Aerospace Science and Technology 46 (2015) 256–263

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Contribution of simulated space VLBI to the Chang'E-1 orbit determination and *EOPs* estimation

Erhu Wei [a,b], Wei Yan [c], Shuanggen Jin [d,e,*], Jianan Wei [f], Hakan Kutoglu [d], Xuechuan Li [a], Jozsef Adam [g], Sandor Frey [h], Jingnan Liu [i]

[a] *School of Geodesy and Geomatics, Wuhan University, Wuhan 430079, China*
[b] *Collaborative Innovation Center for Geospatial Technology, Wuhan 430079, China*
[c] *National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China*
[d] *Department of Geomatics Engineering, Bulent Ecevit University, Zonguldak 67100, Turkey*
[e] *Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, China*
[f] *Faculty of Built Environment, University of New South Wales, NSW 2052, Australia*
[g] *Budapest University of Technology and Economics, Budapest H-1521, Hungary*
[h] *Institute of Geodesy, Cartography and Remote Sensing, Budapest H-1592, Hungary*
[i] *GNSS Research Center, Wuhan University, Wuhan 430079, China*

ARTICLE INFO

Article history:
Received 22 July 2014
Received in revised form 25 February 2015
Accepted 22 July 2015
Available online 29 July 2015

Keywords:
Chang'E-1
Space VLBI
Orbit determination
EOPs

ABSTRACT

Space very long baseline interferometry (SVLBI) is an extension of ground based VLBI to space, which has advantages, such as improving the precision and geometry structure of time delay observables with interconnecting multiple spatial coordinate systems directly. In this paper, a mathematical model of relativistic simulated SVLBI observables for estimating Chang'E-1 (CE-1) transfer orbit and Earth orientation Parameters (*EOPs*) is derived and discussed. A comparison of parameter estimation precision between ground ΔVLBI measurements and simulated SVLBI observables is carried out to verify the contribution of SVLBI. The optimal observation condition of CE-1 for SVLBI simulated observables is determined based on the analysis of parameter estimation results under CE-1 current observation condition. By using simulated SVLBI time delay observables under the optimal observation condition, the estimation precision of CE-1 orbit can achieve a level of 2 m. On the other side, the precision of some *EOPs* components can be improved when compared with their predicted values by fixed remaining components as known values. The method discussed in this paper provides a new attempt of deep space probe orbit determination and *EOPs* estimation.

1. Introduction

Being the most accessible planetary body, the Moon has always been an important target of international deep space exploration. In addition, it is significant for deciphering the geological evolutionary history of solar system [1]. In recent years, with remarkable achievements made in the field of Earth satellites and manned space flights, China has been actively carrying out its autonomous deep space exploration projects. Up to now, Chang'E-1/2 and 3 (CE-1/2/3), the three important projects of China's Lunar Exploration, have been implemented successfully. A solid foundation has been laid for subsequent lunar science missions, such as lunar sample returning and so on.

During the lunar exploration, precise orbit determination of the probe is a key element due to its direct relation to various scientific experiments and applications, such as the lunar probe entering the mission orbit, global lunar image map, lunar DEM, lunar gravity field and so on. In CE-1 project, the unified s-band monitoring system (USB) and very long baseline interferometry (VLBI) were used for probe orbit determination, in which VLBI played an important role due to its high precision observations when the probe entered the transfer orbit. Current orbit accuracy of CE-1 is just 1–2 km during the transfer orbit and several hundred meters during the mission orbit, which would not meet the demand of high-precision applications, such as lunar geodesy [2–4]. Therefore, improving probe orbit determination pre-

---

[*] Corresponding author. Tel.: +90 534 9218865, fax: +86 21 64384618.
E-mail addresses: ehwei@sgg.whu.edu.cn (E. Wei), sgjin@shao.ac.cn, sgjin@beun.edu.tr (S. Jin).

