated in Li et al. (2012a) are used to validate the contribution of the receiver. The results show that it can be neglect in IFCB estimation (Montenbruck et al. 2012b). In order to improve the computation efficiency, a differenced approach was presented in Li et al. (2012b). The ambiguity term in (1) can be removed by differencing the DIF measurements at the two adjacent epochs. The ED IFCB can be obtained as:

\[
\Delta \delta(m) = DIF(B_1, B_2, B_3)(m) - DIF(B_1, B_2, B_3)(m - 1) \tag{2}
\]

where “\(^\Delta\)” indicates the ED operator, and \(\Delta \delta(m)\) is the ED IFCB at the epoch of \(m\). Assuming there are \(n\) stations in the network, which improves the redundancy of the solution, the ED IFCB \(\Delta \delta(m)\) can be calculated by averaging \(\Delta \delta(m)\) over the entire network:

\[
\Delta \delta(m) = \frac{1}{n} \sum_{k=1}^{n} \Delta \delta(m)_k \tag{3}
\]

The absolute IFCB accumulates from the estimated ED IFCB based on the IFCB at the reference epoch. It can be given as:

\[
\delta(m) = \delta_0 + \sum_{j=1}^{m} \Delta \delta(j) \tag{4}
\]

where \(\delta_0\) is the IFCB at the reference epoch, \(\Delta \delta(j)\) is the ED IFCB at epoch \(j\), and \(m\) is the number of epochs between the reference epoch and epoch \(m\).

2.2. Modeling

Two models are introduced to describe the IFCB in this paper. One is a high-order harmonics-based model which is similar to the model shown in Li et al. (2012b), the other is a segmented quadratic function based model. The harmonics-based model has been used to model the Block IIF IFCB. Its performance shows that it can describe the Block IIF IFCB well (Montenbruck et al. 2012a; Li et al. 2012b). The COMPASS satellites, especially the GEO satellite operates in its own orbit that differs from Block IIF satellite so that they have themselves features, for example the orbital period and operation altitude. The IFCB is understood to come from the sun illumination (Montenbruck et al. 2012a; Li et al. 2012b). Based on the theory of the sun illumination and the features of the GEO satellites, we assume that there must be a difference between the IFCBs of the Block IIF and the COMPASS GEO satellite. In addition, the period is most important in IFCB modeling. Currently, there is no information about the periodic variation of IFCB of COMPASS GEO satellite.

Based on the above discussion and consideration, a high-order harmonics-based composite function (Li et al.
2012b) is still used to model the IFCB of COMPASS GEO satellites. The model is written as:

\[
\delta(t) = a + b \cdot t + \sum_{i=1}^{4} \lambda_i \cdot \sin \left( \frac{2\pi}{T_i} \cdot t + \theta_i \right),
\]

\((t = 0 \sim 24h)\)  

(5)

where \(a\) is a constant, \(b\) is the linear term, \(t\) is the observation time, \(\theta_i\) is phase offset, \(\lambda_i\) is the amplitude; \(T_i\) is the period. Considering the features of the COMPASS GEO satellite, a segmented quadratic function also is used to model the IFCB of COMPASS GEO satellite. The model is written as:

\[
\delta(t) = \begin{cases} 
\begin{align*} 
a_1 + b_1 \cdot t_1 + c_1 \cdot t_1^2 & \quad t_1 = 0 \sim 12h \\
a_2 + b_2 \cdot t_2 + c_2 \cdot t_2^2 & \quad t_2 = 12 \sim 24h 
\end{align*} 
\end{cases}
\]

(6)

where \(a_1\) and \(a_2\) are the constants, \(b_1\) and \(b_2\) are the linear terms; \(t_1\) and \(t_2\) is the observation time; \(c_1\) and \(c_2\) are the acceleration of the IFCB variation.

3. Validation and results

To study the features of the IFCB of the COMPASS GEO satellites and test the empirical model, the 15 months data (DOY 121, 2011-214, 2012) from five stations are processed. In data processing, phase wind-up effects and antenna offset are considered. Data are sampled at 30 s. Correctional rate is defined to study the correction ability of the model. It is given by:

\[
P = \left( 1 - \frac{\text{RMS}_D}{\text{RMS}_E} \right) \cdot 100\%
\]

(7)

where \(P\) is the correctional rate, \(\text{RMS}_D\) is the RMS of the difference between the modeled and estimated IFCB, \(\text{RMS}_E\) is the RMS of the estimated IFCB.

