

# **Global Terrestrial Reference Systems and Frames: Application to the International Terrestrial Reference System/Frame**

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# OUTLINE

- **What is a Terrestrial Reference System (TRS), why is it needed and how is it realized ?**
- **Concept and Definition**
- **TRS Realization by a Frame (TRF)**
- **International Terrestrial Reference System (ITRS) and its realization: the International Terrestrial Reference Frame (ITRF)**
- **ITRF2008 Geodetic & Geophysical Results**
- **How to access the ITRF ?**
- **GNSS associated reference systems and their relationship to ITRF:**
  - **World Geodetic System (WGS84)**
  - **Galileo Terrestrial Reference Frame (GTRF)**

# Defining a Reference System & Frame:

## Three main conceptual levels :

- **Ideal Terrestrial Reference System (TRS):**  
Ideal, mathematical, theoretical system
- **Terrestrial Reference Frame (TRF):**  
Numerical realization of the TRS to which users have access
- **Coordinate System:** cartesian (X,Y,Z), geographic ( $\lambda, \phi, h$ ),  
...
  - The TRF is a materialization of the TRS inheriting the mathematical properties of the TRS
  - As the TRS, the TRF has an **origin, scale & orientation**
  - TRF is constructed using space geodesy observations



# Ideal Terrestrial Reference System

A tridimensional reference frame (mathematical sense)  
Defined in an Euclidian affine space of dimension 3:

Affine Frame (O,E) where:

O: point in space (**Origin**)

E: vector base: orthogonal with the same length:

- vectors co-linear to the base (**Orientation**)
- unit of length (**Scale**)

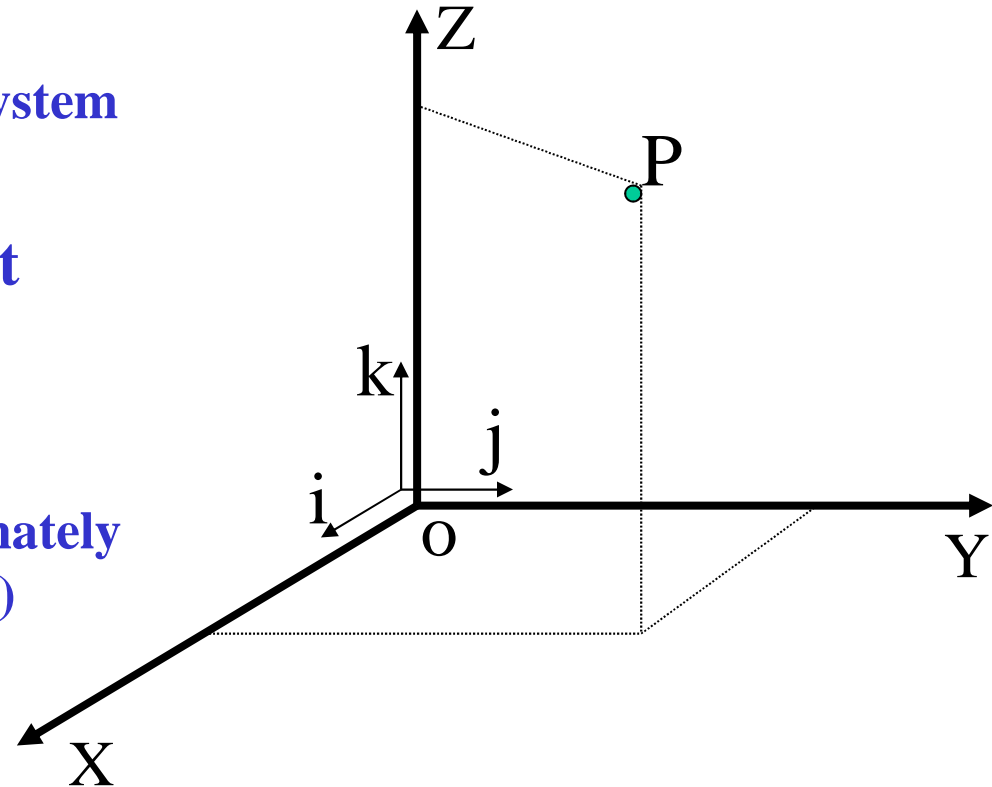
$$\lambda = \|\vec{E}_i\|_{i=1,2,3}$$

$$\vec{E}_i \cdot \vec{E}_j = \lambda^2 \delta_{ij}$$

$$(\delta_{ij} = 1, \quad i = j)$$

# Terrestrial Reference Frame in the context of space geodesy

- **Origin:**
  - Center of mass of the Earth System
- **Scale (unit of length): SI unit**
- **Orientation:**
  - Equatorial (Z axis is approximately the direction of the Earth pole)



# Transformation between TRS (1/2)

7-parameter similarity:

$$\boxed{X_2 = T + \lambda \cdot \mathcal{R} \cdot X_1}$$

Translation Vector  $T = (T_x, T_y, T_z)^T$

Scale Factor  $\lambda$

Rotation Matrix  $\mathcal{R} = R_x \cdot R_y \cdot R_z$

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos R1 & \sin R1 \\ 0 & -\sin R1 & \cos R1 \end{pmatrix}$$

$$R_y = \begin{pmatrix} \cos R2 & 0 & -\sin R2 \\ 0 & 1 & 0 \\ \sin R2 & 0 & \cos R2 \end{pmatrix}$$

$$R_z = \begin{pmatrix} \cos R3 & \sin R3 & 0 \\ -\sin R3 & \cos R3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

## Transformation between TRS (2/2)

In space geodesy we use the linearized formula:

$$X_2 = X_1 + T + DX_1 + R.X_1 \quad (1)$$

with:  $T = (T_x, T_y, T_z)^T$ ,  $\lambda = (1 + D)$ , and  $\mathcal{R} = (I + R)$

where  $R = \begin{pmatrix} 0 & -R3 & R2 \\ R3 & 0 & -R1 \\ -R2 & R1 & 0 \end{pmatrix}$

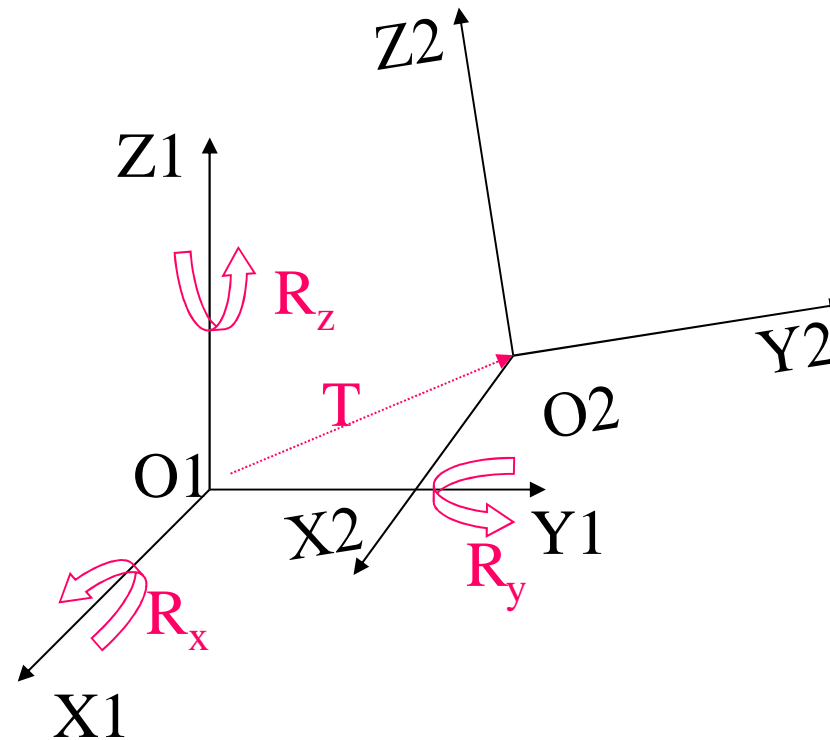
since  $T$  is less than 100 meters,  $D$  &  $R$  less than  $10^{-5}$

The terms of 2nd ordre are neglected: less than  $10^{-10} \approx 0.6$  mm.

Differentiating equation 1 with respect to time, we have:

$$\dot{X}_2 = \dot{X}_1 + \dot{T} + \overbrace{D\dot{X}_1}^{\approx 0} + \dot{D}X_1 + \overbrace{R\dot{X}_1}^{\approx 0} + \dot{R}X_1$$

## From one RF to another ?

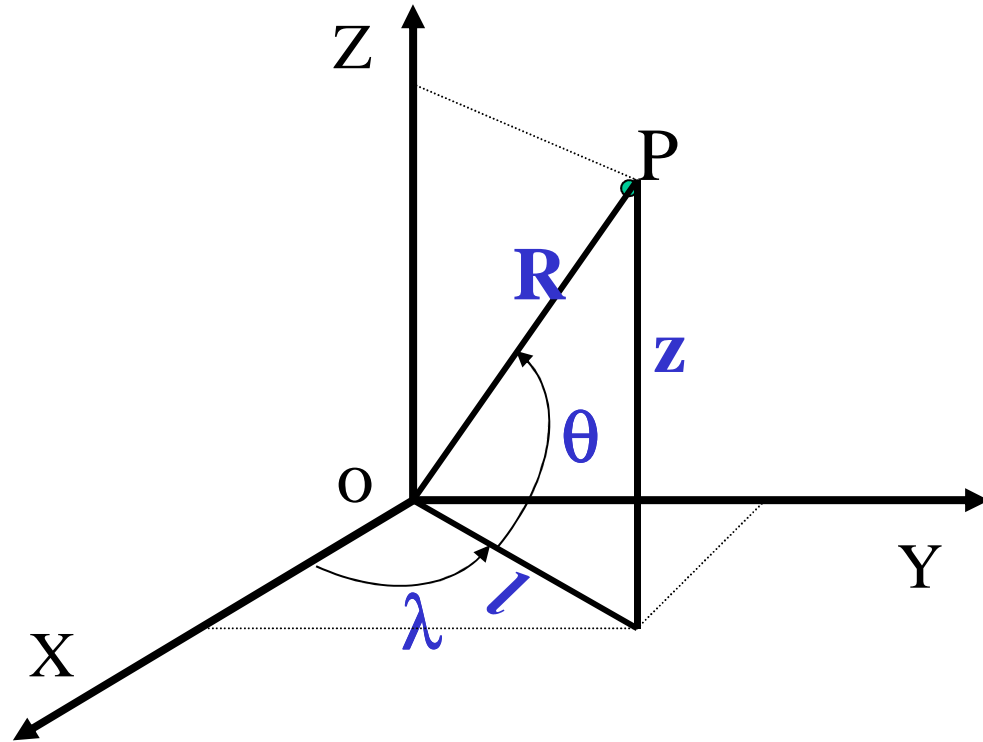


$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_2 = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_1 + \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix} + \begin{pmatrix} D & -R_z & R_y \\ R_z & D & -R_x \\ -R_y & R_x & D \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_1$$



# Coordinate Systems

- Cartesian:  $X, Y, Z$
- Ellipsoidal:  $\lambda, \varphi, h$
- Mapping:  $E, N, h$
- Spherical:  $R, \theta, \lambda$
- Cylindrical:  $l, \lambda, Z$



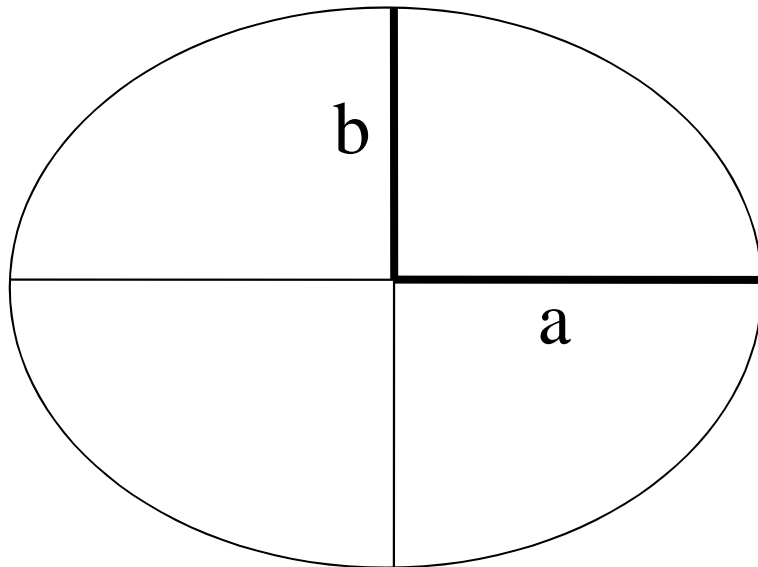
$$OP \begin{cases} l \cos \lambda \\ l \sin \lambda \\ z \end{cases}$$

Cylindrical

$$OP \begin{cases} R \cos \theta \cos \lambda \\ R \cos \theta \sin \lambda \\ R \sin \theta \end{cases}$$

Spherical

# Ellipsoidal and Cartesian Coordinates: Ellipsoid definition



**a: semi major axis**

**b: semi minor axis**

**f: flattening**

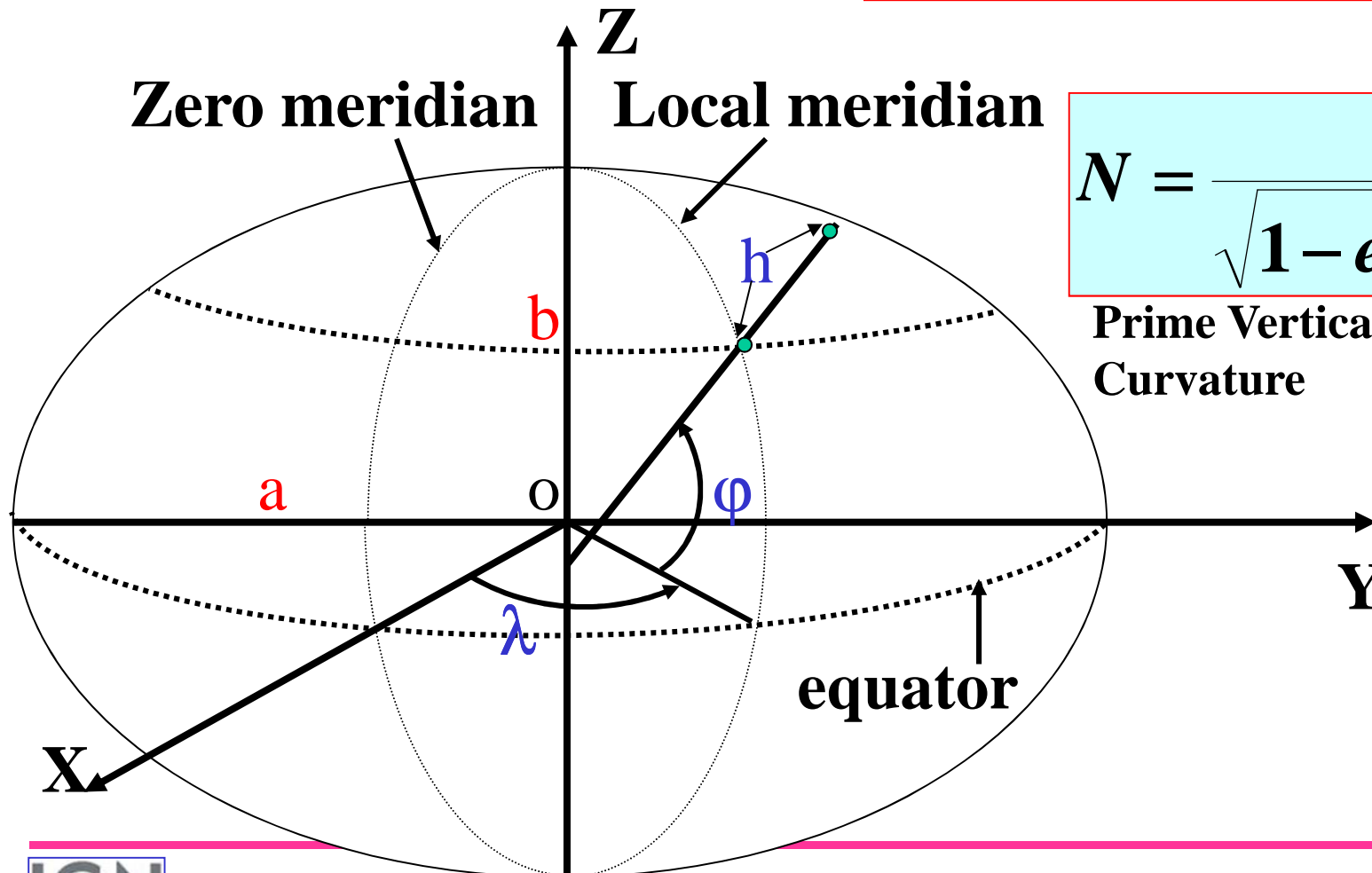
**e: eccentricity**

$$e^2 = \frac{a^2 - b^2}{a^2}, \quad f = \frac{a - b}{a}$$

**(a,b), (a,f ), or (a,e<sup>2</sup>) define entirely and geometrically the ellipsoid**

# Ellipsoidal and Cartesian Coordinates

$$X = (N + h) \cos \lambda \cos \varphi$$
$$Y = (N + h) \sin \lambda \cos \varphi$$
$$Z = [N(1 - e^2) + h] \sin \varphi$$



$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi}}$$

Prime Vertical Radius of Curvature

$$(X, Y, Z) \implies (\lambda, \varphi, h)$$

$$f = 1 - \sqrt{1 - e^2}$$

$$R = \sqrt{X^2 + Y^2 + Z^2}$$

$$\lambda = \operatorname{arctg}\left(\frac{Y}{X}\right)$$

$$\mu = \operatorname{arctg}\left[\frac{Z}{\sqrt{X^2 + Y^2}} \left( (1 - f) + \left(\frac{e^2 a}{R}\right) \right)\right]$$

$$\varphi = \operatorname{arctg}\left[\frac{Z(1 - f) + e^2 a \sin^3 \mu}{(1 - f)[X^2 + Y^2 - e^2 a \cos^3 \mu]}\right]$$

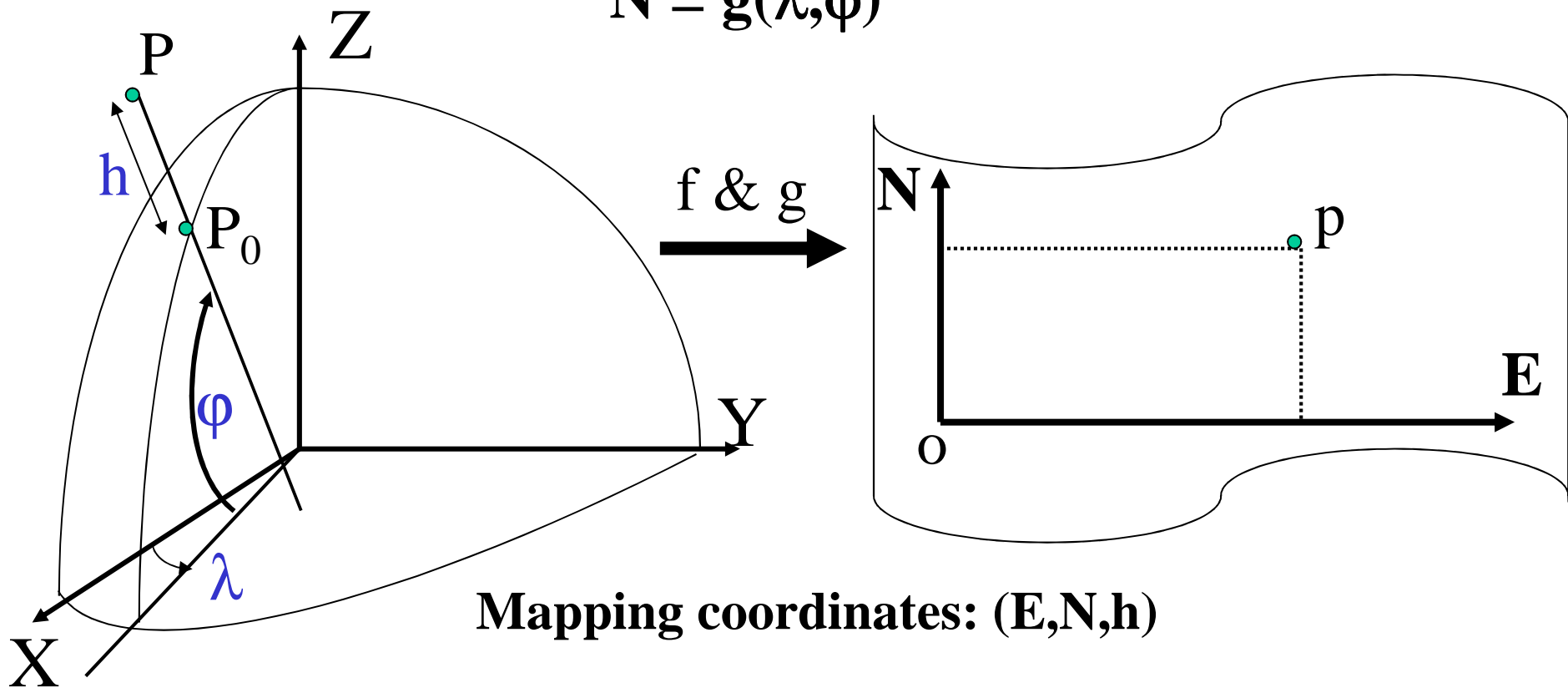
$$h = \sqrt{X^2 + Y^2} [\cos \varphi + Z \sin \varphi] - a \sqrt{1 - e^2 \sin^2 \varphi}$$

