Orbit Fitting based on Helmert Transformation

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Abstract: Orbit fitting is used in many GPS applications. E.g., in Precise Point Positioning (PPP), GPS orbits(SP3 orbits) are normally retrieved either from IGS or from one of its Analysis Centers (ACs) with 15 minutes' sampling, which is much bigger than the normal observation sampling. Therefore, algorithms should be derived to fit GPS orbits to the observation time. Many methods based on interpolation were developed. Using these methods the orbits fit well at the sampling points. However, these methods ignore the physical motion model of GPS satellites. Therefore the trajectories may not fit the true orbits at the periods in between 2 sampling epochs. To solve this problem, we develop a dynamic approach, in which a model based on Helmert transformation developed in GPS orbit fitting. In this orbit fitting approach, GPS orbits at sampling points are treated as pseudo-observations. Thereafter, Helmert transformation is built up between the pseudo-observations and dynamically integrated orbits at each epoch. A set of Helmert parameter together with corrections of GPS initial orbits are then modeled as unknown parameters. Results show that the final fit orbits have the same precision as the IGS final orbits.

Key words: Precise Point Positioning, IGS Orbits, Orbit Fitting, Helmert Transformation

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Introduction

Currently, GPS orbit products of IGS achieves a precision better than 5 cm(IGS: http://igscb.jpl.nasa.gov/components/prods.html), which provides and enhances a lot of applications, e.g. PPP. In PPP, orbits and clocks of GPS satellites are normally fixed to a certain value. They may be retrieved either from IGS or from one of its ACs. Currently, the sampling rate of GPS orbits provided by IGS is 15 minute. For high frequency data processing, algorithms should be derived to get the orbits at the observation time.

Many methods based on interpolation such as Chebyshev polynomial[1-2], Lagrange polynomial[3], Newton polynomial[4] etc., were developed. These methods use polynomial to fit the trajectories of the GPS orbit. The orbits normally fit well at the sampling point. However, there is one problem of these methods that they ignore the dynamical physical motion model of GPS satellites, i.e., the Newtonian second law. Consequently, the orbits may not fit well at the epochs in between 2 sampling epochs.

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Starting from this point, a model based on Helmert transformation is presented in GPS orbit fitting. Based on this model an orbit fitting approach is developed. Firstly, we get satellites' initial orbits from the IGS final orbits. An orbit integration is then performed based on the derived initial orbits. Afterwards, Helmert transformation between IGS orbits and integrated orbits is set up at sampling epochs of the IGS orbits. Accumulating all the equations at each epoch, the parameters of the model, including Helmert transformation parameters and corrections of initial orbits, are then estimated. Using the updated initial orbits, orbit integration can be performed again to get the new integrated orbits. This procedure can be iterated. Results show that the final integrated orbits have the same precision of the IGS final orbits.

1. Orbit integration

According to the Newtonian second law, satellites' motion equation and satellites' initial orbits at epoch t_0 can be written as[5-6],

$$\begin{cases} \dot{x} = F(x,t) \\ x\big|_{t_0} = x_0 \end{cases} \tag{1}$$

Where, $x_0 = \begin{pmatrix} r_0 & \dot{r}_0 & p_0 \end{pmatrix}^T$ are initial orbits including positions, velocities and dynamic parameters (e.g. the solar radiation pressure parameters (SRP)) of the satellite. $F(x,t)$ is the modeling equation of the complete set of forces acting on an orbiting satellite[7-9]. With proper integration method such as Adams-Cowell numerical integration, dynamic integrated orbits x^* can be computed based on x_0.

In equation (1), we can define $\delta = x - x^*$. Based on the Taylor expansion, we get

$$\dot{\delta} = \left.\frac{\partial F(x,t)}{\partial x}\right|_{*} \delta \tag{2}$$

The solution of (2) can be expressed as,

$$\delta = \Psi(t,t_0)\delta_0 \tag{3}$$

Where, $\delta_0 = x_0 - x^*$ is the orbit corrections at initial epoch t_0. Substitute (3) into (2), we have the following equation,

$$\begin{cases} \dot{\Psi}(t,t_0) = \dfrac{\partial F}{\partial x}\Psi(t,t_0) \\ \Psi(t_0,t_0) = I \end{cases} \tag{4}$$