http://dx.doi.org/10.1016/j.ast.2015.07.016
1270-9638/

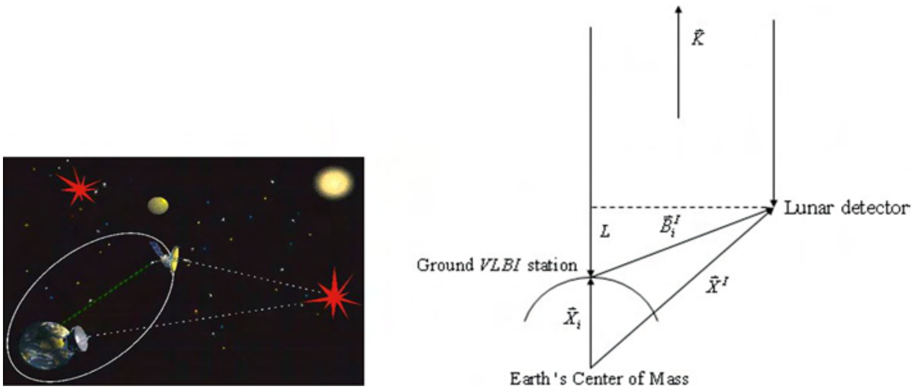

Fig. 1. Geometry of SVLBI observation.

cision has become a main challenge of China's Lunar Exploration Program.

To improve probe orbit determination precision, the Earth orientation Parameters (*EOPs*) and the relativistic effect corrections were introduced to the orbit determination of CE-1 by a derived differential VLBI (ΔVLBI) time delay mathematical model [5,6]. An important conclusion has been drawn that *EOPs* is crucial for lunar probe orbit determination and must be estimated because the priori precision of *EOPs* would cause an error of orbit estimation with a level of one hundred meters. As the consequence, the probe orbit parameters are simultaneously estimated with *EOPs* under CE-1 ground measurements, and the results showed that the precision of CE-1 orbital parameters and *EOPs* can be improved significantly when compared with their priori values by this method. Thus, a win–win approach of improving the precision of CE-1 orbital parameters and *EOPs* has been provided.

However, the precision of CE-1 ΔVLBI time delay measurements with just nanoseconds is still lower than the precision of traditional VLBI observables with picoseconds. As a result, the estimation precision of unknown parameters will be affected. On the other hand, geometry structure of CE-1 observables will be gradually deteriorated with the increase of the orbit altitude and some other problems exposed when the probe enters the mission orbit, such as the influence of lunar gravitational field, precise coordinate conversion between Earth system and Moon system, and so on. The above factors will make it difficult to obtain more precise orbit by ground ΔVLBI measurements under current observation condition, which will restrict further development of lunar exploration mission and must be resolved.

An intuitive idea is placing an antenna on a satellite in the sky, which means space very long baseline interferometry (SVLBI). SVLBI is an extension of ground based VLBI by observing stable extragalactic radio sources using VLBI telescopes mounted on the probe and on the ground respectively. Because the baseline of SVLBI may achieve a longer diameter than the Earth, more precise observables can be expected. Furthermore, SVLBI has advantages, such as improving the geometry structure of time delay observables, directly interconnecting three coordinate systems involved in geodesy and geodynamics including the terrestrial reference system, celestial reference system and dynamical reference system, calculating orbital parameters and *EOPs* simultaneously and so on [7]. So SVLBI can be attempted to improve the orbit determination of deep space probes. In this paper, the principal of SVLBI and its development status are introduced. Feasibility of the application of SVLBI in CE-1 orbit determination is analyzed. Mathematical model is derived, targeted experiments are implemented and some initial results are presented.

2. Space VLBI

The proposal of SVLBI can be traced back to 1970s. With the development of related theory and techniques, although it is difficult to mount a large antenna on a satellite, there still have been many international SVLBI projects by now, such as VSOP (Japan, 1997), RADIOASTRON (Russia, postponed), ARISE (USA, planning) and so on, in which VSOP is the 1st SVLBI satellite in human history [7,8]. These projects provided valuable data and experience for the development of SVLBI technology.