# Map Projection

Mathematical function from an ellipsoid to a plane (map)

$$\mathbf{E} = \mathbf{f}(\lambda, \varphi)$$

$$\mathbf{N} = \mathbf{g}(\lambda, \varphi)$$

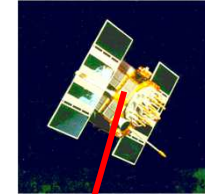


# Why a Reference System/Frame is needed?

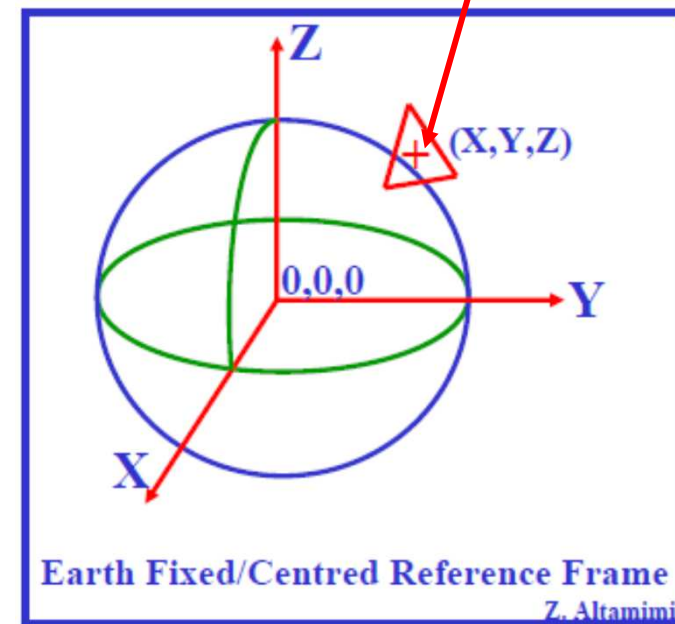
- **Precise Orbit Determination for:**
  - **GNSS: Global Navigation Satellite Systems**
  - **Other satellite missions: Altimetry, Oceanography, Gravity**
- **Earth Sciences Applications**
  - **Tectonic motion and crustal deformation**
  - **Mean sea level variations**
  - **Earth rotation**
  - ...
- **Geo-referencing applications**
  - **Navigation: Aviation, Terrestrial, Maritime**
  - **National geodetic systems**
  - **Cartography & Positioning**

# What is a Reference Frame in practice?

- **Earth fixed/centred RF: allows determination of station location/position as a function of time**
- It seems so simple, but ... we have to deal with:
  - Relativity theory
  - Forces acting on the satellite
  - The atmosphere
  - Earth rotation
  - Solid Earth and ocean tides
  - Tectonic motion
  - ...
- **Station positions and velocities are now determined with mm and mm/yr precision**



**Origin, Scale & Orientation**



# **"Motions" of the deformable Earth**

- **Nearly linear motion:**
  - **Tectonic motion: horizontal**
  - **Post-Glacial Rebound: Vertical & Horizontal**
  
- **Non-Linear motion:**
  - **Seasonal: Annual, Semi & Inter-Annual caused by loading effects**
  - **Rupture, transient: uneven motion caused by Earthquakes, Volcano Eruptions, etc.**



# Crust-based TRF

The instantaneous position of a point on Earth Crust at epoch  $t$  could be written as :

$$X(t) = X_0 + \dot{X} \cdot (t - t_0) + \sum_i \Delta X_i(t)$$

- $X_0$  : point position at a reference epoch  $t_0$   
 $\dot{X}$  : point linear velocity  
 $\Delta X_i(t)$  : high frequency time variations:
- Solid Earth, Ocean & Pole tides
  - Loading effects: atmosphere, ocean, hydrology, Post-glacial-Rebound
  - ... Earthquakes

# Reference Frame Representations

- "Quasi-Instantaneous" Frame: mean station positions at "short" interval:
  - One hour, 6-h, 12-h, one day, one week
  - ==> **Non-linear motion embedded in time series of quasi-instantaneous frames**
- **Long-Term Secular Frame: mean station positions at a reference epoch ( $t_0$ ) and station velocities:  $X(t) = X_0 + V^*(t - t_0)$**

# Implementation of a TRF

- Definition at a given epoch, by selecting 7 parameters, tending to satisfy the theoretical definition of the corresponding TRS
- A law of time evolution, by selecting 7 rates of the 7 parameters, **assuming linear station motion!**
- $\implies$  14 parameters are needed to define a TRF

## How to define the 14 parameters ? « TRF definition »

- **Origin & rate: CoM (Satellite Techniques)**
  - **Scale & rate: depends on physical parameters**
  - **Orientation: conventional**
  - **Orient. Rate: conventional: Geophysical meaning (Tectonic Plate Motion)**
- 
- **==> Lack of information for some parameters:**
    - **Orientation & rate (all techniques)**
    - **Origin & rate in case of VLBI**
    - **==> Rank Deficiency in terms of Normal Eq. System**

# Implementation of a TRF in practice

The normal equation constructed upon observations of space techniques is written in the form of:

$$N.(\Delta X) = K \quad (1)$$

where  $\Delta X = X_{est} - X_{apr}$  are the linearized unknowns

Eq. (1) is a singular system: has a rank deficiency equal to the number of TRF parameters not given by the observations.

Additional constraints are needed:

- Tight constraints  $(\sigma \leq 10^{-10})$  m
  - Removable constraints  $(\sigma \cong 10^{-5})$  m
  - Loose constraints  $(\sigma \geq 1)$  m
- Applied over station coordinates
- $(X_{est} - X_{apr}) = 0 \quad (\sigma)$

- **Minimum constraints** (applied over the TRF parameters, see next)

## TRF definition using minimum constraints (1/3)

The standard relation linking two TRFs 1 and 2 is:

$$X_2 = X_1 + A\theta$$

$$X_i = (x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i)^T$$

$$\theta = (T1, T2, T3, D, R1, R2, R3, \dot{T}1, \dot{T}2, \dot{T}3, \dot{D}, \dot{R}1, \dot{R}2, \dot{R}3)^T$$

$\theta$  is the vector of the 7 (14) transformation parameters

Least squares adjustment gives for  $\theta$  :

$$\theta = \overbrace{(A^T A)^{-1} A^T}^{\mathbf{B}} (X_2 - X_1)$$

**A** : design matrix of partial derivatives given in the next slide

# The Design matrix **A**

14 parameters

$$\begin{array}{c}
 \underbrace{\hspace{15em}}_{7 \text{ parameters}} \\
 \underbrace{\hspace{10em}}_{7 \text{ parameters}} \\
 A = \left( \begin{array}{cccccccccccc}
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 1 & 0 & 0 & x_i^0 & 0 & z_i^0 & -y_i^0 & & & & & \\
 0 & 1 & 0 & y_i^0 & -z_i^0 & 0 & x_i^0 & & 0 & & & \\
 0 & 0 & 1 & z_i^0 & y_i^0 & -x_i^0 & 0 & & & & & \\
 & & & & & & & & & & & \\
 & & & & & & & & & & & \\
 & & & & & & & & 1 & 0 & 0 & x_i^0 & 0 & z_i^0 & -y_i^0 \\
 & & & \approx 0 & & & & & 0 & 1 & 0 & y_i^0 & -z_i^0 & 0 & x_i^0 \\
 & & & & & & & & 0 & 0 & 1 & z_i^0 & y_i^0 & -x_i^0 & 0 \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
 \end{array} \right)
 \end{array}$$

Note: **A** could be reduced to specific parameters. E.g. if only rotations and rotation rates are needed, then the first 4 columns of the two parts of **A** are deleted

## TRF definition using minimum constraints (2/3)

- The equation of minimum constraints is written as:

$$B(X_2 - X_1) = 0 \quad (\Sigma_\theta)$$

It nullifies the 7 (14) transformation parameters between TRF 1 and TRF 2 at the  $\Sigma_\theta$  level

- The normal equation form is written as:

$$B^T \Sigma_\theta^{-1} B(X_2 - X_1) = 0$$

$\Sigma_\theta$  is a diagonal matrix containing small variances of the 7(14) parameters, usually at the level of 0.1 mm



## TRF definition using minimum constraints (3/3)

Considering the normal equation of space geodesy:

$$N_{nc}(\Delta X) = K \quad (1)$$

where  $\Delta X = X_{est} - X_{apr}$  are the linearized unknowns

Selecting a reference solution  $X_R$ , the equation of minimal constraints is given by:

$$B^T \Sigma_\theta^{-1} B(\Delta X) = B^T \Sigma_\theta^{-1} B(X_R - X_{apr}) \quad (2)$$

Accumulating (1) and (2), we have:

$$(N_{nc} + B^T \Sigma_\theta^{-1} B)(\Delta X) = K + B^T \Sigma_\theta^{-1} B(X_R - X_{apr})$$

Note: if  $X_R = X_{apr}$ , the 2nd term of the right-hand side vanishes

## Combination of daily or weekly TRF solutions (1/3)

The basic combination model is written as:

$$X_s^i = X_c^i + T_s + D_s X_c^i + R_s X_c^i$$

Inputs:  $X_s^i$ , coordinates of point  $i$  of individual solution  $s$ .

Outputs (unknowns): combined coordinates  $X_c^i$  and transformation parameters  $T_s, D_s, R_s$  from TRF  $s$  to TRF  $c$ .

Note that the translation vector  $T_s$  and the rotation matrix  $R_s$  have each three components around the three axes  $X, Y, Z$ .

The unknown parameters are linearized around their approximate values:  $x_0^i, y_0^i, z_0^i$ , so that  $x_c^i = x_0^i + \delta x^i$  (respectively  $y_c^i, z_c^i$ ).

**Note:** this combination model is valid at a give epoch,  $t_s$ , for both the input and output station coordinates

## Combination of daily or weekly TRF solutions (2/3)

The observation equation system is written as:

$$\begin{pmatrix} I & A_s \end{pmatrix} \begin{pmatrix} \delta\chi_s \\ \delta T_s \end{pmatrix} + B_s = V_s$$

and the normal equation is:

$$\begin{pmatrix} P_s & P_s A_s \\ A_s^T P_s & A_s^T P_s A_s \end{pmatrix} \begin{pmatrix} \delta\chi_s \\ \delta T_s \end{pmatrix} + \begin{pmatrix} P_s B_s \\ A_s^T P_s B_s \end{pmatrix} = 0$$

where  $I$  is the identity matrix,  $A_s$  is the design matrix related to solution  $s$ ,  $\delta\chi_s$  and  $\delta T_s$  are the linearized unknowns of station coordinates and transformation parameters, respectively.  $B_s$  are the (observed - computed) values and  $V_s$  are the residuals.  $P_s$ : weight matrix =  $\Sigma_s^{-1}$ : inverse of variance-covariance matrix.

## Combination of daily or weekly TRF solutions (3/3)

The design matrix  $A_s$  has the following form:

$$A_s = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 0 & 0 & x_0^i & 0 & z_0^i & -y_0^i \\ 0 & 1 & 0 & y_0^i & -z_0^i & 0 & x_0^i \\ 0 & 0 & 1 & z_0^i & y_0^i & -x_0^i & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

## Definition of the combined TRF

- The normal equation system described in the previous slides is singular and has a rank deficiency of 7 parameters.
- The 7 parameters are the defining parameters of the combined TRF  $c$ : origin (3 components), scale (1 component) and orientation (3 components).
- The combined TRF  $c$ , could be defined by, e.g.:
  - Fixing to given values 7 parameters among those to be estimated
  - Using minimum constraint equation over a selected set of stations of a reference TRF solution  $X_R$ .  
Refer to slide 24 for more details...

## Combination of long-term TRF solutions

The basic combination model is extended to include station velocities and is written as:

$$\begin{cases} X_s^i = X_c^i + T_s + D_s X_c^i + R_s X_c^i \\ \dot{X}_s^i = \dot{X}_c^i + \dot{T}_s + \dot{D}_s X_c^i + \dot{R}_s X_c^i \end{cases}$$

where the dotted parameters are their time derivatives.

Inputs:  $X_s^i$ , position of point  $i$ , at epoch  $t_s$  and velocities,  $\dot{X}_s^i$ , of individual solution  $s$ .

Outputs: combined positions  $X_c^i$ , at epoch  $t_s$ , velocities and transformation parameters  $T_s, D_s, R_s$ , at epoch  $t_s$ , from TRF  $s$  to TRF  $c$ .

In the same way as for daily or weekly TRF combination, observation and normal equations could easily be derived.

**Note: this combination model is only valid at a give epoch, both for the input and output station coordinates**

## Stacking of TRF time series

The basic combination model is written as:

$$X_s^i = X_c^i(t_0) + (t_s - t_0)\dot{X} + T_s + D_s X_c^i + R_s X_c^i$$

Inputs: Time series of station positions,  $X_s^i$ , at different epochs  $t_s$ .

Outputs: combined positions  $X_c^i$  at epoch  $t_0$ , velocities and transformation parameters  $T_s, D_s, R_s$  from TRF  $s$  to TRF  $c$ .

**Here also, observation and normal equations are constructed and solved by least squares adjustment.**

# Space Geodesy Techniques

- **Very Long Baseline Interferometry (VLBI)**
- **Lunar Laser Ranging (LLR)**
- **Satellite Laser Ranging (SLR)**
- **DORIS**
- **GNSS: GPS, GLONASS, GALILEO, COMPASS,**  
...

- 
- **Local tie vectors at co-location sites**



# Complex of Space Geodesy instruments



**SLR/LLR**



**VLBI**



**GPS**



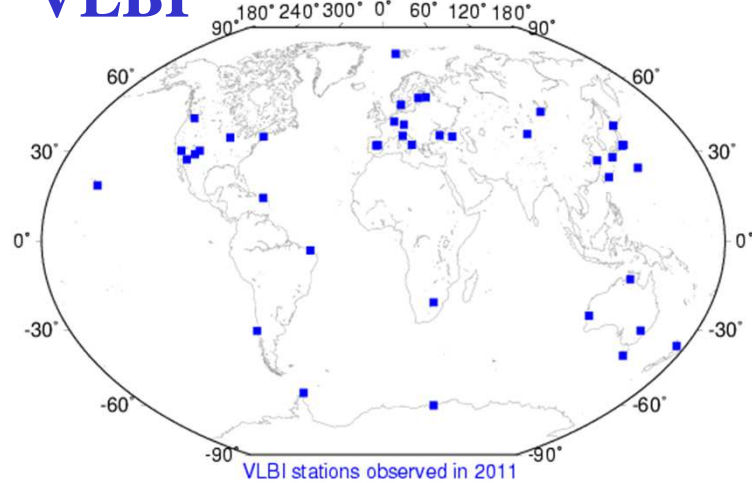
**DORIS**

## Reference frame definition by individual techniques

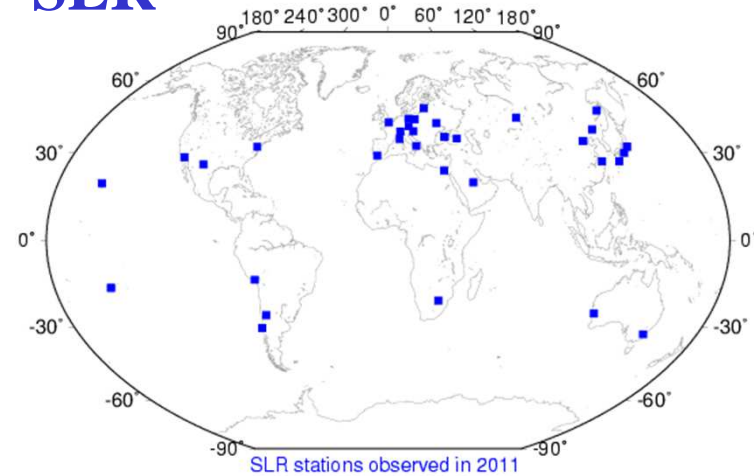
	<b>Satellite Techniques</b>	<b>VLBI</b>
<b>Origin</b>	<b>Center of Mass</b>	<b>-</b>
<b>Scale</b>	<b>GM, c &amp; Relativistic corrections</b>	<b>c Relativistic corrections</b>
<b>Orientation</b>	<b>Conventional</b>	<b>Conventional</b>

# Current networks: stations observed in 2011

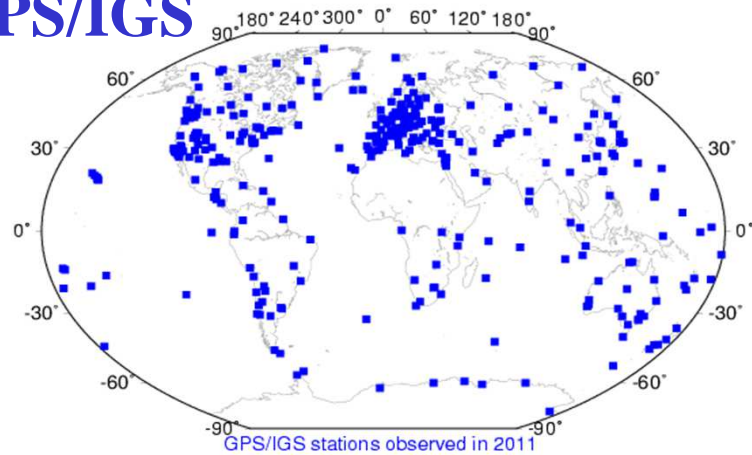
## VLBI



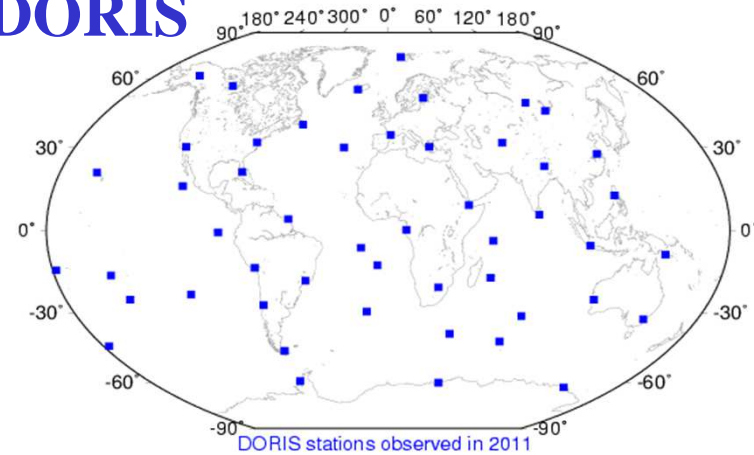
## SLR



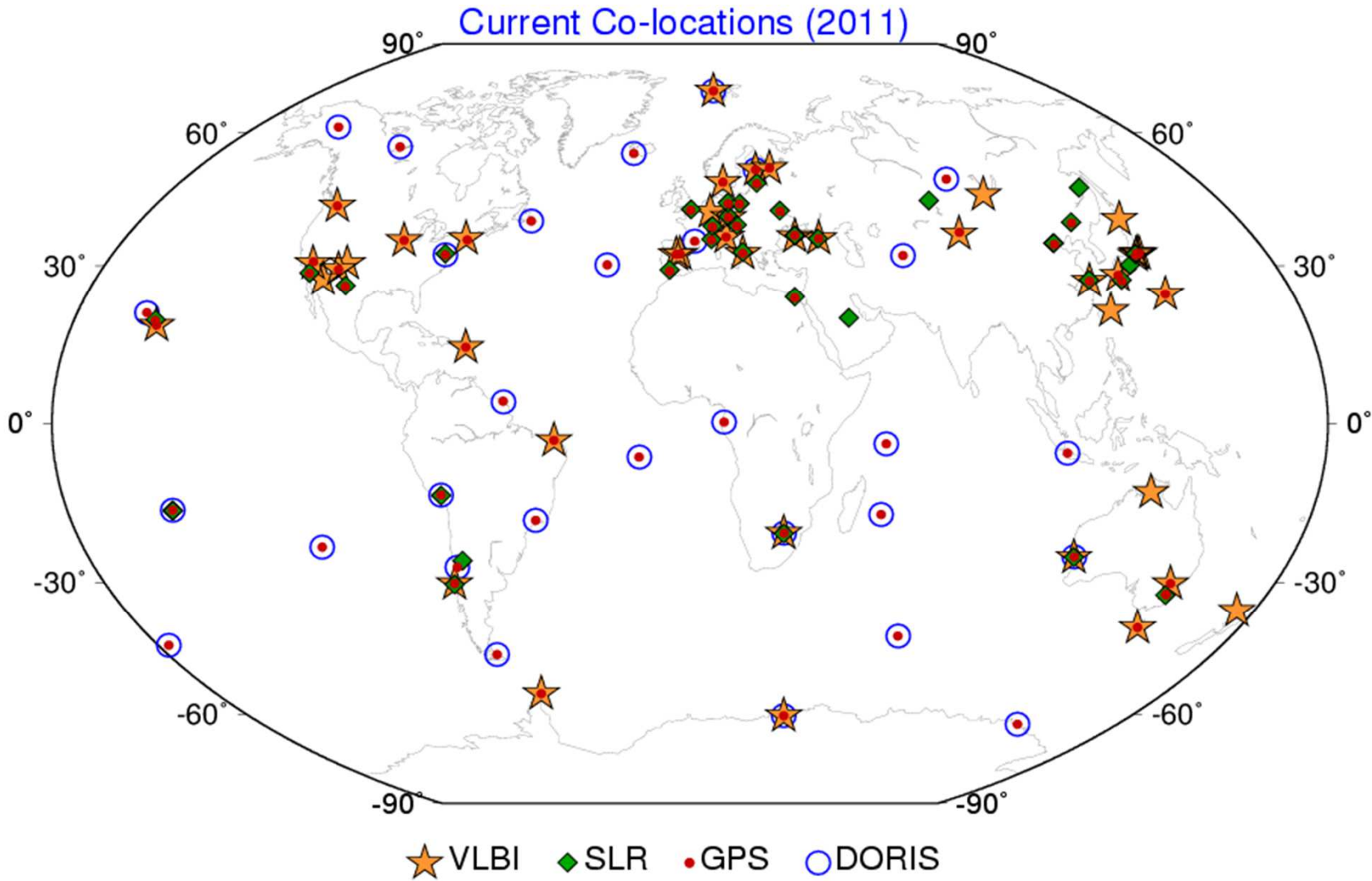
## GPS/IGS



## DORIS



# Current Co-locations (2011)



# **International Association of Geodesy International Services**

- **International Earth Rotation and Reference Systems Service (IERS) (1988)**
- **Intern. GNSS Service (IGS) (1994)**
- **Intern. Laser Ranging Service (ILRS) (1998)**
- **Intern. VLBI Service (IVS) (1999)**
- **Intern. DORIS Service (IDS) (2003)**

