Reduced-dynamic Precise Orbit Determination for Low Earth Orbiters Based on Helmert Transformation

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ABSTRACT: A model based on Helmert transformation is presented in reduced-dynamic Precise Orbit Determination(POD). As an implementation, a reduced-dynamic POD approach was developed. The approach includes two steps: firstly, Kinematic POD and then reduced-dynamic POD. Based on the approach, a set of programs were developed. POD of CHAMP and GRACE was then carried out. Kinematic and reduced-dynamic POD for CHAMP and GRACE satellite over 2 weeks time show that reduced-dynamic orbits of CHAMP have an average 3D RMS of 0.26m compared to PSO orbit of GFZ, and the average 3D RMS of GRACE-A has the same value compared to GNV1B orbit of JPL. The 3D RMS is reduced by around 30% compared with kinematic solutions. The average RMS of the differences in the axis X, Y and Z is (0.14, 0.14, 0.16)m and (0.17, 0.15, 0.13)m for CHAMP and GRACE, respectively.

Keywords: Reduced-dynamic precise Orbit Determination, Helmert transformation, GPS, LEO

1. Introduction

In last years, several gravity satellite missions like CHAMP(**CHA**llenging **M**inisatellite **P**ayload) and GRACE(**G**ravity **R**ecovery **A**nd **C**limate **E**xperiment) were launched, a lot of scientific work of precise orbit determination has been carried out since then (Reigber et al(eds.),2003). To summarize, the POD method can be divided into dynamic method and kinematic method according to the theory and observations it used (Svehla, 2003; Zhu et al, 2004; Chen, 2007). In the dynamic method, the orbit precision mainly depends on initial orbit and dynamic models in satellite motion equation. Benefit from efforts of the International Earth Rotation and Reference Service (IERS) and other communities, dynamic models improve significantly in recent years. In the kinematic method, main challenge is how to reduce influence of weak geometry and phase breaks. Therefore data screening is one of the most important works in kinematic POD (Bock, 2003).

In reduced-dynamic method, the kinematic orbits may be used as pseudo-observations (Beutler, 2004; Bock, 2003), i.e. using kinematic precise orbits to correct dynamic parameters. The mathematical model can be treated simply as Gause-Markov procedure. The main problem of this approach is that the errors of kinematic orbits are accumulated directly into dynamic models. Therefore, reduced-dynamic orbits severely depend on kinematic orbits. To solve this problem, we suggest a model based on Helmert transformation, which connects the dynamic integrated orbits and the kinematic orbits. Using this model, the errors of kinematic orbits can be partly absorbed and therefore the dependence of kinematic orbits of reduced-dynamic POD can be reduced. Based on this model, reduced-dynamic POD of GPS, CHAMP and GRACE satellite were carried out. Results show that 3D RMS of residuals (compared with reference orbits) can be reduced by around 30% compared to kinematic solutions.

2. Orbit integration

According to dynamic POD theory, the satellite motion equation and satellites' initial orbits at epoch t_0 can be written as,

$$\begin{cases} \dot{x} = F(x,t) \\ x\big|_{t_0} = x_0 \end{cases} \tag{1}$$

Where, $x_0 = \begin{pmatrix} r_0 & \dot{r}_0 & p_0 \end{pmatrix}^T$ are initial orbits including positions, velocities and dynamic parameters (e.g. the solar radiation pressure parameters) of the satellite. $F(x,t)$ is the modeling equation of the complete set of forces acting on an orbiting satellite (Beutler,2004; McCarthy and Petit (eds.),2004). With proper integration method such as Adams-Cowell numerical integration, dynamic integrated orbits x^* can be computed based on x_0.