The principal of SVLBI is shown in Fig. 1. The time delay observables of SVLBI can be obtained by observing stable extragalactic radio sources using VLBI telescopes mounted on the deep space probe and on the ground, respectively. The propagation paths of signals from radio source to the two VLBI telescopes can be seen as parallel lines because radio source is too far away from two telescopes. So SVLBI time delay observables can be written as equation (1):

$$\tau = \tau_g + \Delta\tau = -\frac{1}{c}(\vec{B} \cdot \vec{K}) + \Delta\tau \tag{1}$$

where τ_g indicates the geometrical time delay observables and $\Delta\tau$ denotes the non-geometrical time delay observables, including the effects of random error and systematic errors, such as solar radiation pressure, atmospheric refraction and so on, $\vec{B}$ is the baseline vector from ground VLBI station to lunar probe, $\vec{K}$ is the direction vector radio source signals, and c denotes the light speed.

Many researchers have studied the application of SVLBI in the field of geodesy. Adam (1990) [8] studied the estimability of geodetic parameters from SVLBI observables. Kulkarni (1992) [7] researched the feasibility of SVLBI for geodesy and geodynamics. SVLBI time delay observables were simulated by VSOP design orbit. And the model parameters, such as satellite orbital parameters and Earth Rotation Parameters (*ERP*), were estimated. Results have shown that the estimation precision of *ERP* can be improved by SVLBI compared with other geodetic technologies under precise priori information of parameters and the precise modeling of systematic influence. Zheng et al. (1993) [9–11] studied the establishment of reference systems and their connection using SVLBI. Wei (2006) [12] researched the design of Chinese SVLBI system and some simulated computation. These researches show that SVLBI has great potential for the application in geodesy. SVLBI would provide effective methods to overcome the problems involved in the orbit determination strategy of CE-1 using ground based ΔVLBI geometric observables. So it can be expected that the orbit determination precision of CE-1 will be further improved if SVLBI is applied in the orbit determination. In this paper, SVLBI time delay observables are simulated for CE-1 transfer orbit to research the application of SVLBI in probe orbit determination.

3. Models and methods

3.1. Relativistic SVLBI model for CE-1 transfer orbit

In practical, ground stations are fixed in the Earth fixed coordinate system while the probe and observed radio source are defined in the geocentric celestial coordinate system. Therefore it is necessary to unify each component of equation (1) into the same coordinate system for the parameters estimation. Taking into account the relativistic effects, an SVLBI time delay observables mathematical model for CE-1 transfer orbit can be derived as equation (2) by unifying each component's coordinate systems into *J2000.0* ICRS.

$$L = -a\left\{\left(R\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}\right)^T - \begin{bmatrix} X^I \\ Y^I \\ Z^I \end{bmatrix}\right\} \cdot \left\{\begin{bmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{bmatrix}\right\} + c\Delta\tau \tag{2}$$

where (X_i, Y_i, Z_i) are the coordinates of ground VLBI station fixed in Earth-fixed system, such as *ITRF 2000*; (X^I, Y^I, Z^I) are the coordinates of CE-1 defined under *J2000.0* ICRS. (α, δ) are the coordinates of radio source fixed in *J2000.0* ICRS; a is a relativistic effect correction and is seen as a constant in this paper with its first-order expansion as $a = c^2 \cdot (c^2 - U(X_\oplus))^{-1}$, in which $U(X_\oplus)$ is the gravitational potential of Sun, Moon and other planets (excluding Earth) in Barycentric Celestial Reference System (BCRS), $X_\oplus$ is the coordinate of geocenter in *J2000.0* ICRS; R is the rotation matrix of coordinate systems which contains *EOPs*, specifically *xp*, *yp*, *UT1-UTC* (Earth Rotation Parameters *ERPs*), $\Delta\psi$ and $\Delta\varepsilon$ (nutation parameters).