**<http://www.iag-aig.org/>**



# International Terrestrial Reference System (ITRS)

Realized and maintained by the IERS





## **International Earth Rotation and Reference Systems Service (IERS)**

**Established in 1987 (started Jan. 1, 1988) by IAU and IUGG  
to realize/maintain/provide:**

- **The International Celestial Reference System (ICRS)**
- **The International Terrestrial Reference System (ITRS)**
- **Earth Orientation Parameters (EOP)**
- **Geophysical data to interpret time/space variations in the ICRF, ITRF & EOP**
- **Standards, constants and models (i.e., conventions)**

<http://www.iers.org/>

# International Terrestrial Reference System (ITRS): Definition (IERS Conventions)

- **Origin:** Center of mass of the whole Earth, including oceans and atmosphere
- **Unit of length:** meter SI, consistent with TCG (Geocentric Coordinate Time)
- **Orientation:** consistent with BIH (Bureau International de l'Heure) orientation at 1984.0.
- **Orientation time evolution:** ensured by using a No-Net-Rotation-Condition w.r.t. horizontal tectonic motions over the whole Earth

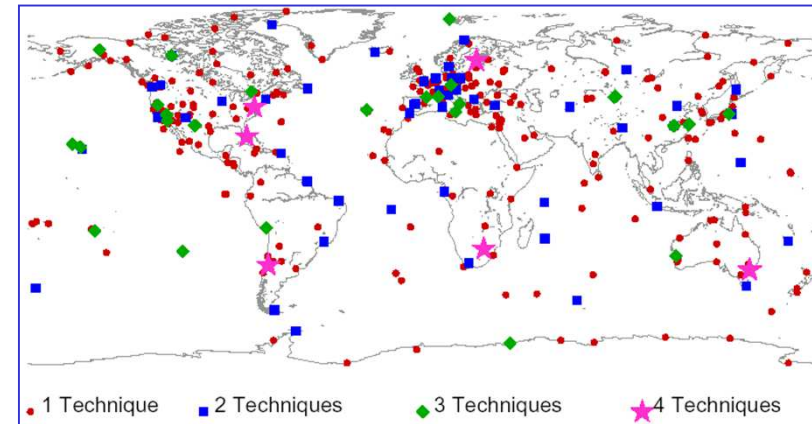
$$h = \int_C X \times V dm = 0$$



# International Terrestrial Reference System (ITRS)

- Realized and maintained by **ITRS Product Center** of the IERS
- Its Realization is called International Terrestrial Reference Frame (**ITRF**)
- Set of station positions and velocities, **estimated by combination** of VLBI, SLR, GPS and DORIS individual TRF solutions
- **Based on Co-location sites**

Adopted by IUGG in 1991 for all Earth Science Applications



More than 800 stations located on more than 500 sites

Available: **ITRF88, ..., 2000, 2005**

**Latest : ITRF2008**

<http://itrf.ign.fr>

# Co-location site

- Site where two or more instruments are operating
- Surveyed in three dimensions, using classical or GPS geodesy
- Differential coordinates (DX, DY, DZ) are available

$$\mathbf{DX}_{(\text{GPS,VLBI})} = \mathbf{X}_{\text{VLBI}} - \mathbf{X}_{\text{GPS}}$$

SLR/LLR



VLBI

GNSS



DORIS

# Strenghts :

## Contribution of Geodetic Techniques to the ITRF

Mix of techniques  
is fundamental to  
realize a frame that  
is stable in origin,  
scale, and with  
sufficient coverage

Technique Signal Source Obs. Type	<b>VLBI</b> Microwave Quasars Time difference	<b>SLR</b> Optical Satellite Two-way absolute range	<b>GPS</b> Microwave Satellites Range change	<b>DORIS</b>
<b>Celestial Frame &amp; UT1</b>	<b>Yes</b>	No	No	No
<b>Polar Motion</b>	<b>Yes</b>	<b>Yes</b>	<b>Yes</b>	Yes
<b>Scale</b>	<b>Yes</b>	<b>Yes</b>	No (but maybe in the future!)	<b>Yes</b>
<b>Geocenter ITRF Origin</b>	No	<b>Yes</b>	Future	<b>Future</b>
<b>Geographic Density</b>	No	No	<b>Yes</b>	<b>Yes</b>
<b>Real-time &amp; ITRF access</b>	Yes	Yes	<b>Yes</b>	Yes
<b>Decadal Stability</b>	<b>Yes</b>	<b>Yes</b>	Yes	Yes

# How the ITRF is constructed ?

- **Input :**

- Time series of mean station positions (at weekly or daily sampling) and daily EOPs from the 4 techniques
- Local ties in co-location sites

- **Output :**

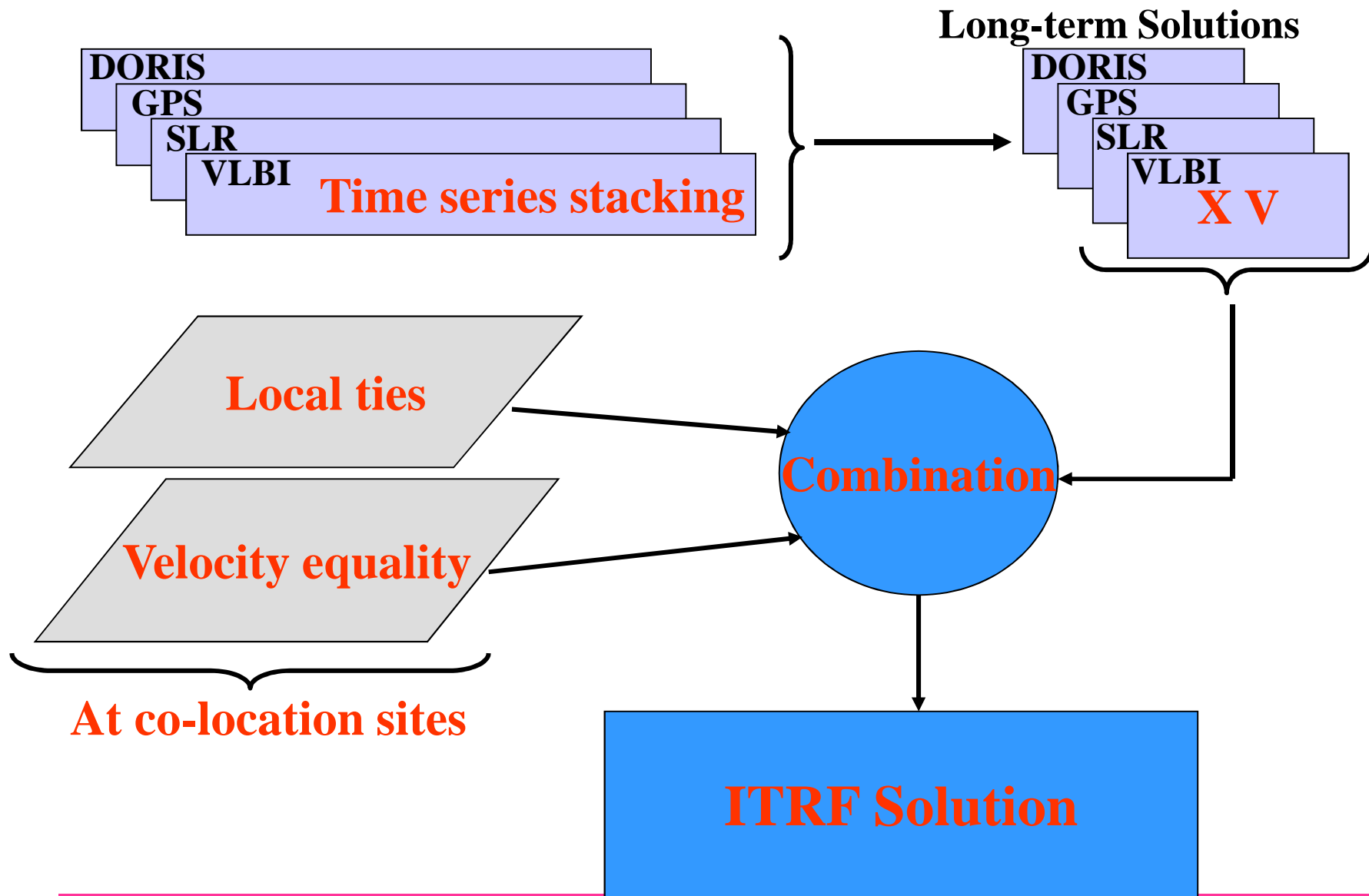
- Station positions at a reference epoch and linear velocities
- Earth Orientation Parameters

## CATREF combination model

$$\left\{ \begin{array}{l} X_s^i = X_c^i + (t_s^i - t_0) \dot{X}_c^i \\ \quad + T_k + D_k X_c^i + R_k X_c^i \\ \quad + (t_s^i - t_k) \left[ \dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i \right] \\ \dot{X}_s^i = \dot{X}_c^i + \dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i \end{array} \right.$$

$$\left\{ \begin{array}{l} x_s^p = x_c^p + R2_k \\ y_s^p = y_c^p + R1_k \\ UT_s = UT_c - \frac{1}{f} R3_k \\ \dot{x}_s^p = \dot{x}_c^p \\ \dot{y}_s^p = \dot{y}_c^p \\ LOD_s = LOD_c \end{array} \right.$$

# ITRF Construction



# SINEX Format

%=SNX 2.01 IGN 10:157:00000 IGN 04:003:00000 09:005:00000 C 01308 2 X V

\*-----

## +SITE/ID

\*CODE PT DOMES T STATION DESCRIPTION APPROX\_LON APPROX\_LAT APP\_H  
ANKR A 20805M002 Ankara, Turkey 32 45 30.4 39 53 14.5 976.0

...

## +SOLUTION/EPOCHS

\*Code PT SOLN T Data\_start Data\_end Mean\_epoch  
ANKR A 5 C 04:003:00000 08:133:00000 06:067:43200

...

## +SOLUTION/ESTIMATE

\*INDEX TYPE CODE PT SOLN REF\_EPOCH UNIT S ESTIMATED VALUE STD\_DEV

.....

19	STAX	ANKR	A	5	06:183:00000	m	2	0.412194852609284E+07	0.17234E-03
20	STAY	ANKR	A	5	06:183:00000	m	2	0.265218790321918E+07	0.12249E-03
21	STAZ	ANKR	A	5	06:183:00000	m	2	0.406902377621100E+07	0.16467E-03
22	VELX	ANKR	A	5	06:183:00000	m/y	2	-.668839830148651E-02	0.14215E-03
23	VELY	ANKR	A	5	06:183:00000	m/y	2	-.270320979559104E-02	0.10069E-03
24	VELZ	ANKR	A	5	06:183:00000	m/y	2	0.971313341105308E-02	0.13542E-03

.....

## +SOLUTION/MATRIX\_ESTIMATE L COVA

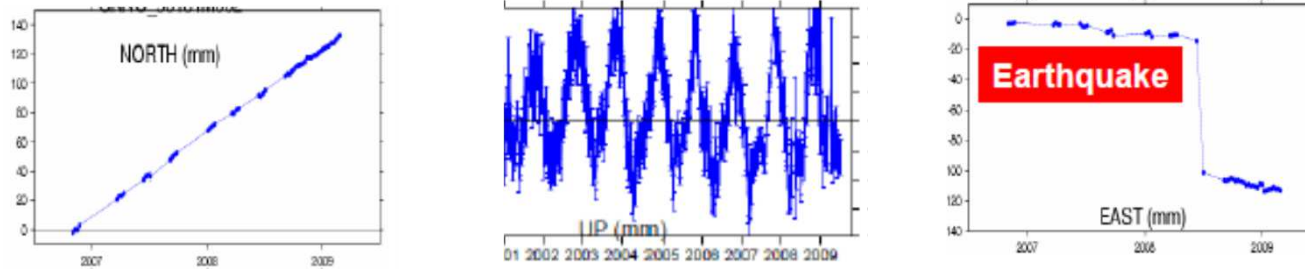
\*PARA1 PARA2 PARA2+0 PARA2+1 PARA2+2

1	1	0.150471439320574E-06		
2	1	-.140657602040892E-06	0.176947767515801E-06	
3	1	-.115071650206259E-06	0.127287839143953E-06	0.122184056413112E-06



# Power of station position time series

- Monitor station behavior
  - Linear, non-linear (seasonal), and discontinuities



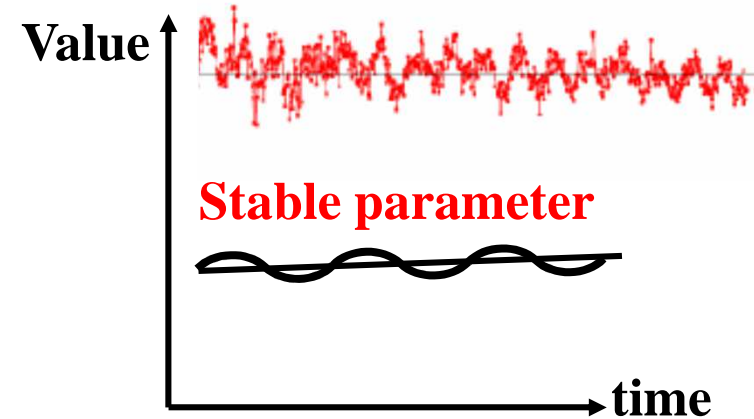
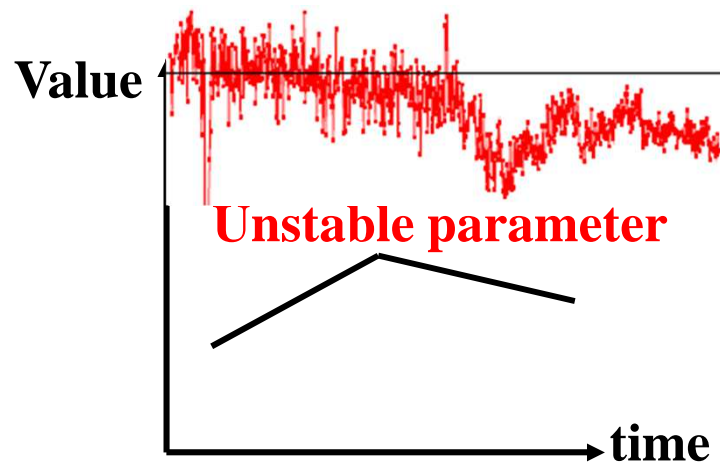
- Monitor time evolution of the frame physical parameter (origin and scale)
- Estimate a robust long-term secular frame

# ITRF and Science Requirement

- Long-term **stable** ITRF: 0.1 mm/yr

==> **Stable**: linear behaviour of the TRF parameters, i.e. with no discontinuity :

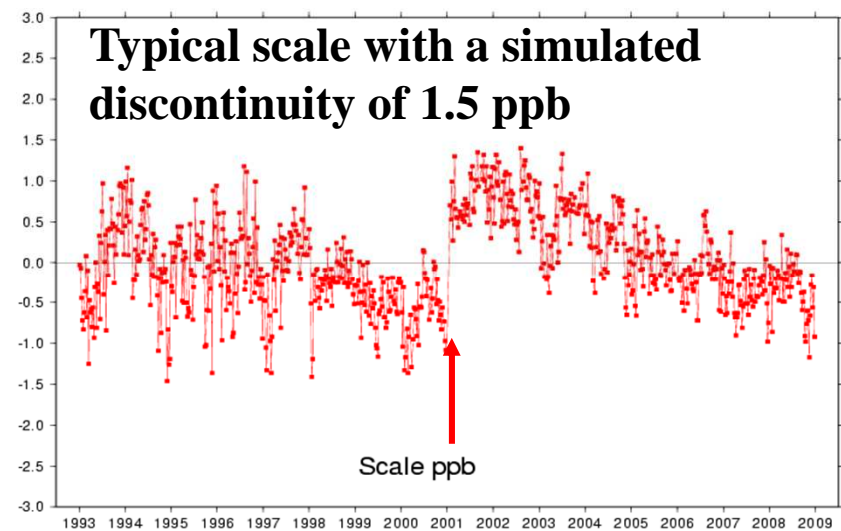
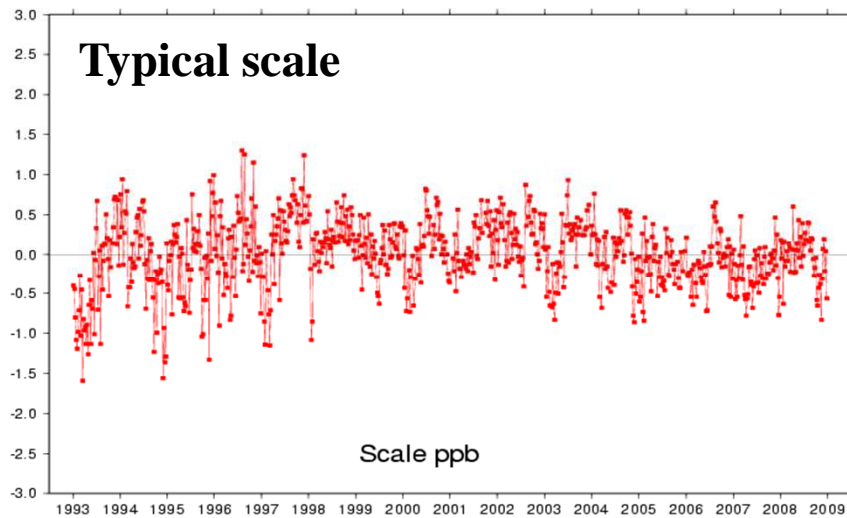
- Origin Components: 0.1 mm/yr
- Scale: 0.01 ppb/yr (0.06 mm/yr)



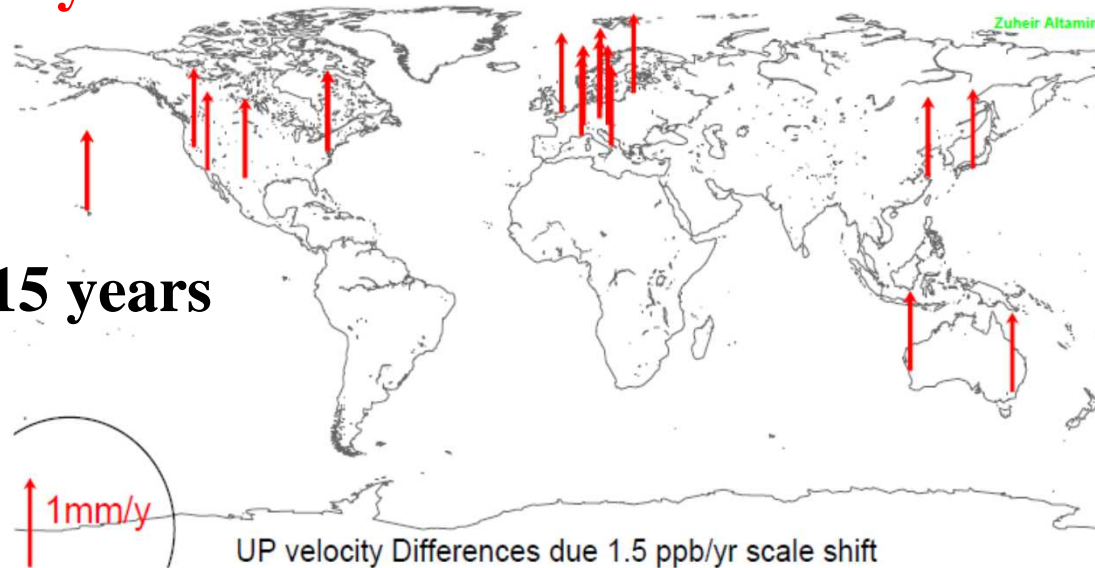
*But stability also means TRF site position predictability*