Where, I is the unit matrix, $\Psi(t,t_0)$ is called transition matrix. It can be expressed in detail as,

$$\Psi\left(t,t_0\right)=\begin{pmatrix}\frac{\partial r}{\partial r_0} & \frac{\partial r}{\partial \dot{r}_0} & \frac{\partial r}{\partial p}\\ \frac{\partial \dot{r}}{\partial r_0} & \frac{\partial \dot{r}}{\partial \dot{r}_0} & \frac{\partial \dot{r}}{\partial p}\\ \frac{\partial p}{\partial r_0} & \frac{\partial p}{\partial \dot{r}_0} & \frac{\partial p}{\partial p}\end{pmatrix} \tag{5}$$

From numeric integration we can get transition matrix as well as integrated orbits x^*.

2. Orbit fitting based on Helmert transformation

Helmert transformation is mostly used to express differences between reference frames[10]. It is used by the International GNSS Service (IGS) community to analyze the systematic differences between ACs and to combine products from different ACs to get the final IGS products[11]. As we know that each software may differ in dynamic models and processing approaches, consequently the software difference results in the difference of the reference frame defined. Considering the systematic differences between our integrated orbits and the IGS final orbits, we can build up Helmert transformation between the dynamic integrated orbits and the IGS final orbits. At epoch t_i it can be expressed as,

$$\begin{pmatrix}X_K^i\\ Y_K^i\\ Z_K^i\end{pmatrix}=\begin{pmatrix}\Delta X\\ \Delta Y\\ \Delta Z\end{pmatrix}+(1+K')R_1(\alpha')R_2(\beta')R_3(\gamma')\begin{pmatrix}X_T^i\\ Y_T^i\\ Z_T^i\end{pmatrix} \tag{6}$$

Where, (X_K^i, Y_K^i, Z_K^i) are the IGS final orbits. (X_T^i, Y_T^i, Z_T^i) are the dynamically integrated orbits expressed in Earth-fixed reference frame, which can be obtained using the following transformation[9],

$$\begin{pmatrix}X_T^i\\ Y_T^i\\ Z_T^i\end{pmatrix}=Q(t_i)R(t_i)W(t_i)\begin{pmatrix}X_I^i\\ Y_I^i\\ Z_I^i\end{pmatrix} \tag{7}$$

Where, (X_I^i, Y_I^i, Z_I^i) are integrated orbits expressed in inertial reference frame, $Q(t_i), R(t_i), W(t_i)$ are the matrices for precession-nutation, Earth rotation and pole wobble, respectively.

We can rewrite the model as,

$$r_K^i = T + (1+K)R1\cdot R2\cdot r_I^i \tag{8}$$

Where, r_K^i, r_I^i are the denotation of the IGS final orbit (in Earth-fixed frame), the dynamically integrated orbit (in inertial frame), T, K represent the translation and the scale parameters of the Helmert transformation, and $R1, R2$ are rotation matrices in equation (6) and (7). In $R2$, parameters contained are: Earth pole x_p, y_p and the rates $\dot{x}_p, \dot{y}_p$, time parameter $UT1-UTC$ ($dUT1$) and the rate $d\dot{U}T1$.

The magnitude of x_p, y_p is less than 1" and the magnitude of $dUT1$ is less than 1 second (15" in angle), Magnitude of $\dot{x}_p, \dot{y}_p$ and $d\dot{U}T1$ are even smaller. Ignoring the effects of $\dot{x}_p, \dot{y}_p$ and $d\dot{U}T1$, equation (8) can be rewritten as,

$$\begin{pmatrix} X_K^i \\ Y_K^i \\ Z_K^i \end{pmatrix} = \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} + (1+K)R_1(\alpha)R_2(\beta)R_3(\gamma)R_{etc} \cdot \begin{pmatrix} X_I^i \\ Y_I^i \\ Z_I^i \end{pmatrix} \tag{9}$$

Where, R_{etc} is the residual matrix comparing equation (9) to equation (8) and,

$$\alpha = \alpha' + y_p, \quad \beta = \beta' + x_p, \quad \gamma = \gamma' + dUT1 \tag{10}$$

By defining $R3(R) = R_1(\alpha)R_2(\beta)R_3(\gamma)$, we can rewrite equation (9) as following,

$$r_K^i = T + (1+K)R3(R) \cdot R_{etc} \cdot r_I^i \tag{11}$$

Where, T, K, R represent Helmert parameters(H) ($\Delta X, \Delta Y, \Delta Z, K, \alpha, \beta, \gamma$).