In equation (1), we can define $\delta = x - x^*$. Based on the Taylor expansion, we get

$$\dot{\delta} = \left.\frac{\partial F(x,t)}{\partial x}\right|_{*} \delta \tag{2}$$

The solution of (2) can be expressed as,

$$\delta = \Psi(t,t_0)\delta_0 \tag{3}$$

Where, $\delta_0 = x_0 - x^*$ is the orbit corrections at initial epoch t_0. Substitute (3) into (2), we have the following equation,

$$\begin{cases} \dot{\Psi}(t,t_0) = \dfrac{\partial F}{\partial x}\Psi(t,t_0) \\ \Psi(t_0,t_0) = I \end{cases} \tag{4}$$

Where, I is the unit matrix, $\Psi(t,t_0)$ is called transition matrix. It can be expressed in detail as,

$$\Psi\left(t,t_0\right)=\begin{pmatrix} \frac{\partial r}{\partial r_0} & \frac{\partial r}{\partial \dot{r}_0} & \frac{\partial r}{\partial p} \\ \frac{\partial \dot{r}}{\partial r_0} & \frac{\partial \dot{r}}{\partial \dot{r}_0} & \frac{\partial \dot{r}}{\partial p} \\ \frac{\partial p}{\partial r_0} & \frac{\partial p}{\partial \dot{r}_0} & \frac{\partial p}{\partial p} \end{pmatrix} \tag{5}$$

From numeric integration we can get transition matrix as well as integrated orbits x^*.

3. Reduced-dynamic POD based on Helmert transformation

Helmert transformation is mostly used to express differences between reference frames (Boucher, et al, 2004). It considers the origin motions and frame rotations. It is also generally used by International GNSS Servise (IGS) community to remove the systematic differences between Analysis Centers (ACs). Considering the systematic differences, we can build up Helmert transformation between dynamic integrated orbits and kinematic orbits. At epoch t_i it can be expressed as,

$$\begin{pmatrix} X_K^i \\ Y_K^i \\ Z_K^i \end{pmatrix}=\begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix}+(1+K')R_1\left(\alpha'\right)R_2\left(\beta'\right)R_3\left(\gamma'\right)\begin{pmatrix} X_T^i \\ Y_T^i \\ Z_T^i \end{pmatrix} \tag{6}$$

Where, (X_K^i, Y_K^i, Z_K^i) are kinematic orbits. (X_T^i, Y_T^i, Z_T^i) are dynamic-integrated orbits in earth-fixed reference frame, which can be obtained using the following transformation (McCarthy and Petit (eds.),2004),

$$\begin{pmatrix} X_T^i \\ Y_T^i \\ Z_T^i \end{pmatrix}=Q(t_i)R(t_i)W(t_i)\begin{pmatrix} X_I^i \\ Y_I^i \\ Z_I^i \end{pmatrix} \tag{7}$$

Where, (X_I^i, Y_I^i, Z_I^i) are integrated orbits in inertial reference frame, $Q(t_i), R(t_i), W(t_i)$ are the matrices for precession, earth rotation and pole wobble, respectively.

We can rewrite the model as,

$$r_K^i = T + (1+K)R1\cdot R2\cdot r_I^i \tag{8}$$

Where, r_K^i, r_I^i are the denotation of kinematic orbit (in earth-fixed frame), dynamic orbit (in inertial frame), T, K represent translation and scale parameters of Herlmert transformation, $R1, R2$ are rotation matrices in equation (6) and (7). In $R2$, parameters contained are: earth pole x_p, y_p and the rates $\dot{x}_p, \dot{y}_p$, time parameter $UT1-UTC$ ($dUT1$) and the rate $d\dot{U}T1$.

The magnitude of x_p, y_p is less than 1" and the magnitude of $dUT1$ is less than 1 second (15" in angle), Magnitude of $\dot{x}_p, \dot{y}_p$ and $d\dot{U}T1$ are even smaller. Ignoring the effects of $\dot{x}_p, \dot{y}_p$ and $d\dot{U}T1$, equation (8) can be rewritten as,

$$\begin{pmatrix} X_K^i \\ Y_K^i \\ Z_K^i \end{pmatrix} = \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} + (1+K) R_1(\alpha) R_2(\beta) R_3(\gamma) R_{etc} \cdot \begin{pmatrix} X_I^i \\ Y_I^i \\ Z_I^i \end{pmatrix} \tag{9}$$

Where, R_{etc} is the residual matrix comparing equation (9) to equation (8) and,

$$\alpha = \alpha' + y_p, \quad \beta = \beta' + x_p, \quad \gamma = \gamma' + dUT1 \tag{10}$$

Rewrite equation (10) as following,

$$r_K^i = T + (1+K) R \cdot R_{etc} \cdot r_I^i \tag{11}$$

Where, T, K, R represent Helmert transformation parameters ($\Delta X, \Delta Y, \Delta Z, K, \alpha, \beta, \gamma$).