# Impact of 1.5 ppb scale discontinuity

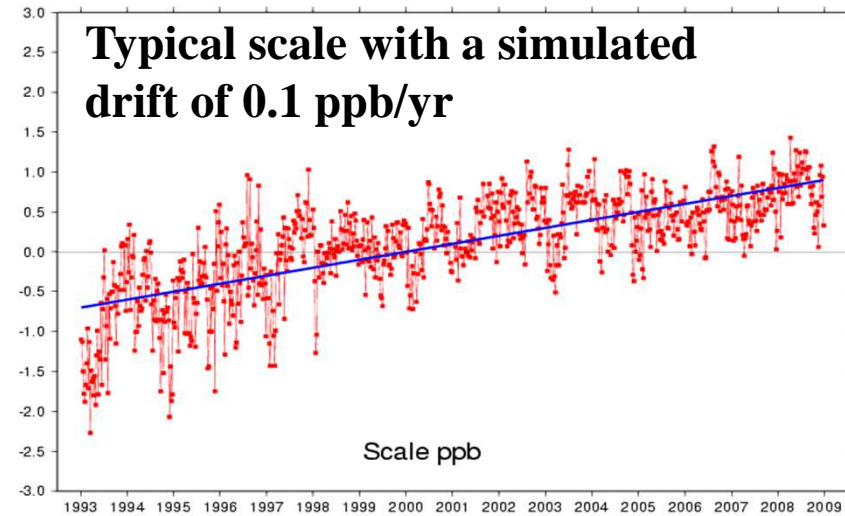
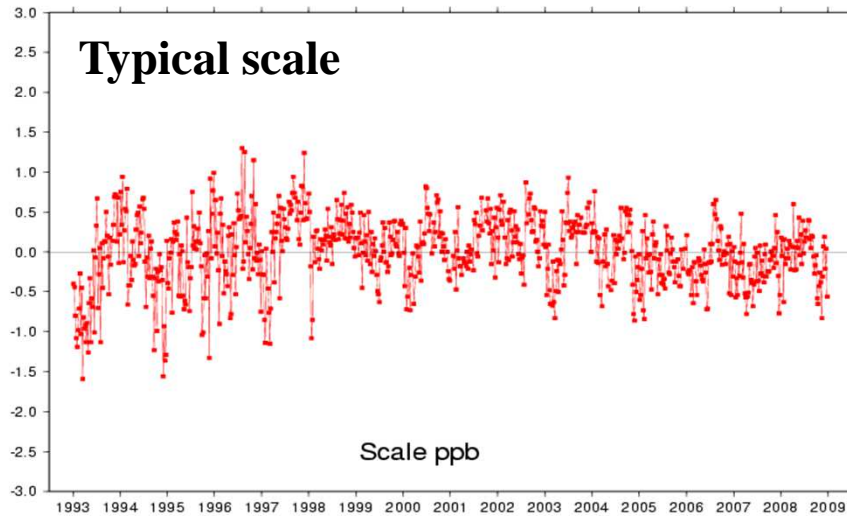


## Vertical velocity Diffs

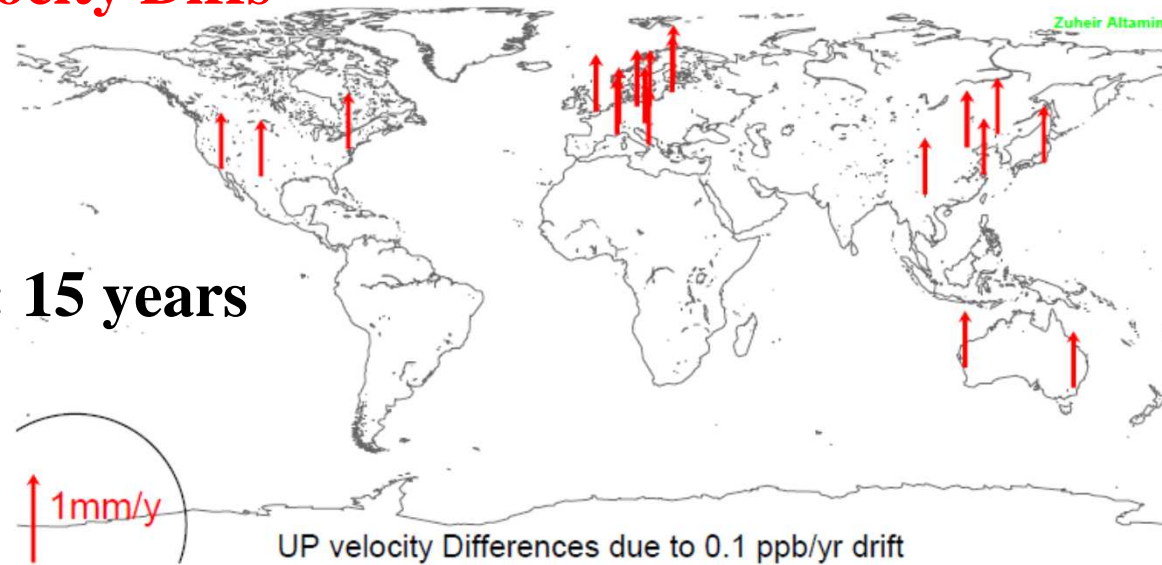


Data-span: 15 years

# Impact of 0.1 ppb/yr scale drift



## Vertical velocity Diffs

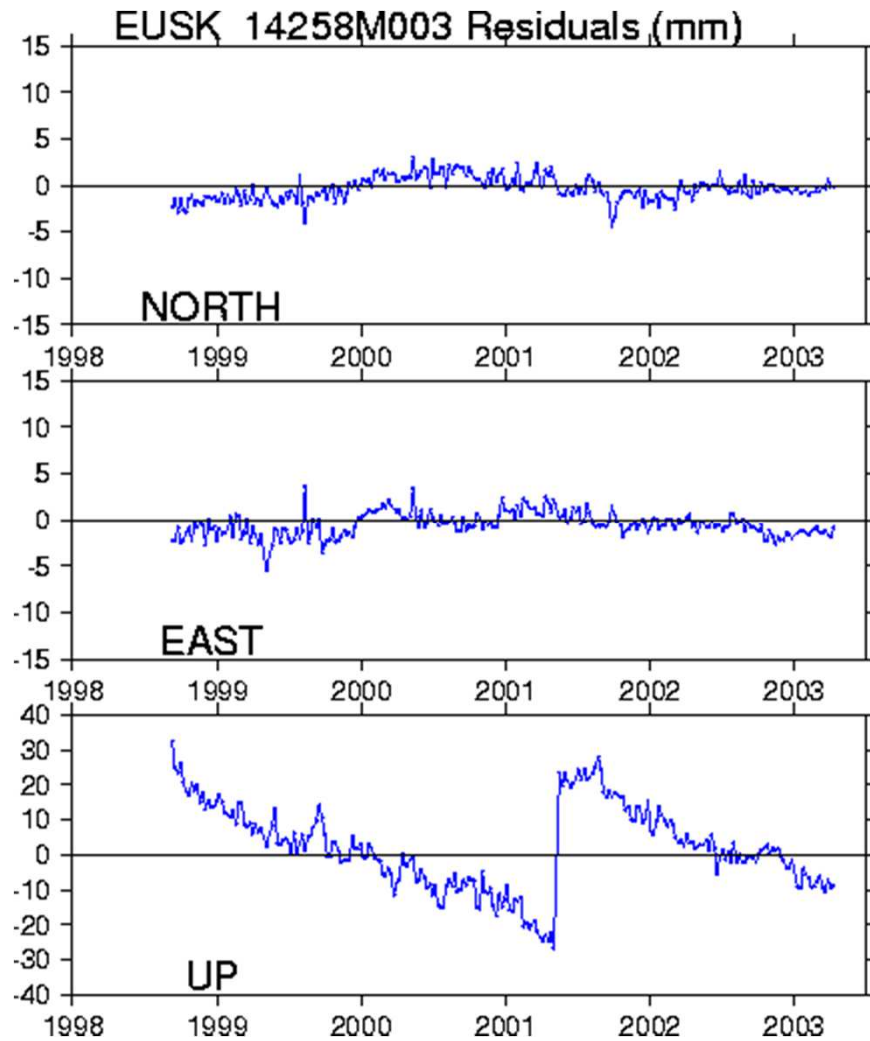


Data-span: 15 years

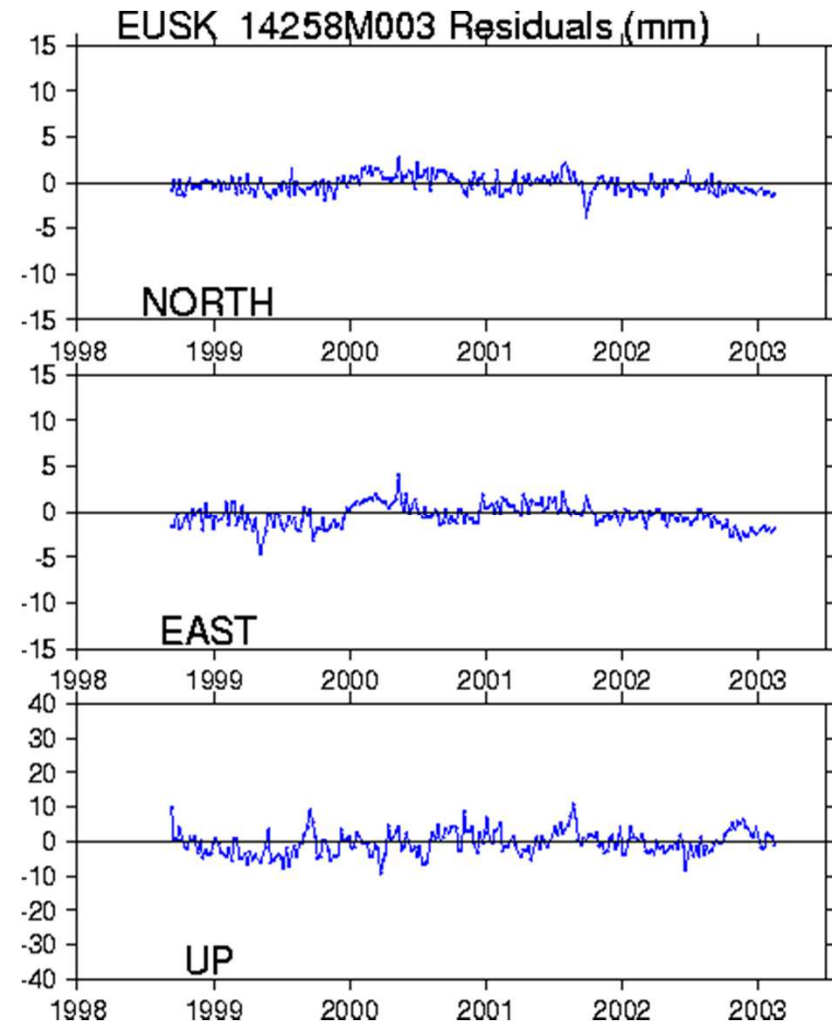
# Some examples of discontinuities and seasonal variations

# Dicontinuity due to equipment change

## Before

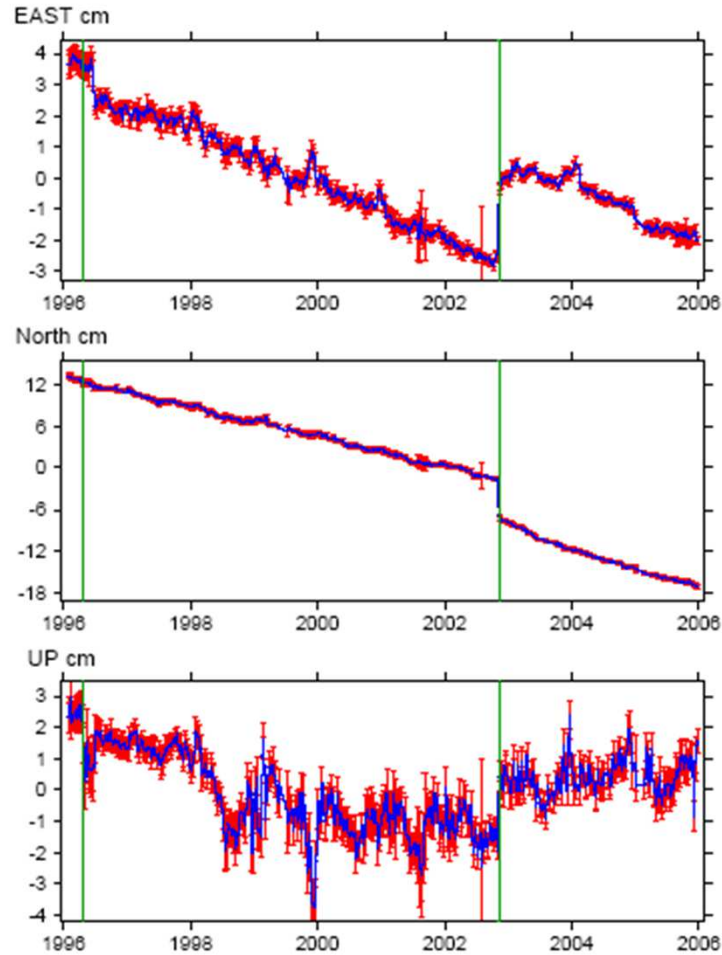


## After

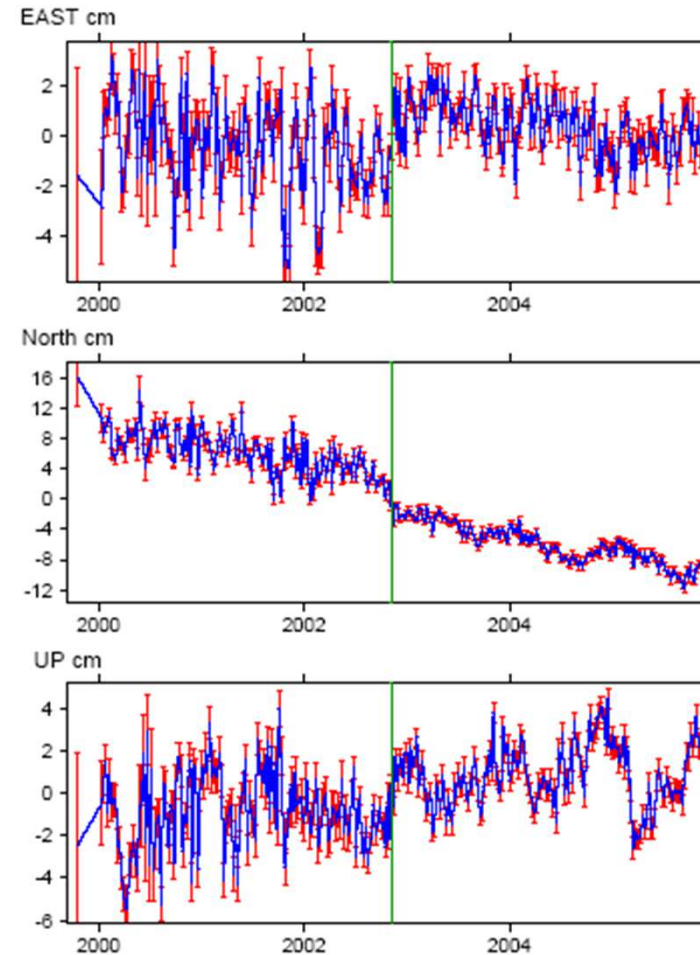


# Denaly Earthquake (Alaska)

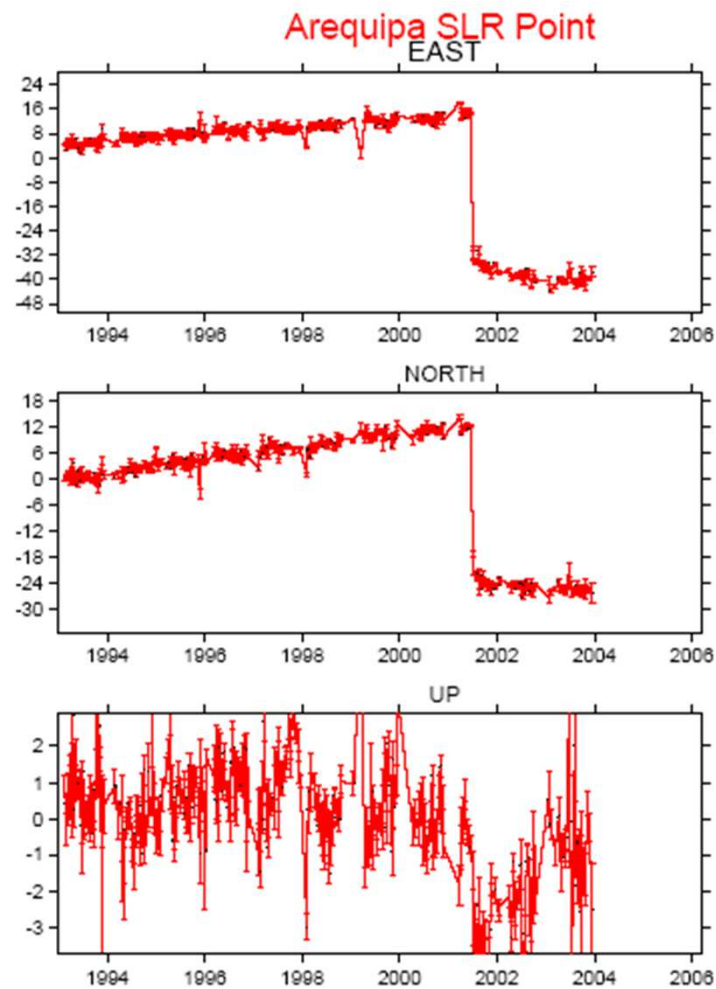
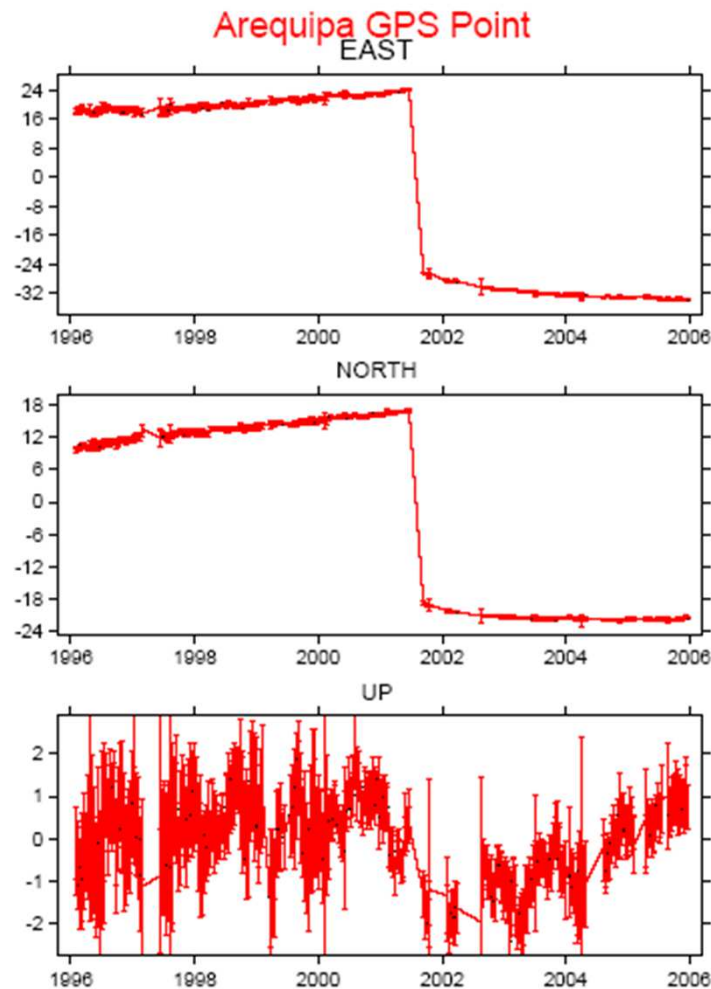
## GPS