3.1. Performance of the model

The determination of the periods of all orders is very important for modeling the IFCB in (5). In order to study the periodic variations of IFCB of COMPASS GEO satellite, a harmonic analysis is performed by using a FFT (fast Fourier transformation) based on the 15 months single-day-estimated IFCBs. The 15 months single-day results of each GEO satellite show that the IFCBs of COMPASS GEO satellites have notable periodic variations with the periods of 12 and 8 h. It can be understood that the sun illumination and thermal changes stated in Montenbruck et al. (2012a) cause of the IFCB variation with 12-h period. It is very interesting that the IFCBs of the COMPASS GEO satellites also has the characterization of periodic variation with 8-h period which is similar to GPS Block IIF satellites (Li et al. 2012b) and cannot be explained using sun illumination. Especially, the COMPASS GEO satellites and the GPS Block IIF satellites operate in different pattern, for example the period and operation altitude. Even though the 8 h period of COMPASS GEO satellite and GPS Block IIF brings plenty of imaginations to us, it is difficult to determine what factors are involved.

Considering the orbital period of COMPASS GEO satellite and the theory of sun illumination, the different groups of period are used to model the IFCBs of GEO satellites. Based on the periods, the coefficients \(\delta_i(i = 1–4)\) and \(\lambda_i(i = 1–4)\) are estimated from a least squares fit for a 24-h data arc over the 15 months. Inserting the estimated coefficients and used periods into (5), the modeled IFCBs are obtained. Comparing the modeled IFCBs with estimated ones, the RMSs and correctional rates are computed. The means of the 15 months single-day correctional rates and RMSs of different groups are shown in Table 1. The correctional rates and RMSs reveal the contribution of the 24-h period to the IFCB is more prominent than that of other periods. From the results, it is observed that the accuracy and correctional rate of the model improves with increase of the order and the 4-order harmonics-based model is better than others. When the correctional rates of the two models with the periods of 24, 12 and 6 h and 24, 12 and 8 h are compared, an apparent improvement from 70.8, 75.4, 76.5 and 73.0% to 71.9, 76.7, 77.0 and 75.9% is noticed. The improvement reveals the reliability of the 8-h period. Comparing the correctional rates of the 3-order harmonics-based model to that of the 4-order harmonics-based model, it can be seen that the correctional rates improves from a mean of 74.65–77.50%. The correctional rates and RMSs of 4-order harmonics-based model indicate that the periods of 24, 12, 8 and 6 h are more fit than the periods of 24, 12, 8 and 4 h for modeling IFCB. The results of the two models also show that the 6-h period is more reasonable than the 4-h period, when they are selected as the fourth-order period. The modeled and estimated IFCBs of DOY 204, 2012 in Fig.1 further validate that the appropriateness of the periods \((T_i(i = 1–4))\) of 24, 12, 8 and 6 h. The RMSs and correctional rates of the model with the periods of 24, 12, 8 and 6 are shown in Figs. 2 and 3. The Figs. 2 and 3 and Table 1 indicate that the model is more favorable to G04 than other GEO satellites. When it is used to model the IFCB of G04, it can reach millimeter level in most cases. From Fig. 2, it can be seen that the most of RMS values of G01, G03 and G05 are better than 2.0 cm.

We assume that the segmented quadratic function can describe the IFCB of the COMPASS GEO satellite well if only sun illumination causes IFCB. Based on this consideration, the harmonic function based model also is used to model IFCB of COMPASS GEO satellite. Its coefficients \(\delta_i, b_i, c_i, a_2, b_2\) and \(c_2\) are estimated from a least squares fit for a 24-h data arc over 15 months. Inserting the estimated coefficients into (6), the modeled IFCBs are obtained. The 15 months RMSs and correctional rates are computed using the estimated and modeled IFCB and shown in Table 2. Its correctional rates and RMSs show that it reaches the accuracy of the 2-order harmonics-based model with the periods of 24 and 12 h, and more than 64.8% IFCB can be corrected by using the quadratic
function based model. Its performance demonstrates there remain contributions of high-order terms after the IFCB is corrected using it. In other words, the performance of the quadratic function based model indicates that the IFCB also comes from other factors besides sun illumination.