The linearization of equation (11) reads as,

$$r_K^i = r_{K,0}^i + \frac{\partial r_K^i}{\partial v} dv \tag{12}$$

Parameters to be estimated are expressed in equation (13), where $dHel$ are corrections of Helmert transformation parameters, dr_I^i are corrections of integrated orbits at current epoch.

$$dv = (dHel, dr_I^i)^T \tag{13}$$

Considering equation (3) and (4), we can transform the parameter dr_I^i to initial orbit corrections dr_0 using equation (14).

$$dr_I^i = \Psi(t_i, t_0) dr_0 \tag{14}$$

Therefore the final parameters can be expressed as:

$$dx = (dHel, dr_0)^T \tag{15}$$

Design matrix is:

$$A_i = \frac{\partial r_K^i}{\partial x} = \left(\frac{\partial r_K^i}{\partial Hel}, \frac{\partial r_K^i}{\partial r_I^i} \cdot \Psi(t_i, t_0) \right) \tag{16}$$

Equation (12) can be formed at each epoch t_i. The solution of these equations may use the classic Least Square Estimation (LSE).

4. Data processing

To implement the reduced-dynamic method introduced above, a set of programs are developed. Data processing procedure contains the following two parts:

- Kinematic orbit determination and,
- Reduced-dynamic orbit determination

The procedure can runs in iterations. It starts with kinematic orbit determination without any a priori orbits. Afterwards, reduced-dynamic orbit determination is performed to provide reduced-dynamic orbits. The reduced-dynamic orbits can be later used in data preprocessing of kinematic orbit determination during iterations. Applying this procedure, POD of CHAMP and GRACE were performed.

4.1. CHAMP POD

Onboard GPS observations and accelerometer measurements in Day of Year (DoY) 191, 2004, were used. Kinematic orbits were first derived with an RMS of about 0.38m compared to GFZ Post-processed Science Orbit (PSO). Dynamic models and parameters settings are listed in table 1.

Tab.1 Dynamic models of CHAMP

Gravity model	EIGEN-CHAMP03S (120*120)
Tide	Solid Earth Tide,Ocean Tide CSR 3.0
N-body	JPL ephemeris DE405
Accelerometer data	ACC File(official bias and scale parameter)
Empirical parameter	9 parameters per arc-pass

The performance of different parameter sets were studied first. Different sets of Hermert parameters are listed in table 2, where the first case (Ex.1) is conventional orbit fitting (e.g. D. Svehla, 2003; H. Bock,2003). Figure 1 shows the RMS of the final reduced-dynamic orbits compared to GFZ PSO.

Tab.2 Parameter settings, with × indicates that the parameter is set up

Parameter	Ex. 1	Ex. 2	Ex. 3	Ex. 4	Ex. 5	Ex. 6	Ex. 7	Ex. 8
T		×		×		×		×
K			×	×			×	×
R					×	×	×	×

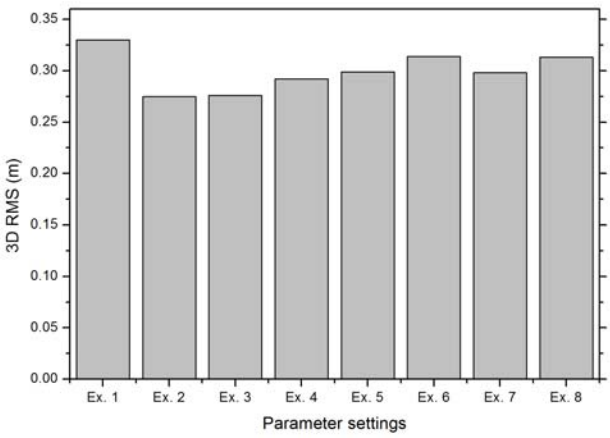

Fig.1 RMS of orbits difference between CHAMP reduced-dynamic orbits and GFZ PSO, under different parameter settings

As can be seen from Figure 1, the precision of reduced-dynamic orbits has been improved compared to kinematic orbits. This is due to the implementation of empirical parameters. We obtain better orbits with the parameter settings in case 2 to case 7, where in case 2 and case 3 the orbits precision is the best. The results are similar in case 5 and case 7, case 6 and case 8, which means that the scale parameter is not sensitive in our model. Figure 2 shows the orbits difference between our final reduced-dynamic orbits and PSO in case 2. The RMS is 0.27 m, which is reduced by 29% compared to kinematic orbits. RMS in axis X, Y and Z is (0.16,0.13,0.17)m, and the average of orbits difference is (1.1,1.1,0.02)cm.