Compared with the mathematical models in Adam (1990) [8] and Kulkarni (1992) [7], the derived mathematical model in this paper expands the interconnection parameters of reference system from *ERPs* to *EOPs* while considering the influence of relativistic effects on SVLBI time delay observables, which makes the mathematical model more valid to ensure the theoretical rigor of probe parameters estimation. In equation (2), (X_i, Y_i, Z_i) can be precisely obtained by GPS or other geodetic technologies and (α, δ) can be received from astronomical ephemeris, as such (X^I, Y^I, Z^I) and *EOPs* are regarded as unknown parameters in this study.

3.2. Estimation of unknown parameters

According to the mathematical model derived above, the error equation of SVLBI time delay observables can be written as:

$$V = Ax - l \tag{3}$$

where A is the design matrix formed by partial derivatives of unknown parameters, x is the correction of unknown parameters (including CE-1 orbital parameters and *EOPs*) and l is the difference between observables and model values. Then unknown parameters can be estimated using the Least Squares Adjustment Principle [13,14].

Estimability of unknown parameters can be analyzed by studying the linear correlation of each column of matrix A. If the linear correlation between columns of A exists, the matrix will show column rank deficient and the normal matrix will consequently be singular, which means that not all of the parameters are estimable. According to correlation test results, there is no linear correlation existing among each parameter involved in SVLBI time delay observables. The estimated parameters of the derived model are as follows:

$$(X_S, Y_S, Z_S, x_p, y_p, UT1\text{-}UTC, \Delta\psi, \Delta\varepsilon)$$

It should be noticed that there were linear correlations between ground station coordinates, *ERP*, radio source ascension, and probe orbit right ascension of the ascending node (RAAN) in the mathematical model [8] to influence the estimated precision of unknown parameters because nutation parameters are not involved. The estimability analysis in this paper illustrates the function of nutation parameters for improving the linear correlation of unknown parameters. The model derived in this paper is suitable for studying the influence of *EOPs* to probe orbital parameters and the orbit strategy of estimating *EOPs* and probe orbital parameters simultaneously by SVLBI time delay observables.

4. Experiments and results

Because there was no VLBI antenna on CE-1, simulated SVLBI time delay observables are used in this paper for the estimation of CE-1 orbital parameters and *EOPs*. To compare with CE-l's engineering orbit determination precision, SVLBI time delay observables are firstly be simulated under CE-1 observation condition.

4.1. Parameter estimation under the observation condition of CE-1

1) The observation condition of CE-1

(1) 4 Ground VLBI stations of China VLBI Net (CVN), including VLBI antennas in Shanghai (SH), Beijing (BJ), Kunming (KM) and Urumqi (UM), which are fixed in *ITRF 2000* coordinates.

(2) Epochs with a sample interval of about 5 s.

(3) CE-1 a priori orbital ephemeris fixed in *J2000.0 ICRS* coordinates with a sample interval of 1 min, which are interpolated by Chebyshev polynomials [15] into 5 s.

(4) The radio sources fixed in *J2000.0 ICRS* coordinates, which are selected according to the average direction of CE-1 in *J2000.0 ICRS* during the selected orbital arc;

(5) A priori values of *EOPs* are from IERS.

2) Simulation of SVLBI time delay observables

According to equation (1), the geometrical time delay τ_g and non-geometrical time delay $\Delta\tau$ are involved in SVLBI time delay observables, in which τ_g can be simulated conveniently by the mathematical model equation (2) according to observation condition of CE-1.

Non-geometrical time delay $\Delta\tau$ is mainly depending on the random error and the systematic errors, in which random error is a kind of stochastic quantity with the characteristics of normal distribution, so it is also called white noise. And the random error is very important to SVLBI simulated time delay observables because of the authenticity of simulation and the mathematical characteristics of Least Squares Adjustment Principle. On the other hand, effects of solar radiation pressure, atmospheric refraction, clock errors and relativistic effect will be concerned in the simulated observables as the major systematic errors because they are significant to affect SVLBI system.