The linearization of equation (11) reads as,

$$r_K^i = r_{K,0}^i + \frac{\partial r_K^i}{\partial v} dv \tag{12}$$

Parameters to be estimated are expressed in equation (13), where dH are corrections of Helmert transformation parameters, dr_I^i are corrections of integrated orbits at current epoch.

$$dv = (dH, dr_I^i)^T \tag{13}$$

Considering equation (3) and (4), we can transform the parameter dr_I^i to initial orbit corrections dr_0 using equation (14).

$$dr_I^i = \Psi(t_i, t_0) dr_0 \tag{14}$$

Therefore the final parameters can be expressed as:

$$dx = (dH, dr_0)^T \tag{15}$$

Design matrix is:

$$A_i = \frac{\partial r_K^i}{\partial x} = \left(\frac{\partial r_K^i}{\partial H}, \frac{\partial r_K^i}{\partial r_I^i} \cdot \Psi(t_i, t_0) \right) \tag{16}$$

Equation (12) can be formed at each epoch t_i. Accumulating all the epochs (normally 96 epochs) in the IGS final orbits, we get the observation equations,

$$Adx = L \tag{17}$$

Where,

$$A = \begin{pmatrix} A_1 \\ A_2 \\ \cdots \\ A_n \end{pmatrix} \qquad L = \begin{pmatrix} L_1 \\ L_2 \\ \cdots \\ L_n \end{pmatrix} = \begin{pmatrix} r_K^1 - r_{K,0}^1 \\ r_K^2 - r_{K,0}^2 \\ \cdots \\ r_K^n - r_{K,0}^n \end{pmatrix} \tag{18}$$

Normal equation can be written as,

$$Ndx = b \tag{19}$$

Where,

$$N = A^T A, \qquad b = A^T L \tag{20}$$

According to the Least Square Estimation (LSE) theory, the final normal equation equals to the accumulation of the normal equation at each epoch, i.e.,

$$N = \sum_{i=1}^{n} \left(A_i^T A_i \right), \qquad b = \sum_{i=1}^{n} A_i^T L_i \tag{21}$$

The solution of equation (19) can be iterated. Using the new initial orbits from (19), new integrated orbits can be generated. Based on the final estimated initial orbits, final integrated orbits can be generated, which is our final fit orbits.

3. Data processing

As a validation of the model and the processing approach, we fit the GPS orbits on the day 062 of year 2005 to the IGS final orbits. Initial satellites' positions and velocities of orbit integration are derived from the IGS orbits by interpolation. Initial SRP parameters are set to zero. Dynamic models are listed in table 1.

Table 1 Dynamic models

Gravity model	EGM96 (8*8)
Tide	Solid Earth Tide

N-body	Sun & Moon
Solar radiation pressure	BERNE[12-13]

According to Chen[14], satellites' orbits achieve similar precision under different parameter sets of our model. Therefore here we estimate only Helmert translation parameters. To sum up, the estimated parameters include Helmert translation parameters, corrections of initial orbits (positions, velocities and SRP parameters).

Figure 1 shows the orbit differences between our final fit orbits and the IGS orbits of GPS PRN01, where we see smoothing periodical variations. The period fits well with GPS' revolution period. The range of the variations is within (-4,4) cm in each direction.

Figure 2 to figure 4 summarize the statistics of the orbit differences of each satellite. Figure 2 shows the absolute value of the mean orbit differences, where we see that most of them are smaller than 2mm. This shows us that the systematic errors between our fit orbits and the IGS orbits are absorbed quite well by using our model.

Figure 3 shows the mean RMS of the orbit differences. The RMS is less than 3 cm in each direction and 3D RMS is less than 4 cm for each satellite, which is similar with the current precision of the IGS final orbits. Figure 4 shows the mean differences of the distance from satellite to the Earth's center (3D range), where we see that all of them are smaller than 4 cm.