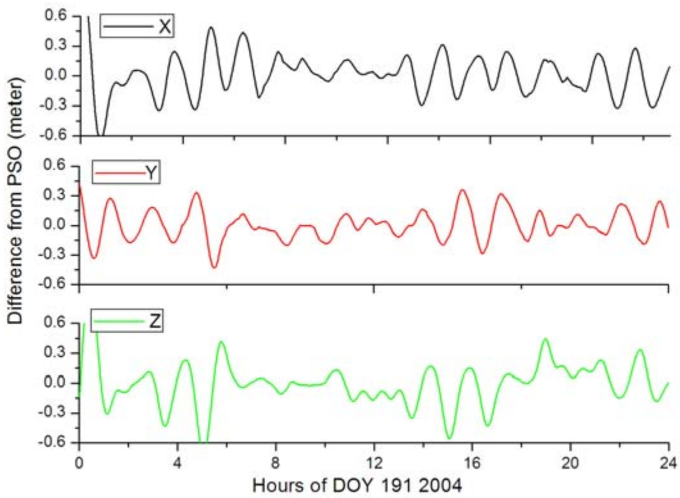

Fig.2 Difference between CHAMP reduced-dynamic orbits and GFZ PSO, 3DRMS=0.27m

4.2. GRACE POD

GRACE-A onboard GPS observations and accelerometer measurements in DoY 094, 2003, were used. Kinematic orbits were first derived with an RMS of about 0.39m compared to GNV1B orbits of JPL. Gravity model used is EIGEN-GRACE02S, and other dynamic parameters are the same as in table 1.

Under the parameter setting case 2 (Ex.2) in table 2, reduced-dynamic POD for GRACE was carried out. Figure 3 shows the difference between reduced-dynamic orbits and GNV1B orbits. The RMS is 0.26 m, which is reduced by 33% compared to kinematic orbits. RMS in axis X, Y and Z is (0.14, 0.14, 0.16)m, and the average of orbits difference is (0.1,1.0,0.2)cm.

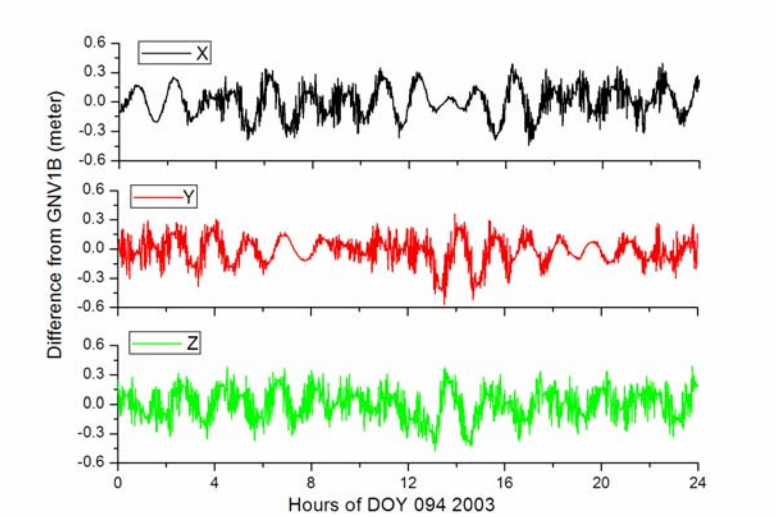

Fig.3 Difference between GRACE reduced-dynamic orbits and GNV1B orbits,

3DRMS=0.26m

4.3. Further validation

As a further investigation of our model, data of CHAMP and GRACE-A from DoY 124 to DoY 137, 2003 were processed. Figure 4 and 5 show the RMS statistic of the reduced-dynamic orbits compared to reference orbits.