3) Simulation steps

(1) Visibility testing. The purpose of visibility testing is to confirm that CE-1 and ground station can receive the signals from target radio source at some epochs simultaneously.

(2) If both CE-1 and ground station can receive the signals from radio source simultaneously, the geometrical time delay τ_g can be simulated. Otherwise the visibility testing for next epoch will be carried on.

(3) Simulation of the random error at these epochs.

(4) Simulation of the systematic errors at the epochs by calculating the residuals of solar radiation pressure, atmospheric

Table 1
Estimation precision of CE-1 orbital parameters and *EOPs* during transfer orbit by ΔVLBI [5].

	σ_{X_p} (mas)	σ_{y_p} (mas)	$\sigma_{UT1\text{-}UTC}$ (ms)	$\sigma_{\Delta\psi}$ (mas)	$\sigma_{\Delta\varepsilon}$ (mas)	σ_{Xs} (m)	σ_{Ys} (m)	σ_{Zs} (m)
31 Oct.	0.133	0.116	0.012	0.269	0.736	371.79	631.71	330.22
1 Nov.	0.070	0.070	0.024	0.005	0.002	459.32	717.64	430.66
2 Nov.	0.035	0.035	0.012	0.027	0.009	778.47	966.63	571.95
3 Nov.	0.011	0.011	0.004	0.011	0.003	1506.07	1208.95	712.99
4 Nov.	0.013	0.013	0.004	0.024	0.004	128.13	132.96	155.33
5 Nov.	0.035	0.035	0.002	0.027	0.008	80.4	75.67	105.14

Table 2
Estimation precision CE-1 orbital parameters and *EOPs* on 31 October 2007 from each baseline measurements.

Baseline	Baseline length (km)	σ_{X_p} (mas)	σ_{y_p} (mas)	$\sigma_{UT1\text{-}UTC}$ (ms)	$\sigma_{\Delta\psi}$ (mas)	$\sigma_{\Delta\varepsilon}$ (mas)	σ_{Xs} (m)	σ_{Ys} (m)	σ_{Zx} (m)
SH-BJ	1114	0.244	0.214	0.026	0.493	1.349	867.47	1552.53	771.86
SH-KM	1920	0.233	0.204	0.024	0.470	1.287	730.61	1299.40	658.41
BJ-KM	2158	0.199	0.174	0.021	0.402	1.101	855.76	1539.93	777.70
SH-UM	3249	0.170	0.149	0.018	0.344	0.941	509.73	916.94	464.60