## DORIS

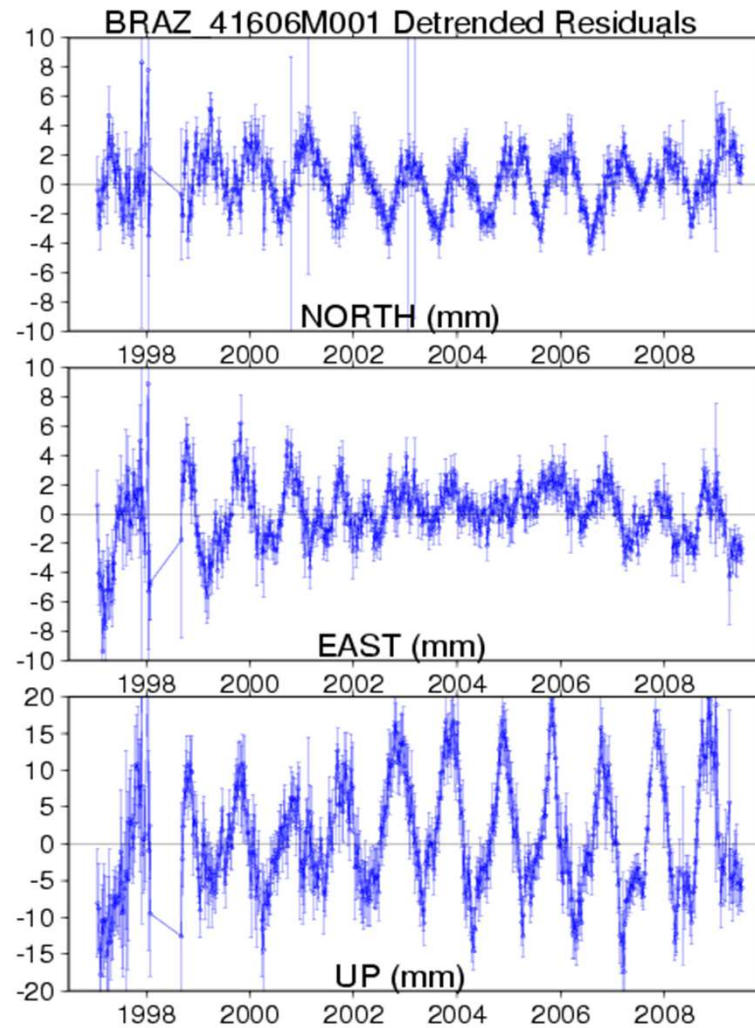


# Arequipa Earthquake



# Example of seasonal variations

## BRAZ GPS antenna



## ITRF2008

- **Time Series of Station Positions :**
  - **Daily (VLBI)**
  - **Weekly (GPS, SLR & DORIS)**
- **and Earth Orientation Parameters:**
  - Polar Motion ( $x_p, y_p$ )**
  - Universal Time (UT1) (Only from VLBI)**
  - Length of Day (LOD) (Only from VLBI)**

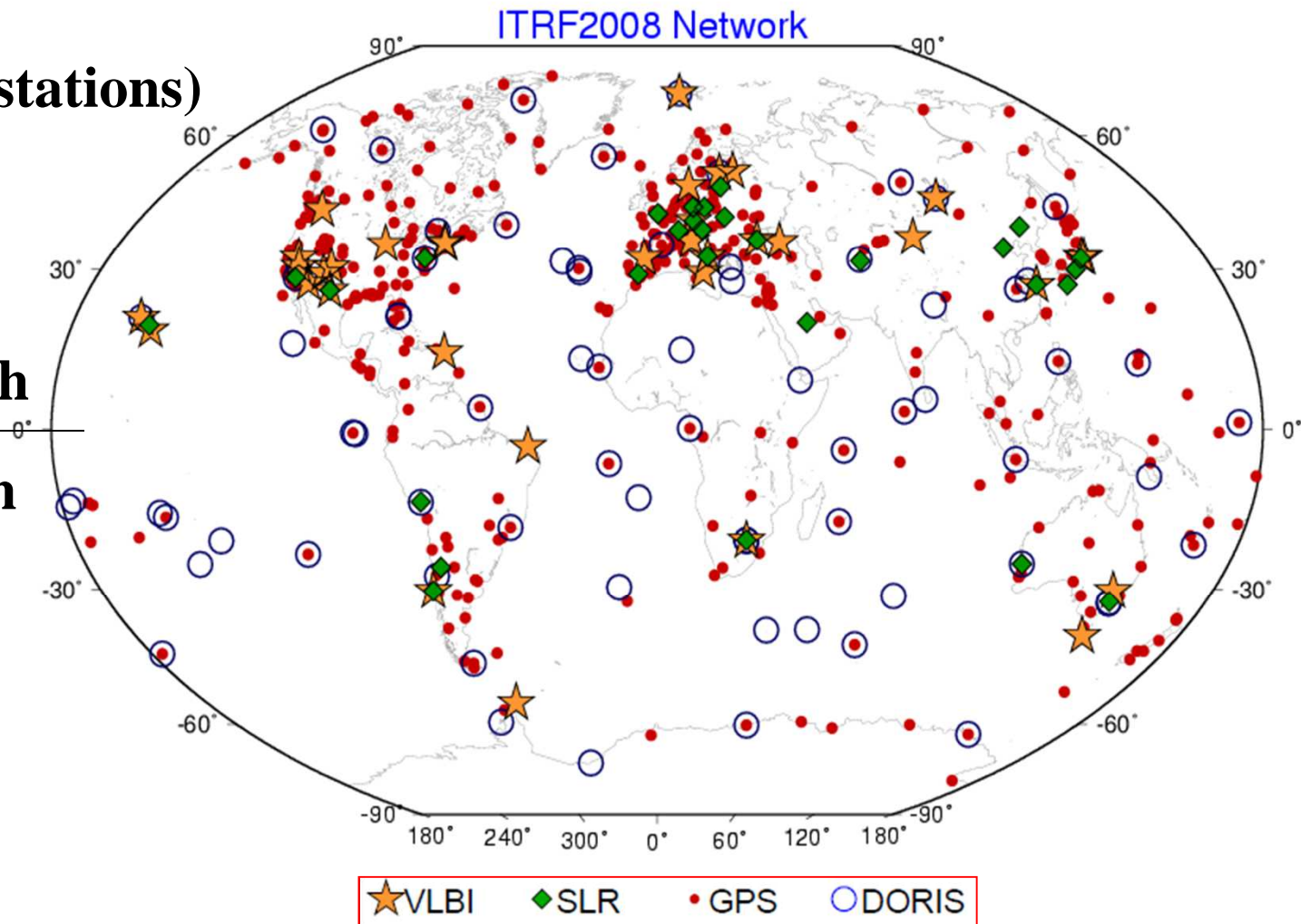


# ITRF2008 Network

580 sites (920 stations)

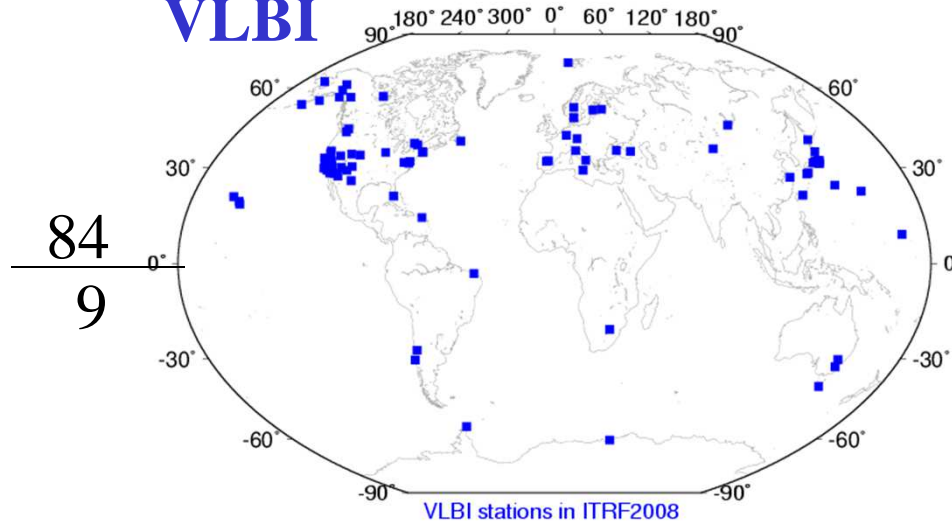
461 Sites North

118 Sites South

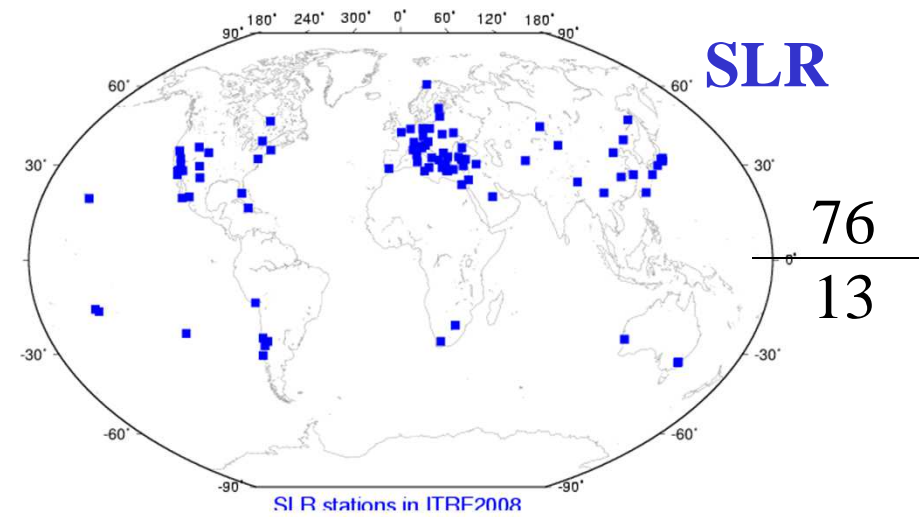


# ITRF2008: Site distribution per technique

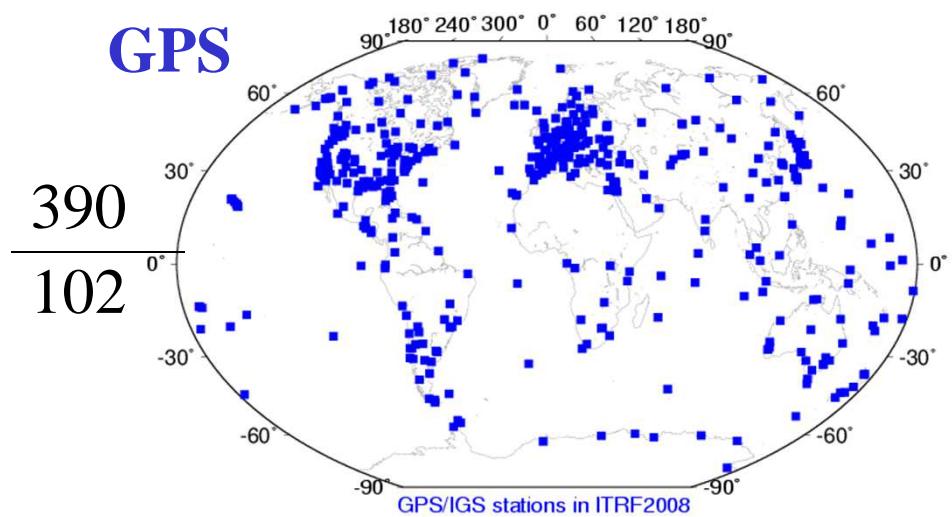
**VLBI**



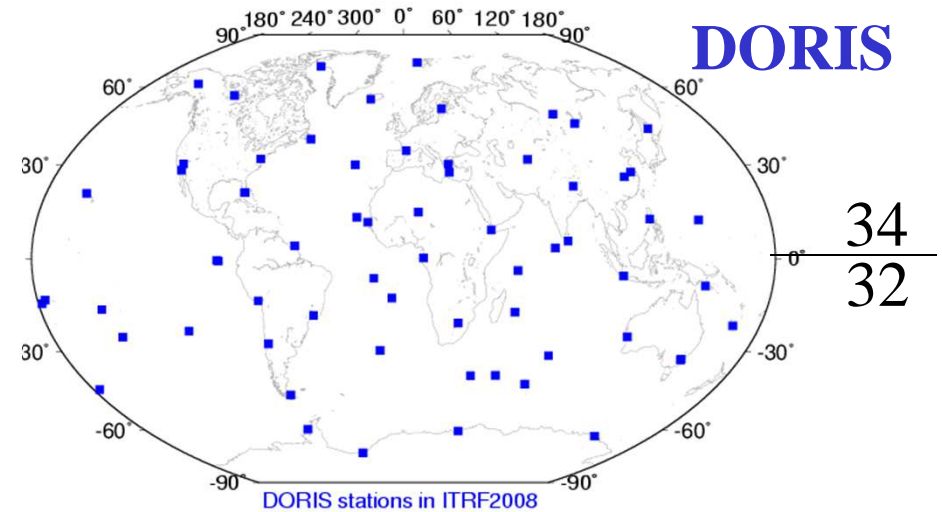
**SLR**



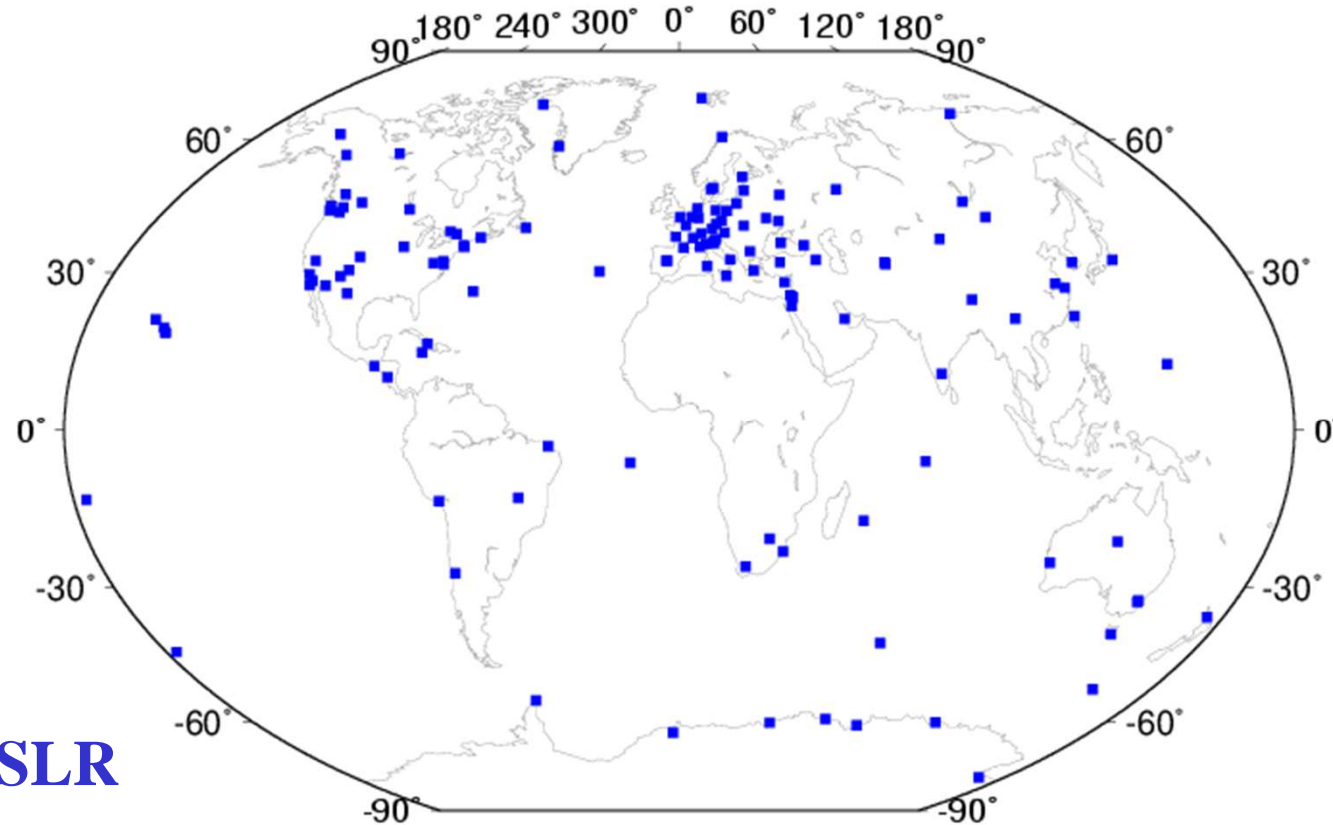
**GPS**



**DORIS**



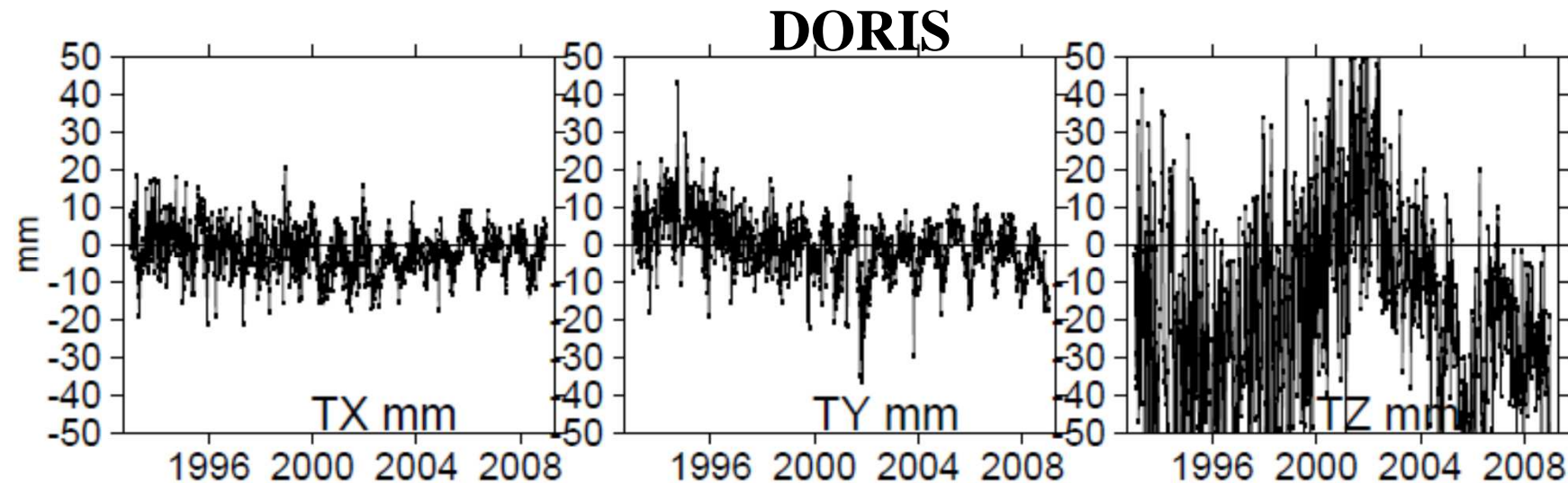
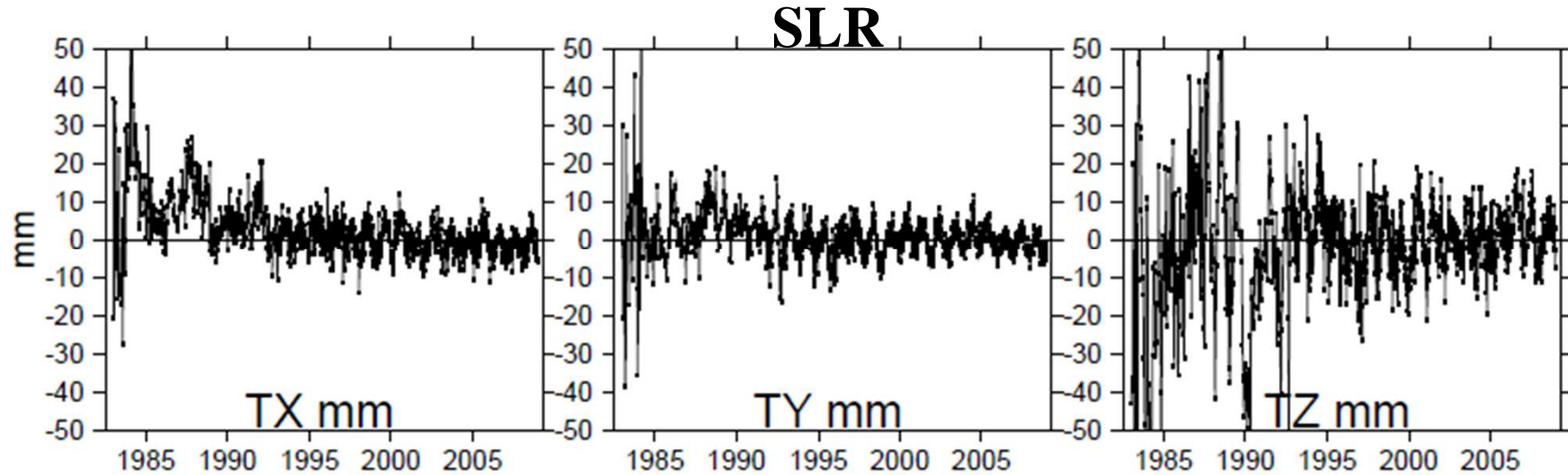
# ITRF2008 Datum Specification