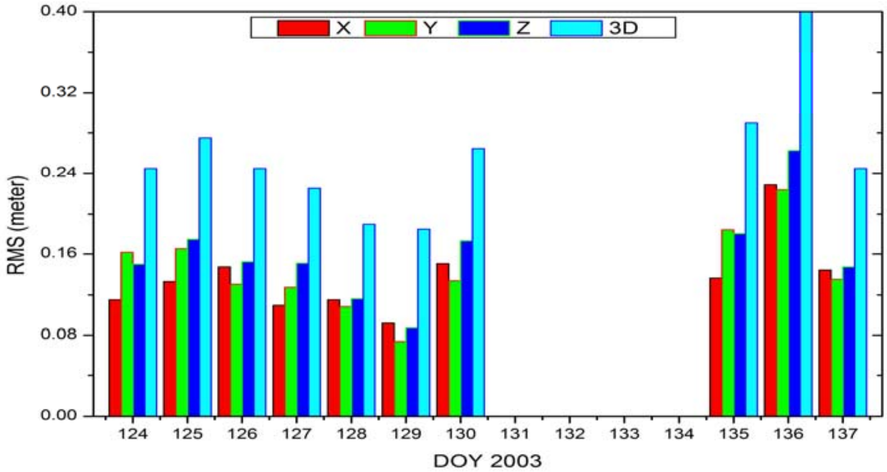

Fig.4 RMS of the difference between CHAMP Reduced-dynamic orbits and PSO

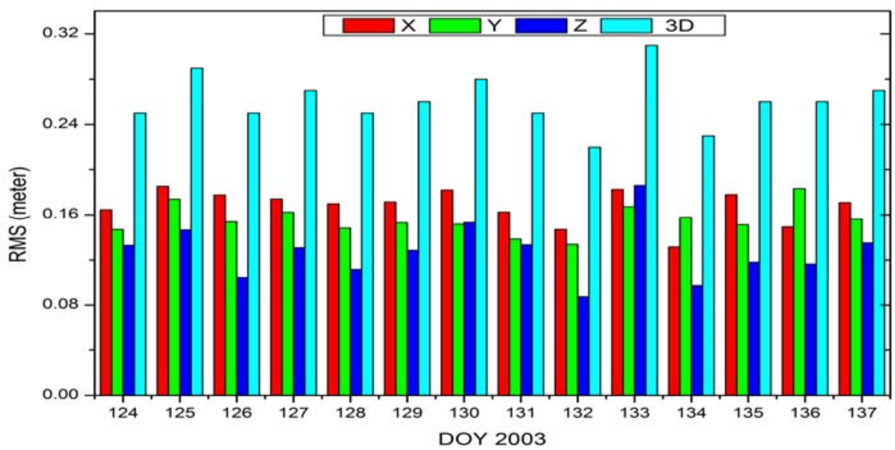

Fig.5 RMS of the difference between GRACE Reduced-dynamic orbits and GNV1B

The comparison for CHAMP during DoY 131-134 are not performed, because the GFZ PSO are missing in these days.

As can be seen from figure 4 and figure 5, the RMS of orbits difference in each axis is in the range from 6 cm to 18 cm. In DoY 136, biggest RMS of CHAMP orbit is seen. At the same day, we also see the biggest RMS in CHAMP kinematic orbits. The average 3DRMS of both CHAMP and GRACE orbits is 0.26 m, and the average RMS in axis X, Y and Z are (0.14,0.14,0.16)m and (0.17,0.15,0.13)m, respectively.

5. Conclusions

As POD performed for CHAMP and GRACE shows, the proposed model reduced the observing errors of kinematic orbits and therefore improves the orbits precision by around 30%.

The investigation of different parameter settings of our model shows that the introduction of Hermert transformation parameters improves orbits precision, and orbits achieve the highest precision under the parameter setting with only transformation parameters being estimated. Test of our model also shows that the scale parameter has no influence on orbits when rotation parameters are estimated.

The reduced-dynamic orbits derived from our research have obvious periodicities, which follow the revolution periods of the satellites. This is mostly due to the errors in accelerometer measurements, which are corrected using the official calibration parameters. By adding calibration parameters of accelerometer measurements in our approach, the performance of our model will be further improved.

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