3.2. Amplitude and phase

"Performance of the model" shows that the 4-order harmonics-based model with periods ($T_i (i = 1–4)$) of 24, 12, 8 and 6 h is better than other models in modeling IFCBs of COMPASS GEO satellites, thus its amplitudes ($\omega_i (i = 1–4)$) and phases ($\theta_i (i = 1–4)$) are selected to study the features of the IFCB. The 15 months single-day amplitudes ($\omega_i (i = 1–4)$) of G01, G03, G04 and G05 are illustrated in Figs. 4–7, respectively. The results in those figures show that the variation of the amplitudes of the 24-h period is more prominent than that of other orders. It further demonstrates the contribution of 24 h period to IFCB. From those figures, it can be seen that the amplitudes of GEO satellites vary without apparent features, which differ from that of the GPS Block IIF satellites shown in Li et al. (2012b), even though the GEO satellites operate in different pattern including orbital period and operation altitude. The amplitudes of the 24- and 12-h periods should behave a similar trend, if the theory of sun illumination and thermal changes

<table>
<thead>
<tr>
<th>Group of period (h)</th>
<th>Mean of correctional rate</th>
<th>Mean of RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G01</td>
<td>G03</td>
</tr>
<tr>
<td>$T_1 = 12$, $T_2 = 6$</td>
<td>49.9</td>
<td>49.6</td>
</tr>
<tr>
<td>$T_1 = 24$, $T_2 = 12$</td>
<td>68.4</td>
<td>73.3</td>
</tr>
<tr>
<td>$T_1 = 12$, $T_2 = 6$, $T_3 = 8$</td>
<td>53.8</td>
<td>53.2</td>
</tr>
<tr>
<td>$T_1 = 24$, $T_2 = 12$, $T_3 = 6$</td>
<td>70.8</td>
<td>75.4</td>
</tr>
<tr>
<td>$T_1 = 24$, $T_2 = 12$, $T_3 = 8$</td>
<td>71.9</td>
<td>76.7</td>
</tr>
<tr>
<td>$T_1 = 24$, $T_2 = 12$, $T_3 = 8$, $T_4 = 4$</td>
<td>73.2</td>
<td>77.9</td>
</tr>
<tr>
<td>$T_1 = 24$, $T_2 = 12$, $T_3 = 8$, $T_4 = 6$</td>
<td>74.7</td>
<td>79.5</td>
</tr>
</tbody>
</table>

Fig. 1. Estimated and modeled IFCBs for COMPASS GEO satellite, DOY 204, 2012.

Fig. 2. RMSs (m) of the modeled IFCBs compared with the estimated ones, for the time DOY 121, 2011-214, 2012. The axis is labeled DD.MM.YY.
(Montenbruck et al. 2012a) are strictly considered. But the IFCBs of the COMPASS GEO satellites do not present on this feature. From the factors caused IFCB, it can be explained that the irregular amplitudes come from irregular effect to clock and signal so that it is difficult to note the more apparent information about the features of IFCB.

The amplitude series is fitted by using the quadratic function. The RMSs of the difference between the fitted and estimated amplitudes are computed and shown in Table 3. In fitting, the average span of amplitude series is about 5 months. The RMSs further indicate that the amplitude of the most of periods vary without apparent pattern. It is exceptional that we notice that the amplitudes with 12- and 8-h periods of G04 have the characterization of quadratic curve. Their RMS values in Table 3 validate this.

The single-day phases \( \theta_i (i = 1-4) \) of G01, G03, G04 and G05 are illustrated in Figs. 8–11, respectively. From phases in those figures, it is observed that only the phases of some spans and some periods behave a special trend, for example the phase of the 6-h period of G04, for the time DOY 121, 2011-011, 2012, and the phase of the 24-h period of G05, for the time DOY 63-214, 2012. Like the amplitudes, the phases of 24- and 12-h periods should vary with a linear trend at least, if only the sun illumination and estimated amplitudes are computed and shown in Table 3. In fitting, the average span of amplitude series is about 5 months. The RMSs further indicate that the amplitude of the most of periods vary without apparent pattern. It is exceptional that we notice that the amplitudes with 12- and 8-h periods of G04 have the characterization of quadratic curve. Their RMS values in Table 3 validate this.

The single-day phases \( \theta_i (i = 1-4) \) of G01, G03, G04 and G05 are illustrated in Figs. 8–11, respectively. From phases in those figures, it is observed that only the phases of some spans and some periods behave a special trend, for example the phase of the 6-h period of G04, for the time DOY 121, 2011-011, 2012, and the phase of the 24-h period of G05, for the time DOY 63-214, 2012. Like the amplitudes, the phases of 24- and 12-h periods should vary with a linear trend at least, if only the sun illumination and

**Table 2**

Means of the correctional rate (percentage) and RMS (cm).