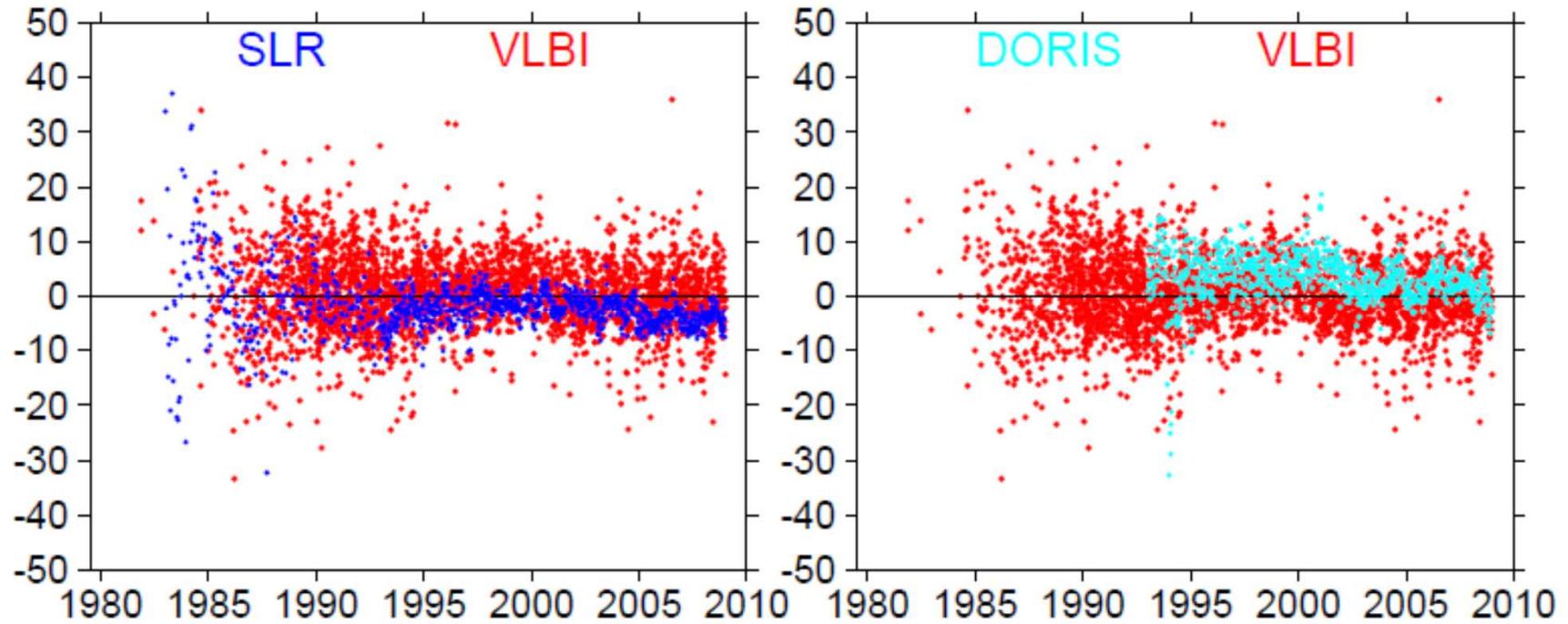
- **Origin: SLR**
- **Scale : Mean of SLR & VLBI**
- **Orientation: Aligned to ITRF2005**  
using 179 stations located at 131 sites:

**104 at northern hemisphere and 27 at southern hemisphere**

# SLR & DORIS origin components wrt ITRF2008



# Scales wrt ITRF2008



## Transformation Param Fm ITRF2008 To ITRF2005

<b>T<sub>x</sub></b>	<b>T<sub>y</sub></b>	<b>T<sub>z</sub></b>	<b>Scale</b>
<b>mm</b>	<b>mm</b>	<b>mm</b>	<b>ppb</b>
<b>-0.5</b>	<b>-0.9</b>	<b>-4.7</b>	<b>0.94</b>
<b>± 0.2</b>	<b>± 0.2</b>	<b>± 0.2</b>	<b>± 0.03</b>

**At epoch  
2005.0**

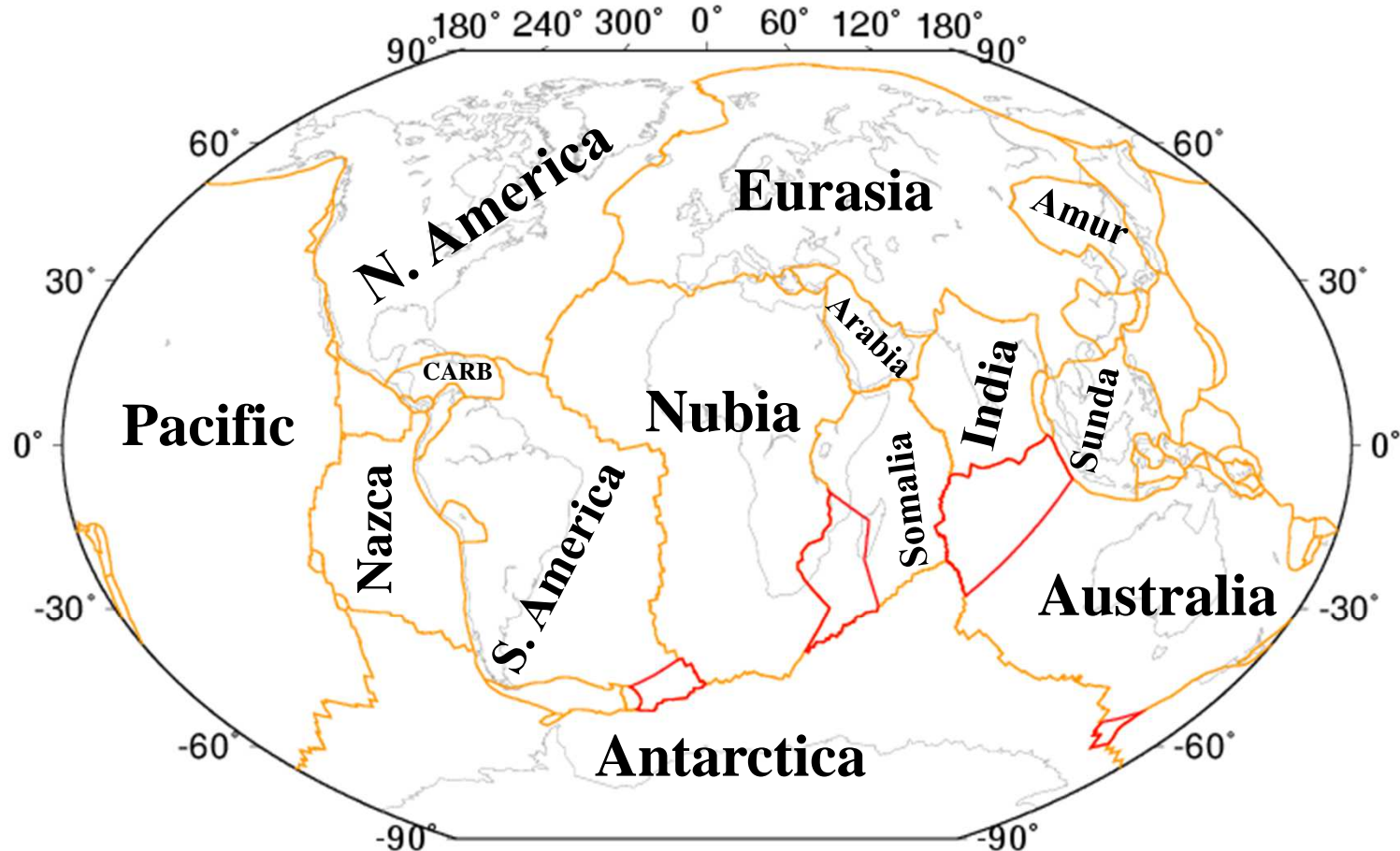
<b>T<sub>x</sub> rate</b>	<b>T<sub>y</sub> rate</b>	<b>T<sub>z</sub> rate</b>	<b>Scale rate</b>
<b>mm/yr</b>	<b>mm/yr</b>	<b>mm/yr</b>	<b>ppb/yr</b>
<b>0.3</b>	<b>0.0</b>	<b>0.0</b>	<b>0.00</b>
<b>± 0.2</b>	<b>± 0.2</b>	<b>± 0.2</b>	<b>± 0.03</b>

# How to estimate an absolute plate rotation pole ?

$$\dot{X} = \omega_p \times X$$

- TRF definition
- Number and distribution over sites over the plate
- Quality of the implied velocities
- Level of rigidity of the plate

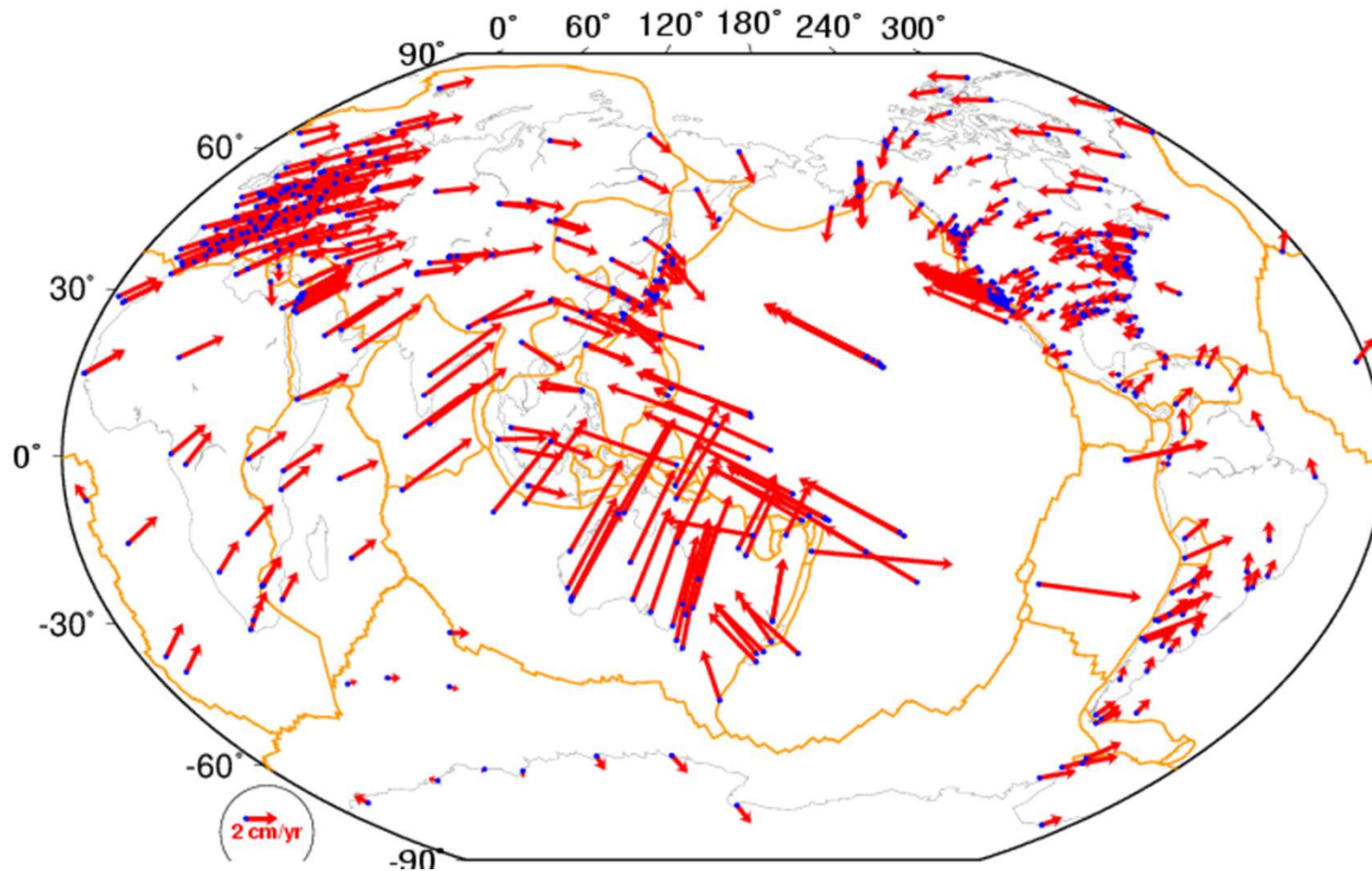
**Plate boundaries: Bird (2003) and MORVEL, DeMets et al. (2010)**





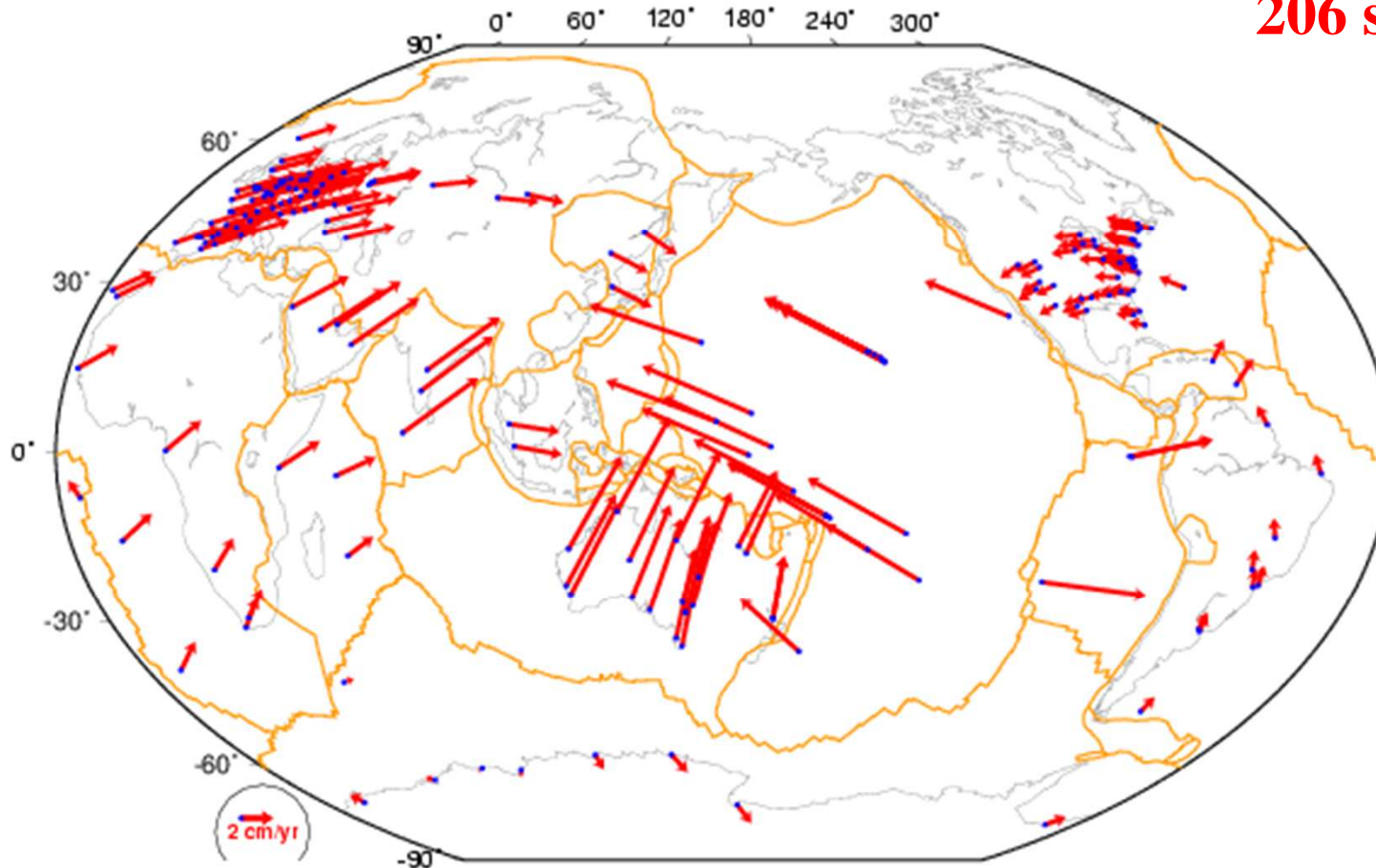
# ALL ITRF2008 Site Velocities: time-span > 3 yrs

509 sites



# ITRF2008-PMM: Selected Site Velocities

206 sites



# ITRF2008 Plate Motion Model

Inversion model:

$$\dot{X}_i = \omega_p \times X_i + \dot{T}$$

**Results:**

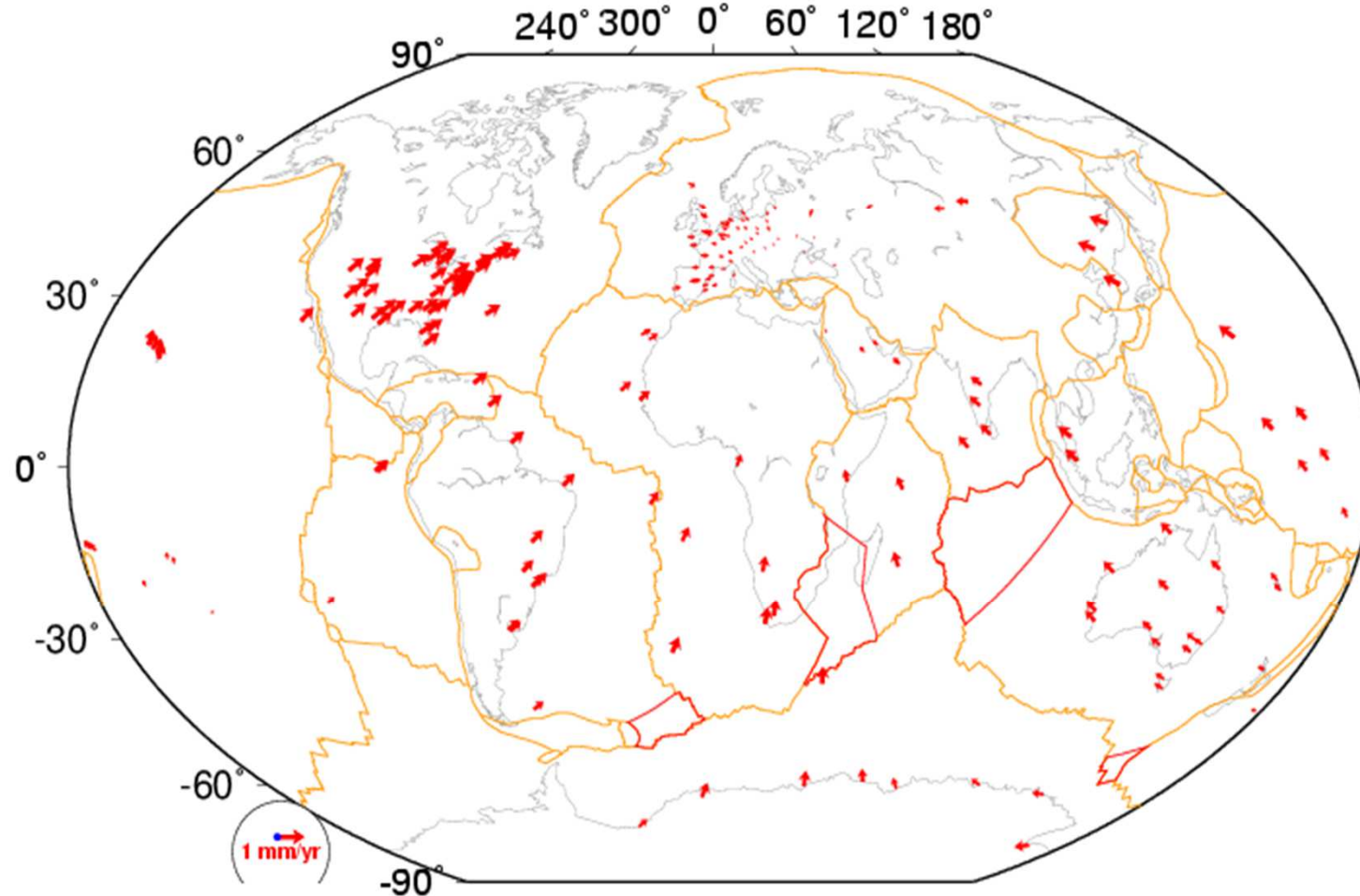
- Angular velocities for 14 plates
- Translation rate components