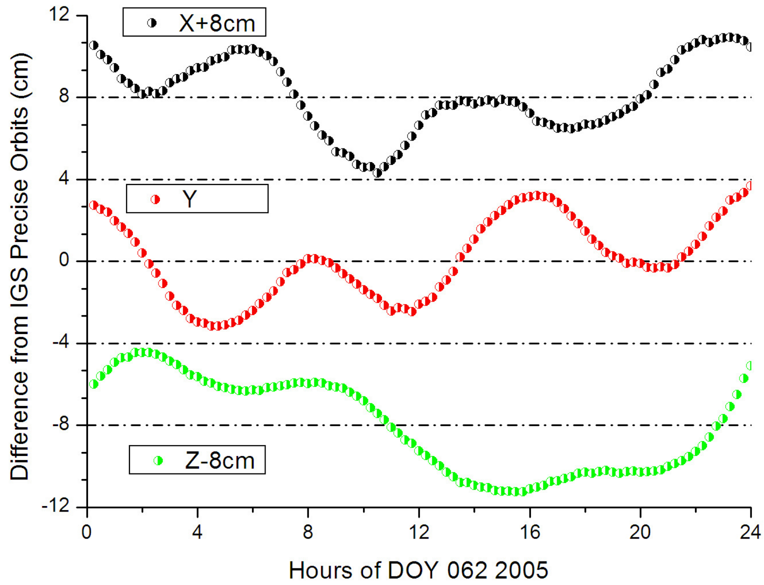

Fig.1 Orbit differences between the fit orbits and the IGS orbits (PRN 01)