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Correctional rate</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>G01</td>
<td>64.8</td>
<td>2.77</td>
</tr>
<tr>
<td>G03</td>
<td>71.4</td>
<td>2.51</td>
</tr>
<tr>
<td>G04</td>
<td>70.5</td>
<td>2.03</td>
</tr>
<tr>
<td>G05</td>
<td>68.9</td>
<td>2.65</td>
</tr>
</tbody>
</table>

Fig. 3. Correctional rates (%) of the modeled IFCBs compared with the estimated ones, for the time DOY 121, 2011-214, 2012.

Fig. 4. Amplitudes of G01 for DOY 121, 2011-214, 2012.

Fig. 5. Amplitudes of G03 for DOY 152, 2011-214, 2012.
thermal changes cause IFCBs of COMPASS GEO satellites. Unfortunately, the phase also does not present this feature. The irregular amplitudes and phases are disadvantageous for long-term prediction of the precise IFCB.

3.3. Performance of the model in short-term prediction of IFCB

The performances of the models show that the model with the periods of the 24, 12, 8 and 6 h is enough to be used in post-processing application. The ten estimated coefficients of the used model can replace the IFCB series so that the service of satellite clock is simplified. In real-time application, the IFCB can be provided by predicting the coefficients of the model. The “Amplitude and phase” shows that the amplitudes and phases have no special characterization which is disadvantageous for the long-term prediction of IFCB, although more than 74.7% IFCB can be corrected by the 4-order harmonics-based model with the periods of 24, 12, 8 and 6 h. To test the performance of the model in short-term prediction, ten-day (DOY 215-224, 2012) data are processed. The first single-day data is used to estimate the coefficients of the model from a least square fit. The estimated coefficients are used to compute the IFCB of other days. The RMSs of the difference between the computed and estimated IFCBs are illustrated in Fig.12. The figure shows that the prediction can reach an accuracy of 3 cm and about 61% IFCB can be corrected by prediction.

4. Conclusions and future works

In order to study the features of the IFCBs of COMPASS GEO satellites and describe their variations, the 15-month data are processed. The results show that there are apparent IFB variations in triple-frequency signals of COMPASS GEO satellite. Based on the 15-month single-day-estimated IFCBs, a harmonic analysis is performed by using a FFT. The notable periodic variations of COMPASS GEO satellite with the 12- and 8-h periods are noticed.

Combined with the 24-h orbital period of COMPASS GEO satellite, different harmonics-based models with different orders and different periods and the quadratic

![Fig. 6. Amplitudes of G04 for DOY 121, 2011-214, 2012.](image)

![Fig. 7. Amplitudes of G05 for DOY 63-214, 2012.](image)

Table 3
Amplitude and RMS of fitted amplitudes with respect to estimated ones; the unit is centimeter.

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>RMS (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G01</td>
<td>G03</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>3.28</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>1.58</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>0.89</td>
</tr>
<tr>
<td>$\lambda_4$</td>
<td>0.72</td>
</tr>
</tbody>
</table>

![Fig. 8. Phases of G01 for DOY 121, 2011-214, 2012.](image)
function based model are used to model the IFCB. The performances of the harmonics based model show that the high-order model is better than that of other orders and the 4-order harmonics based model with the periods of 24, 12, 8 and 6 h can model the IFCBs of COMPASS GEO satellites well. Its correctional rates show that more than 74.7% IFCB can be corrected using the presented model. Although the amplitudes and phases have no special characterization which is disadvantageous for long-term prediction of IFCB, it is very useful and meaningful to model the IFCB, because its products can replace the IFCB series and simply service of the IFCB. The correctional rates of the quadratic function based model show that it only reaches the accuracy of the second-order harmonics-based model with the periods of 24 and 12 h. The performances of the quadratic function based model and the second-order harmonics based model reveal that the other factors may exist that disturb the satellite clock or signals besides sun illumination.

To test the performance of the model in short-term prediction of IFCB, a simple experiment is carried out. From the results, it can be seen that the model can correct about 61% IFCB, if the first daily data is used to estimate the coefficients of the model and the estimated coefficients are used to compute the IFCB of the next day. The performance further validates the irregular variations of the amplitudes and phases. The variation of amplitudes and phases should be still monitored and the further study on
the other factors that disturb the clock or signals needs to be continued in the future.

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