Table 2. Translation rate components

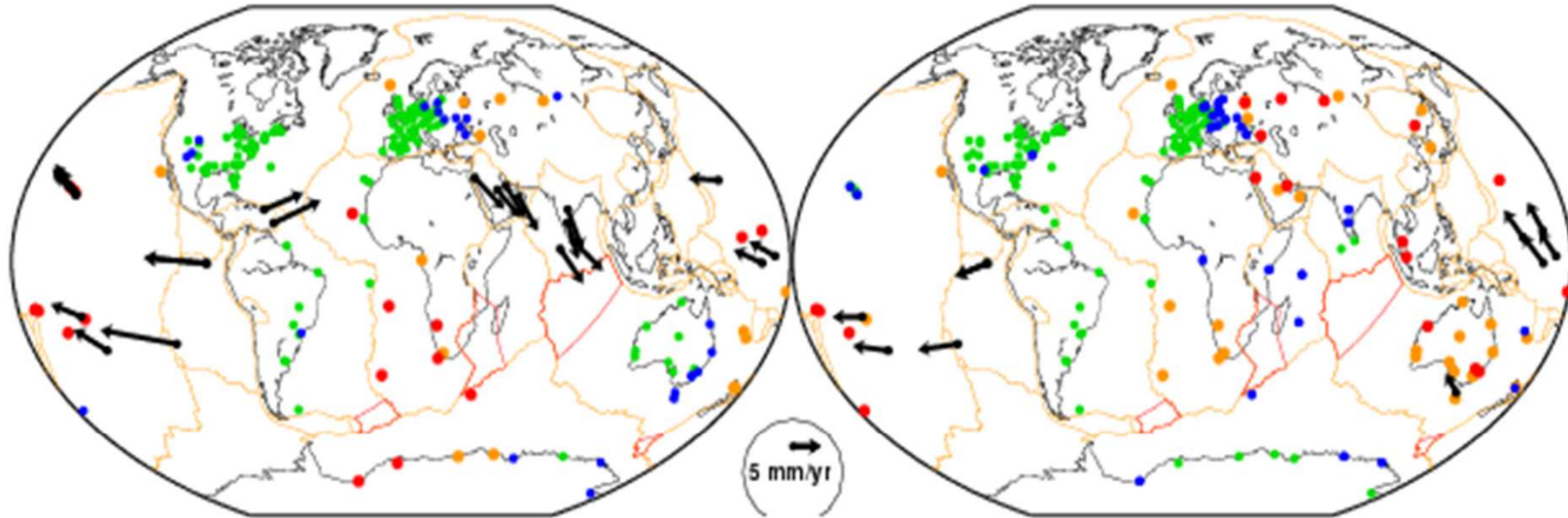
Number of sites			$\dot{T}_x$	$\dot{T}_y$	$\dot{T}_z$
Total	EURA	NOAM	mm/a		
206	69	44	0.41	0.22	0.41
			$\pm 0.27$	$\pm 0.32$	$\pm 0.30$

- More details in JGR paper by Altamimi et al. (2012)

# Impact of the translation rate



# Comparison between ITRF2008 & NNR-NUVEL-1 & NNR-MORVEL56 After rotation rate transformation



## NNR-NUVEL-1A

RMS:

East : 2.5 mm/yr

North: 2.1 mm/yr

- Green: < 2 mm/yr
- Blue : 2-3 mm/yr
- Orange: 3-4 mm/yr
- Red : 4-5 mm/yr
- ←● Black : > 5 mm/yr

## NNR-MORVEL56

RMS:

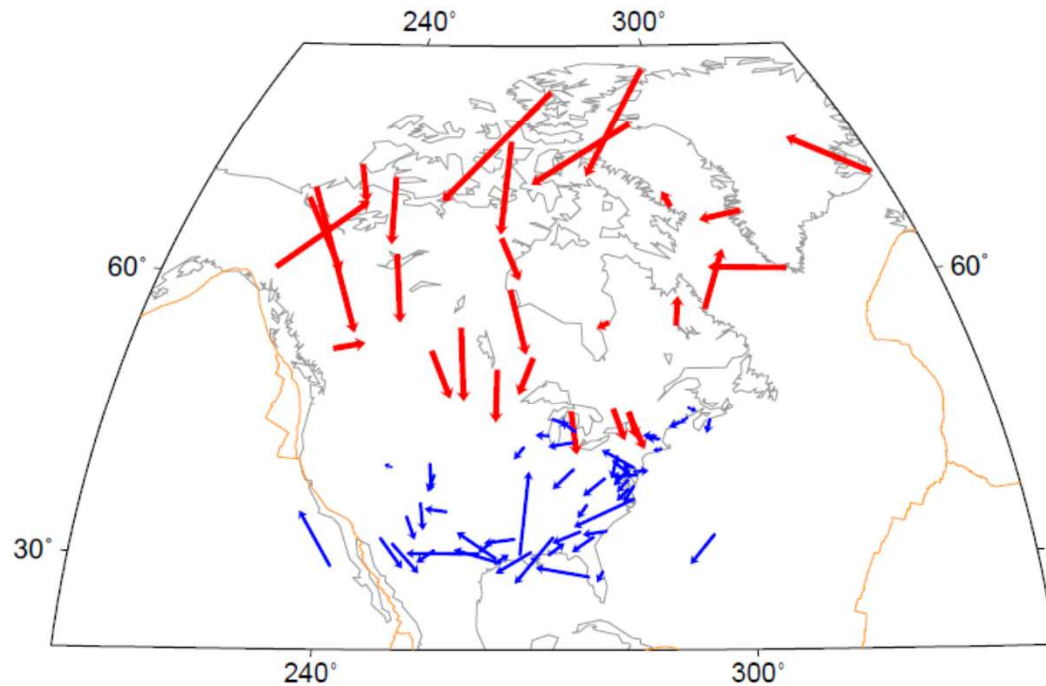
East : 1.8 mm/yr

North: 1.9 mm/yr

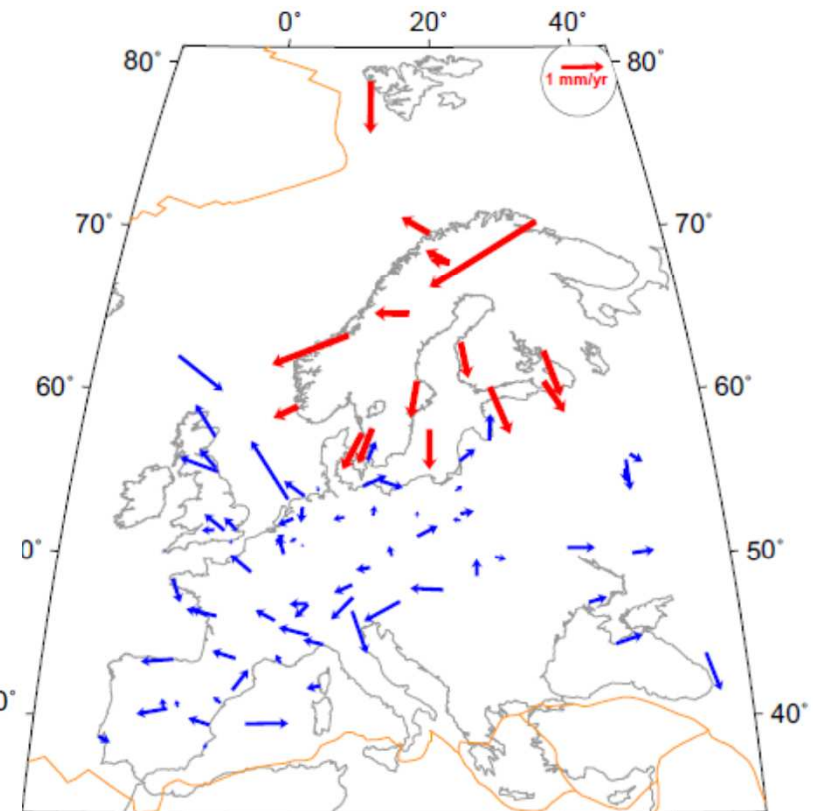
# Plate motion and Glacial Isostatic Adjustment

**Blue** : points used

**Red** : points rejected



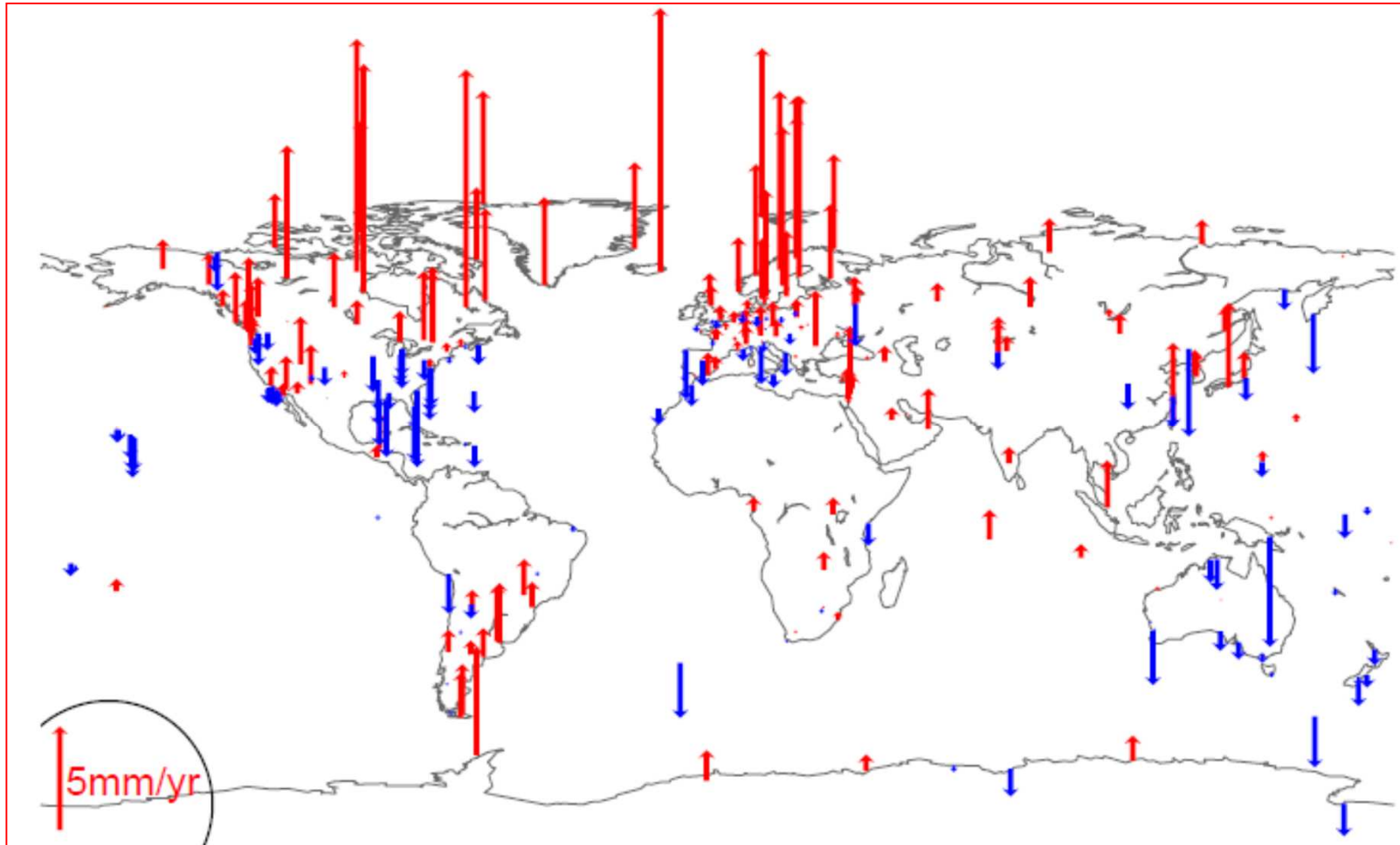
**NOAM**



**EURA**

**Residual velocities after removing NOAM & EURA rotation poles**

# ITRF2008 Vertical velocity field



# ITRF transformation parameters

Table 4.1: Transformation parameters from ITRF2008 to past ITRFs. “ppb” refers to parts per billion (or  $10^{-9}$ ). The units for rates are understood to be “per year.”

ITRF								
Solution	$T1$	$T2$	$T3$	$D$	$R1$	$R2$	$R3$	Epoch
	(mm)	(mm)	(mm)	(ppb)	(mas)	(mas)	(mas)	
ITRF2005	-2.0	-0.9	-4.7	0.94	0.00	0.00	0.00	2000.0
rates	0.3	0.0	0.0	0.00	0.00	0.00	0.00	
ITRF2000	-1.9	-1.7	-10.5	1.34	0.00	0.00	0.00	2000.0
rates	0.1	0.1	-1.8	0.08	0.00	0.00	0.00	
ITRF97	4.8	2.6	-33.2	2.92	0.00	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	
ITRF96	4.8	2.6	-33.2	2.92	0.00	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	
ITRF94	4.8	2.6	-33.2	2.92	0.00	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	
ITRF93	-24.0	2.4	-38.6	3.41	-1.71	-1.48	-0.30	2000.0
rates	-2.8	-0.1	-2.4	0.09	-0.11	-0.19	0.07	
ITRF92	12.8	4.6	-41.2	2.21	0.00	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	
ITRF91	24.8	18.6	-47.2	3.61	0.00	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	
ITRF90	22.8	14.6	-63.2	3.91	0.00	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	
ITRF89	27.8	38.6	-101.2	7.31	0.00	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	
ITRF88	22.8	2.6	-125.2	10.41	0.10	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	



# Access & alignment to ITRF

- **Direct use of ITRF coordinates**
- **Use of IGS Products (Orbits, Clocks): all expressed in ITRF**
- **Alternatively:**
  - **Process GNSS data together with IGS/ITRF global stations in free mode**
  - **Align to ITRF by**
    - **Constraining station coordinates to ITRF values at the central epoch of the observations**
    - **Using minimum constraints approach**

## Transformation from an ITRF to another at epoch $t_c$

- **Input :**  $\mathbf{X}$  (ITRF<sub>xx</sub>, epoch  $t_c$ )
- **Output:**  $\mathbf{X}$  (ITRF<sub>yy</sub>, epoch  $t_c$ )
- **Procedure:**
  - Propagate ITRF transformation parameters from their epoch (2000.0, slide 72) to epoch  $t_c$ , for both ITRF<sub>xx</sub> and ITRF<sub>yy</sub>:

$$P(t_c) = P(2000.0) + \dot{P}(t_c - 2000.0)$$

- Compute the transformation parameters between ITRF<sub>xx</sub> and ITRF<sub>yy</sub>, by subtraction;
- Transform using the general transformation formula given at slide 8:

$$\mathbf{X}(\text{ITRF}_{yy}) = \mathbf{X}(\text{ITRF}_{xx}) + \mathbf{T} + \mathbf{D} \cdot \mathbf{X}(\text{ITRF}_{xx}) + \mathbf{R} \cdot \mathbf{X}(\text{ITRF}_{xx})$$

## How to express a GPS network in the ITRF ?

- Select a reference set of ITRF/IGS stations and collect RINEX data from IGS data centers;
- Process your stations together with the selected ITRF/IGS ones:
  - Fix IGS orbits, clocks and EOPs
  - Eventually, add minimum constraints conditions in the processing
  - ==> Solution will be expressed in the ITRFyy consistent with IGS orbits
  - Propagate official ITRF station positions at the central epoch ( $t_c$ ) of the observations:
$$X(t_c) = X(t_0) + \dot{X}(t_c - t_0)$$
  - Compare your estimated ITRF station positions to official ITRF values **and check for consistency!**

## From the ITRF to Regional Reference Frames

- **Purpose: geo-referencing applications ( $\sigma \sim \text{cm}$ )**
- **There are mainly two cases/options to materialize a regional reference frame:**
  1. **Station positions at a given epoch, eventually updated frequently. Ex.: North & South Americas**
  2. **Station positions & minimized velocities or station positions & deformation model. Ex.: Europe (ETRS89) New Zealand, Greece (?)**
    - **Case 1 is easy to implement (see previous slide)**
    - **Case 2 is more sophisticated & needs application of:**
      - **Transformation formula (ETRS89)**
      - **Deformation model**

## **GNSS and their associated reference systems**

<b><u>GNSS</u></b>	<b><u>Ref. System/Frame</u></b>
• <b>GPS (broadcast orbits)</b>	<b>WGS84</b>
• <b>GPS (precise IGS orbits)</b>	<b>ITRS/ITRF</b>
• <b>GLONASS</b>	<b>PZ-90</b>
• <b>GALILEO</b>	<b>ITRS/ITRF/GTRF</b>
• <b>COMPASS</b>	<b>CGCS 2000</b>
• <b>QZSS</b>	<b>JGS</b>
• <b>All are “aligned” to the ITRF</b>	
• <b>WGS84 <math>\approx</math> ITRF at the decimeter level</b>	
• <b>GTRF <math>\approx</math> ITRF at the mm level</b>	
• <b><math>\sigma</math>-Position using broadcast ephemerides = 150 cm</b>	

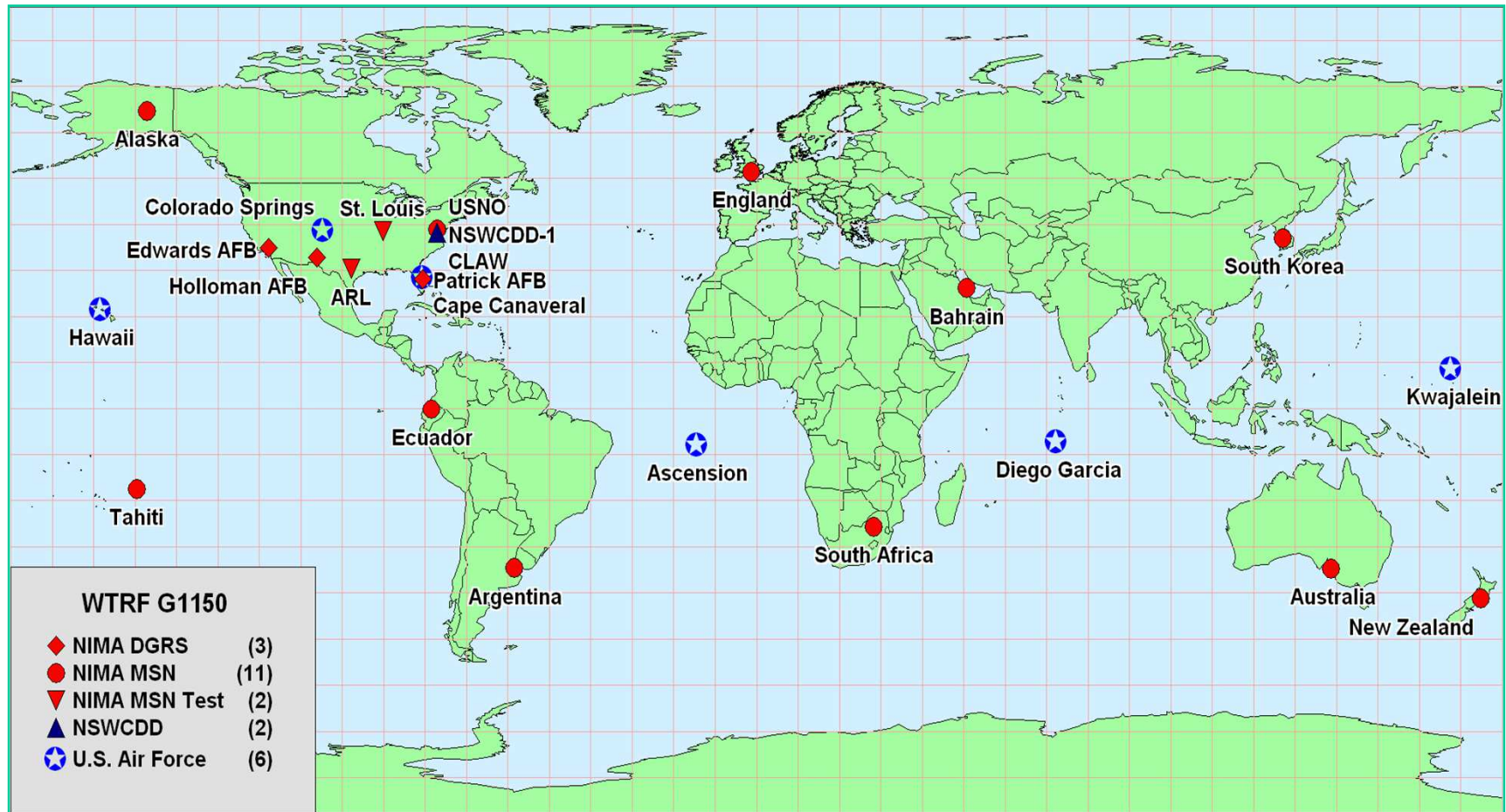
# The World Geodetic System 84 (WGS 84)

- **Collection of models including Earth Gravitational model, geoid, transformation formulae and set of coordinates of permanent DoD GPS monitor stations**
- **WGS 60...66...72...84**
- **Originally based on TRANSIT satellite DOPPLER data**