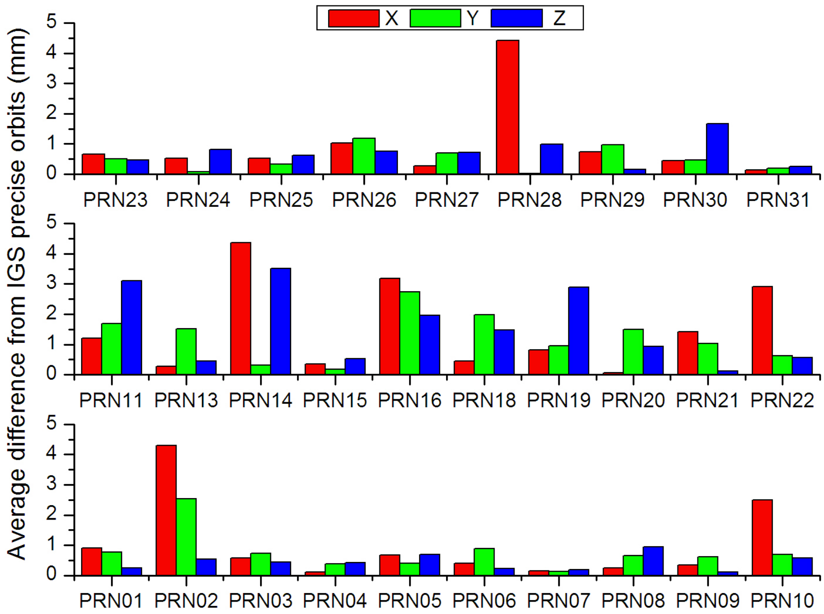

Fig.2 Mean differences between the fit orbits and the IGS orbits

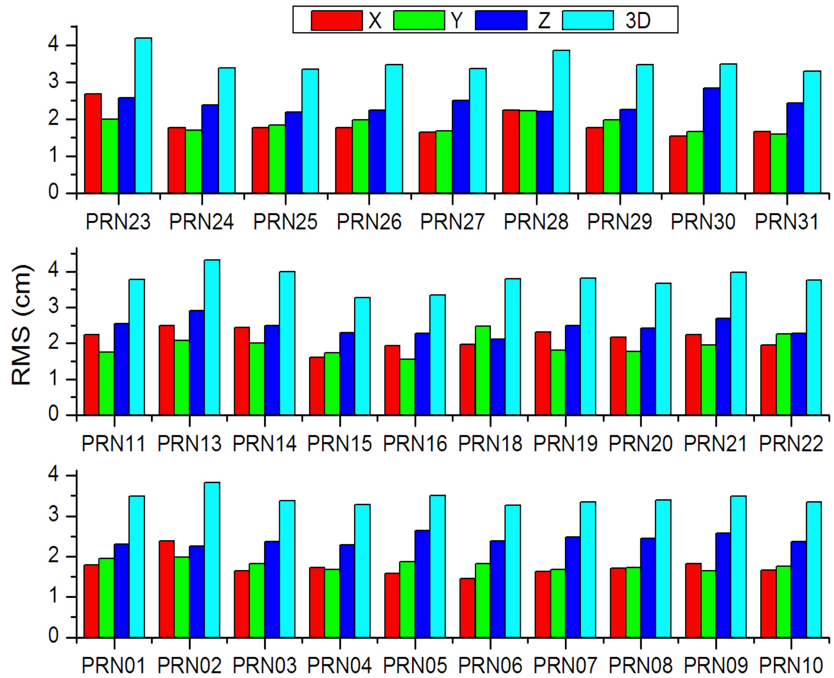

Fig.3 Mean RMS of the differences between the fit orbits and the IGS orbits

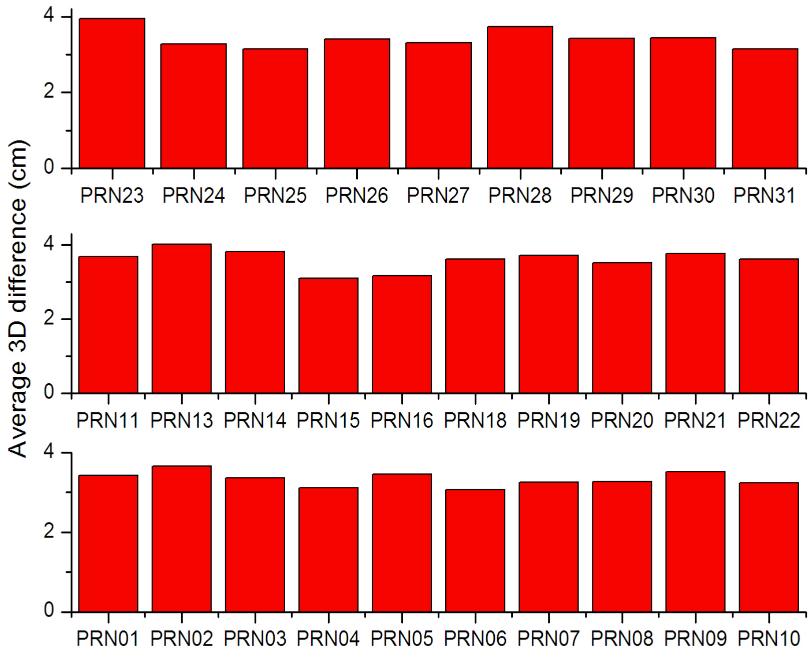

Fig.4 Mean 3D differences between the fit orbits and the IGS orbits

4. Conclusions

As orbit fitting performed for GPS satellites shows, the proposed model achieves orbits with similar precision as IGS final orbits.

Orbit fitting with other Helmert parameter settings was carried out also, and we got similar results as what shown already. In our approach, the fit orbits retain the dynamic properties of the satellite and the sampling of the fit orbits depends on the integration interval (we set it to 9.375 seconds). With the smoothing variation of satellites' orbits within this short period, the misfit problem of the interpolation methods is solved.

The fit orbits derived from our research have obvious periodicities, which follows, in general, the revolution periods of the GPS satellites. This may be due to some dynamic models' deficiency, e.g., SRP model. To better understand the reason, further investigations are needed.

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基于 Helmert 变换的 GPS 动力学轨道平滑

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摘要：精密点定位(PPP)理论的提出以及应用，得益于 GPS 轨道以及钟差产品精度的提高。目前 IGS 公布的 GPS 最终轨道精度已经优于 5cm,钟差的精度优于 0.1ns(3cm)。PPP 一般的做法是固定 IGS 或者其数据分析中心提供的轨道以及钟差参数,从而为测站定位提供参考框架。IGS 提供的最终精密星历的采样率为 15 分钟，远大于一般的观测数据采样率(30 秒)。因而需要采用一定的算法得到观测时刻 GPS 卫星的轨道。目前，很多基于内插原理的算法，例如：切比雪夫多项式、滑动式内插、线性插值等，被提了出来。大量文献显示，这些算法在精密星历采样的时刻吻合较好。

内插的算法不考虑卫星运动的动力学特性。从卫星的运动方程出发，本文提出基于 Helmert 变换的动力学轨道平滑的方法。方法首先基于卫星运动方程得出在给定初始状态下的动力学积分轨道，然后在精密星历采样时刻建立动力学积分轨道与精密星历之间的 Helmert 变换模型，从而实现动力学轨道平滑。计算表明，该算法得到的动力学平滑轨道具有与 IGS 精密星历相当的精度。

关键词：精密点定位，IGS 精密星历，轨道拟合，Helmert 变换