# The World Geodetic System 84 (WGS 84)

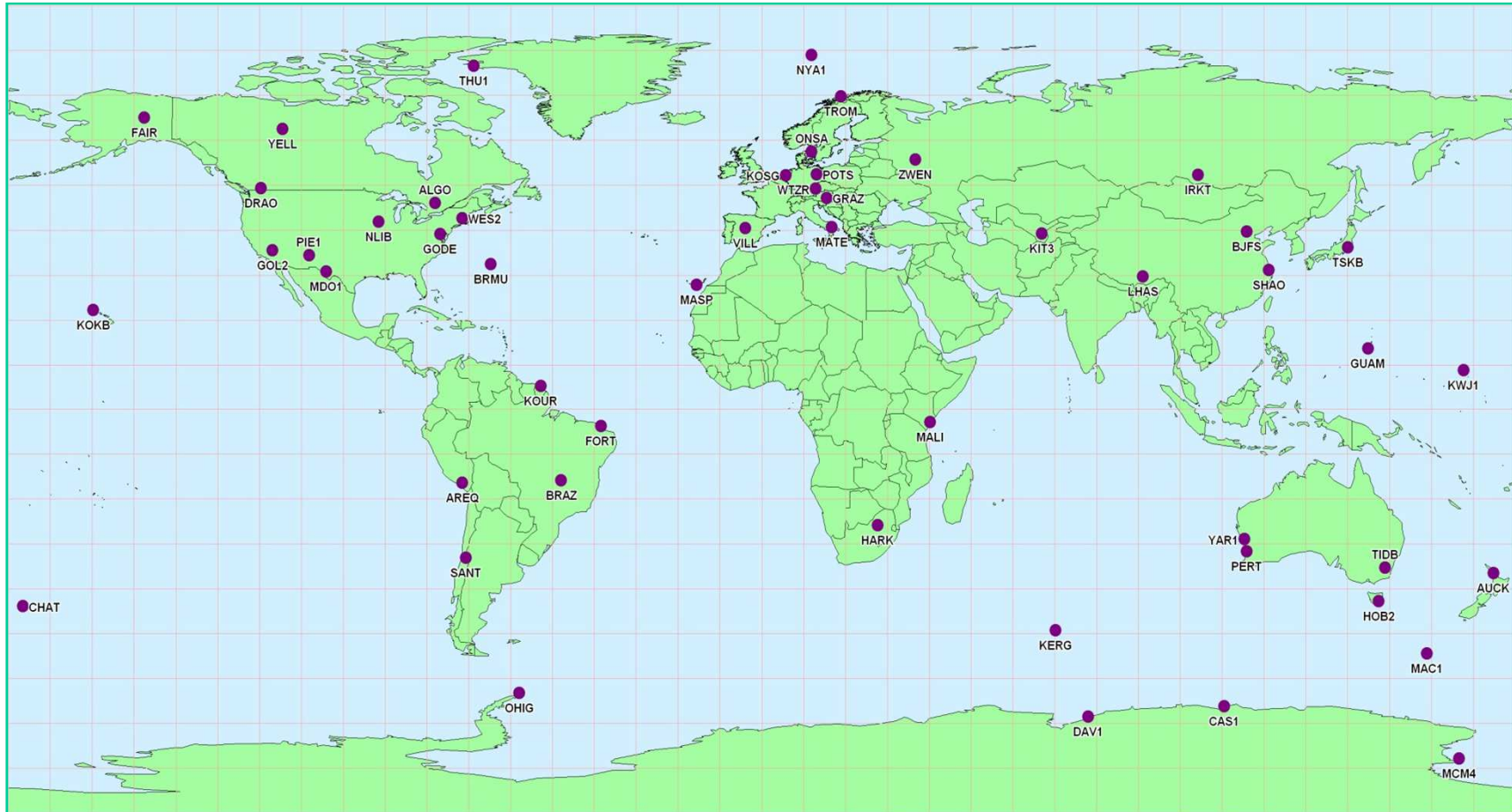
- Recent WGS 84 realizations based on GPS data:
  - G730 in 1994
  - G873 in 1997
  - G1150 in 2002
  - **G1674 in 2012 (aligned to ITRF2008)**
- Coincides with any ITRF at 10 cm level
- No official Transf. Param. With ITRF

# WGS 84-(G1150)





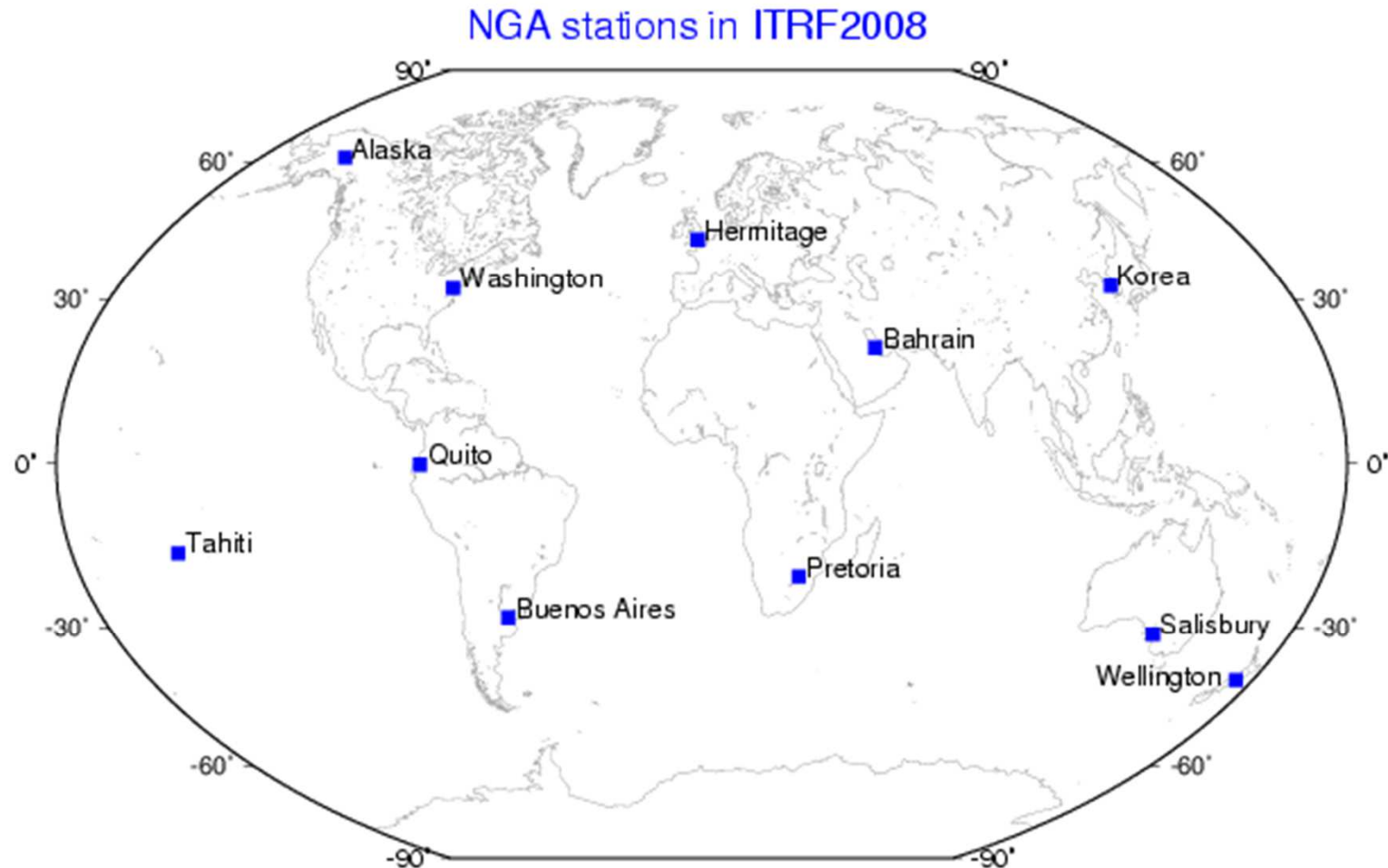
# WGS 84-(G1150)



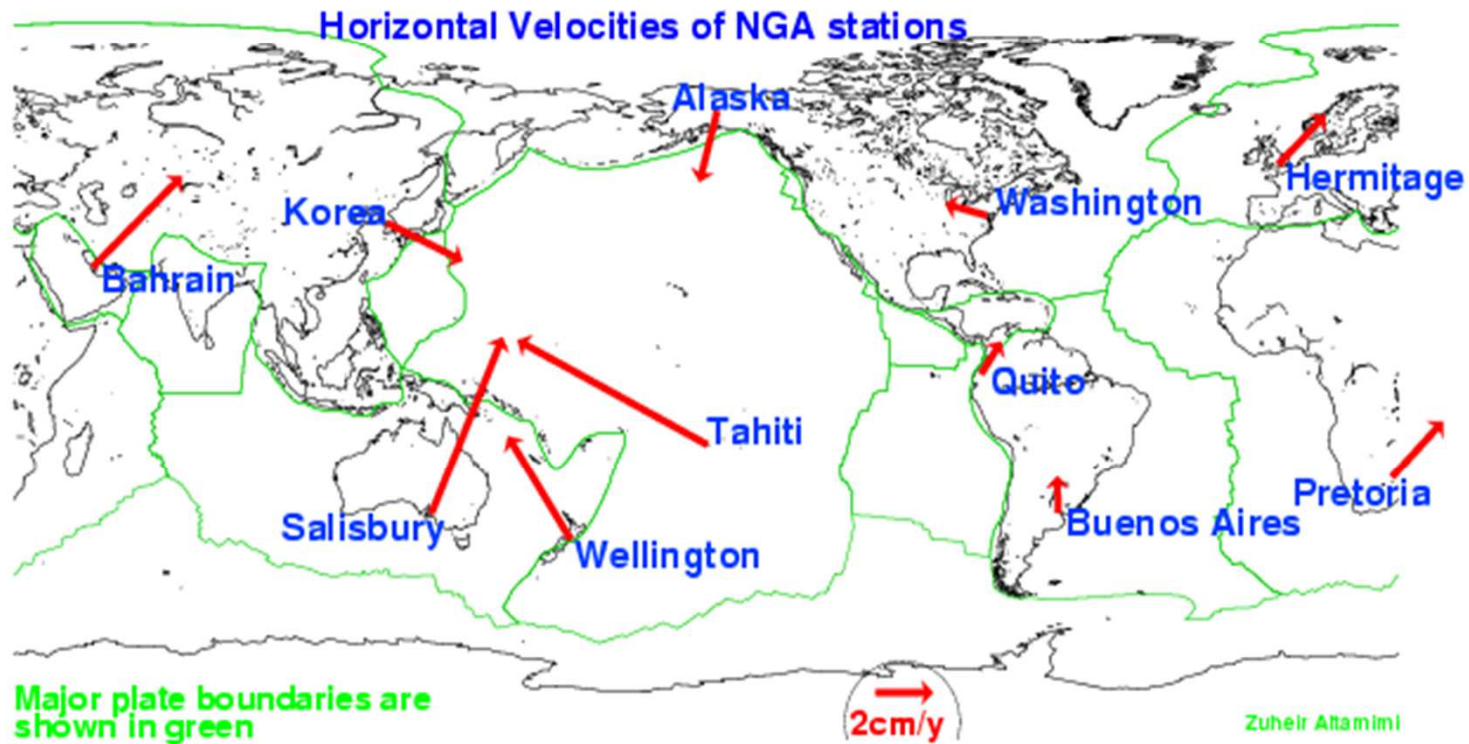
- Coordinates of ~20 stations fixed to ITRF2000
- No station velocities

# WGS84 - NGA Stations in ITRF2008

NGA: National Geospatial-Intelligence Agency



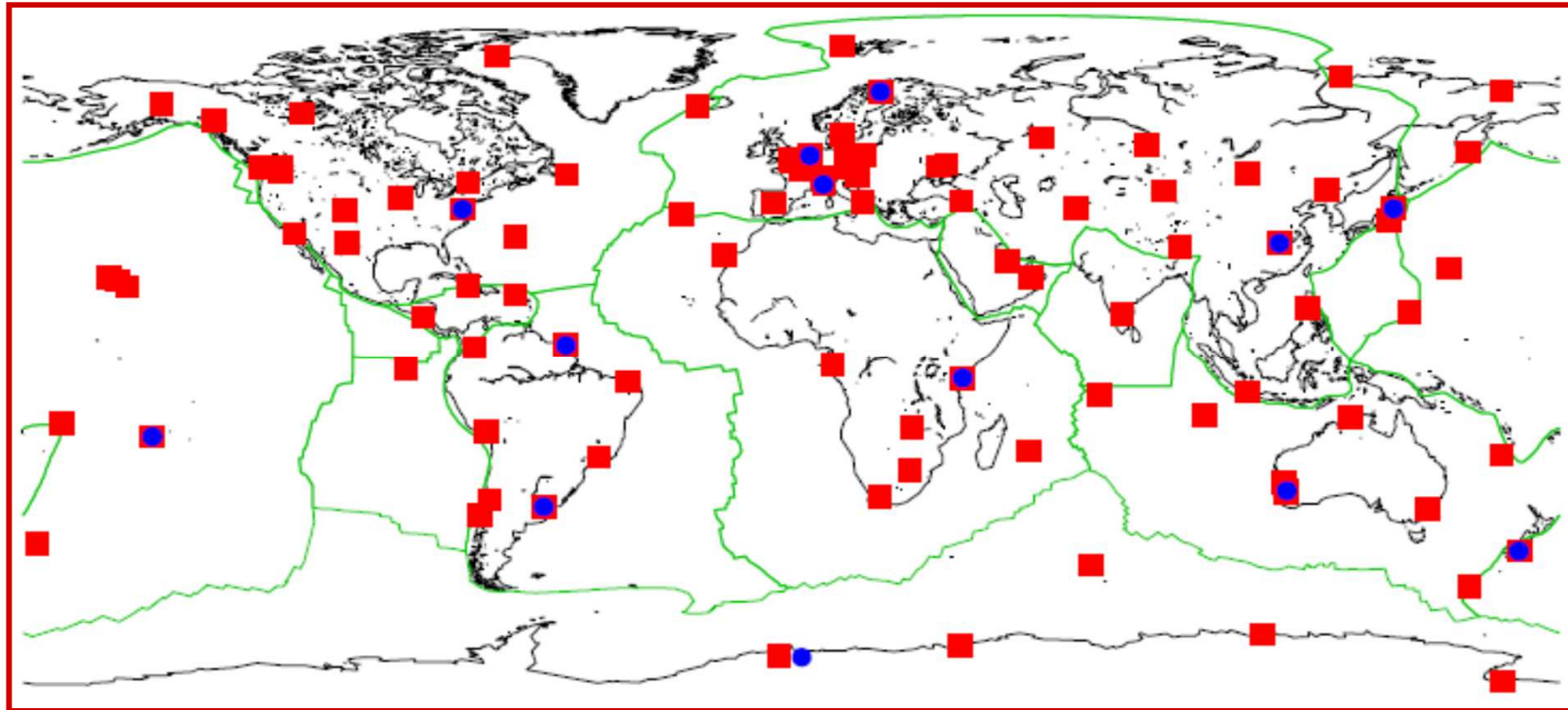
# WGS84 - NGA Stations in ITRF2008



# Galileo Terrestrial Reference Frame (GTRF)

- Galileo Geodesy Service Provider (GGSP)
- GGSP Consortium (GFZ, AIUB, ESOC, BKG, IGN)
  - Define, realize & maintain the GTRF
  - GTRF should be "compatible" with the ITRF at 3 cm level
  - Liaison with IERS, IGS, ILRS
- GTRF is a realization of the ITRS

# The GTRF Experience



● GESS (13)

■ IGS station (~120)

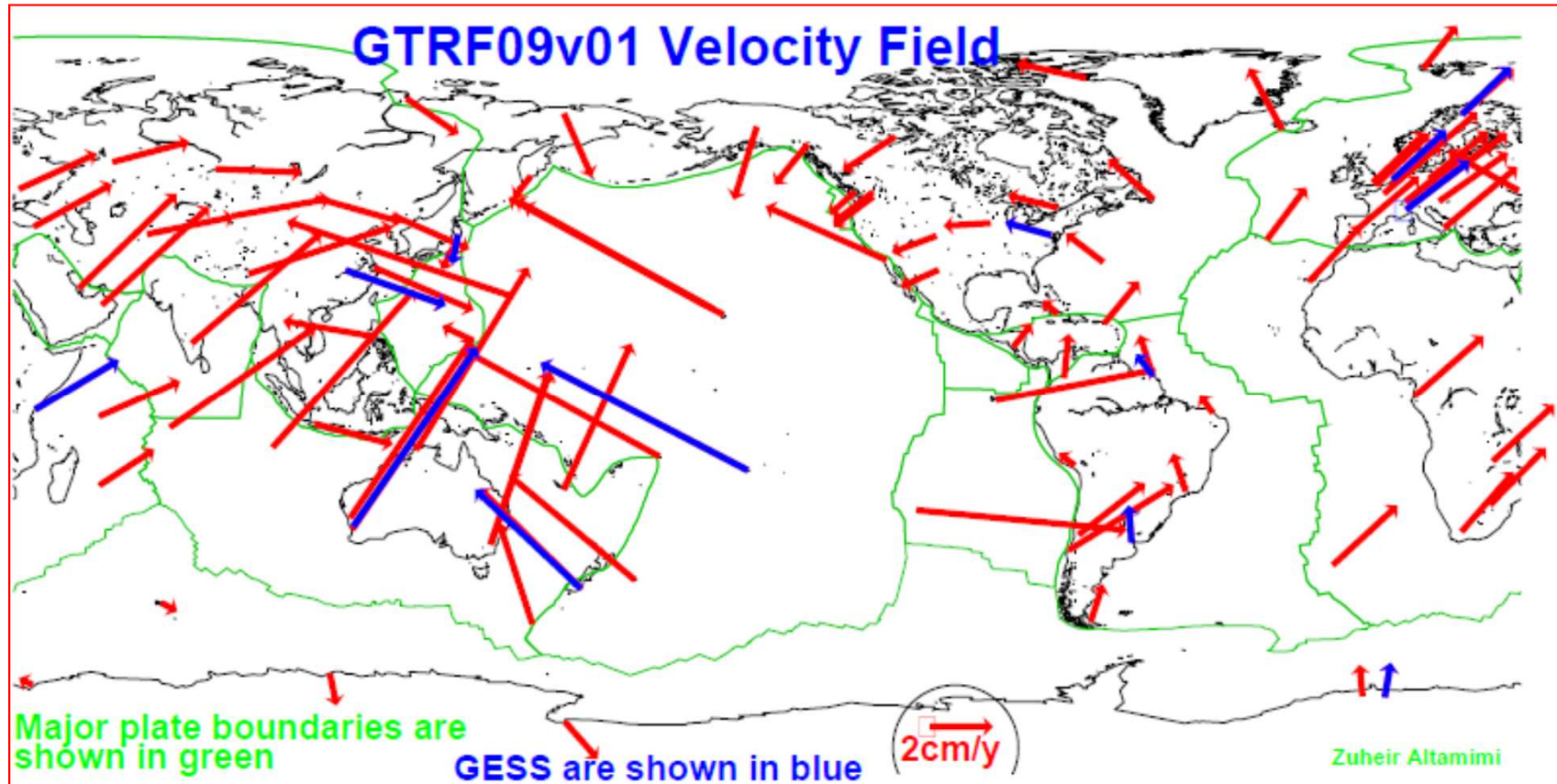
- Initial GSS positions&velocities are determined using GPS observations
- Subsequent GTRF versions using GPS & Galileo observations
- Ultimately Galileo Observations only

# Combination Strategy

- Use Normal Equations from the 3 ACs
- Adequate for weighing
- Weekly and cumulative solutions are transformed into the ITRF using Minimum Constraints

$$\begin{array}{ccc} \boxed{X_R = X_c + A\theta} & \xrightarrow{\theta = 0} & \boxed{(A^T A)^{-1} A^T (X_R - X_c) = 0} \\ \uparrow & & \uparrow \\ \text{ITRF} & & \text{Combined Solution} \\ & & \text{(GTRF)} \end{array}$$

# GTRF09v01 horizontal velocities



## Comparison of GTRF09v01 to ITRF2005

- Transformation parameters

	T1	T2	T3	D	R1	R2	R3	Epoch
	mm	mm	mm	10 <sup>-9</sup>	mas	mas	mas	y
ITRF2005	0.3	-0.3	-0.2	-0.02	-0.003	-0.007	-0.006	7:360
	± 0.2	± 0.2	± 0.2	± 0.03	± 0.007	± 0.008	± 0.008	
Rates	0.0	-0.1	-0.1	0.01	-0.001	-0.002	-0.001	
	± 0.2	± 0.2	± 0.2	± 0.03	± 0.007	± 0.008	± 0.008	

==> Perfect GTRF alignment to the ITRF at the sub-mm level

- RMS difference between stations coordinates and velocities

	N	WRMS-Pos.			Epoch	WRMS-Vel.		
		E	N	U		E	N	U
ITRF2005	89	1.0	1.2	2.6	7:360	0.3	0.3	0.6



## Conclusion (1/2)

- **The ITRF**
  - **is the most optimal global RF available today**
  - **gathers the strengths of space geodesy techniques**
  - **more precise and accurate than any individual RF**
- **Using the ITRF as a common GNSS RF will facilitate the interoperability**
- **Well established procedure available to ensure optimal alignment of GNSS RFs to ITRF**
- **To my knowledge: most (if not all) GNSS RFs are already “aligned” to ITRF**
- **GNSS RFs should take into account station velocities**

# Conclusion (2/2)

**WGS84, PZ90, GTRF**

**Are all connected to (compatible with)  
a Unique System  
The ITRS**