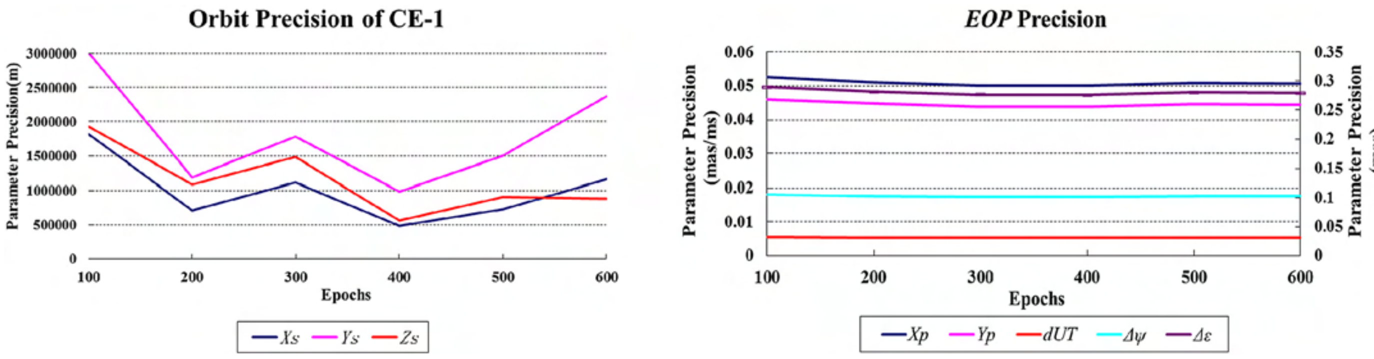

Fig. 2. Parameter estimation precision by the simulated SVLBI observables on 31 Oct., 2007.

refraction, clock errors, relativistic effect and so on. Then non-geometrical time delay $\Delta\tau$ can be generated.

(5) Simulation of SVLBI time delay observables by τ_g and $\Delta\tau$.

4) Estimation results and analysis

CE-1 transfer orbit lasted from 31 Oct. to 5 Nov. 2007. It is necessary to compare the estimation precision of CE-1 orbital parameters and *EOPs* based on ground measurements and SVLBI time delay observables respectively to show the advantage of SVLBI in orbit determination and *EOPs* estimation. The estimation precision of each parameter has been analyzed during the transfer orbit based on ground ΔVLBI time delay observables [5,6]. The results are shown as Table 1.

It can be seen from Table 1 that, although the estimation precision of daily *EOPs* during transfer orbit may achieve a better precision level than IERS, and that the estimation precision of CE-1 orbital parameters is just to a level ranging from several hundred meters to kilometers, especially the Ys component. The precision of $\Delta\varepsilon$ and *UT1-UTC* has not been improved when compared with their predicted values [5], which means the two parameters are sensitive to error of ΔVLBI time delay measurements.

In fact, the results listed in Table 1 are based on ground measurements obtained by 6 baselines of CVN's 4 stations. Space geometry structure of measurements, such as baseline distance, visibility arc to the satellite or to the radio source is critical to parameter estimation precision. The unknown parameters from ΔVLBI measurements of each baseline have been estimated in this paper to analyze the influence of space geometry on estimation precision. The results of 4 baselines measurements on 31 Oct. 2007 are listed in Table 2.

It can be seen clearly from Table 2 that estimation precision of unknown parameters are improved with the increased baseline length. Each parameter's estimation precision is almost twice of the corresponding values listed in Table 1, which cannot meet the demand of the application. The results confirm the limitations of ground based measurements in parameter estimation again. Since the space geometrical structure of ground measurements is unlikely to be improved under current equipment condition, it would cause enormous effect on the orbit determination precision of our deep space probes in the future. SVLBI observables have longer baselines than any ground measurements. In addition, SVLBI observables will not be affected by the atmosphere, such as ionospheric refraction and other similar factors that cannot be ignored on the ground. Thus the study of SVLBI for higher-precision positioning is important for further research.

The study strategy of SVLBI is similar to the research of ground ΔVLBI measurements [5]. Since *EOPs* are very sensitive to the fast flying probe, the whole orbit should be divided into several sub-arcs, the optimal length of which should be determined. On the other hand, a priori precision of *EOPs* should also be considered. These methods are carried on to insure that the estimation precision of CE-1 orbit and *EOPs* is optimal [10]. Fig. 2 illustrates an example of parameter estimation precision by SVLBI simulated observables under different sub-arc's length during CE-1 transfer orbit on 31 October 2007.

Fig. 2 illustrated that the estimation precision of *EOPs* under different lengths of the test orbital arc is quite well and steady. They all achieve the accuracy level of IERS and are improved by nearly an order of magnitude when compared with the results of ground ΔVLBI measurements listed in Table 1 and Table 2.

However, the estimation precision of CE-1 orbital parameters is not-well-adjusted, which is larger than 3000 km. It cannot meet the demand practical application and is less accurate than the results calculated by ground ΔVLBI measurements. It can be con-

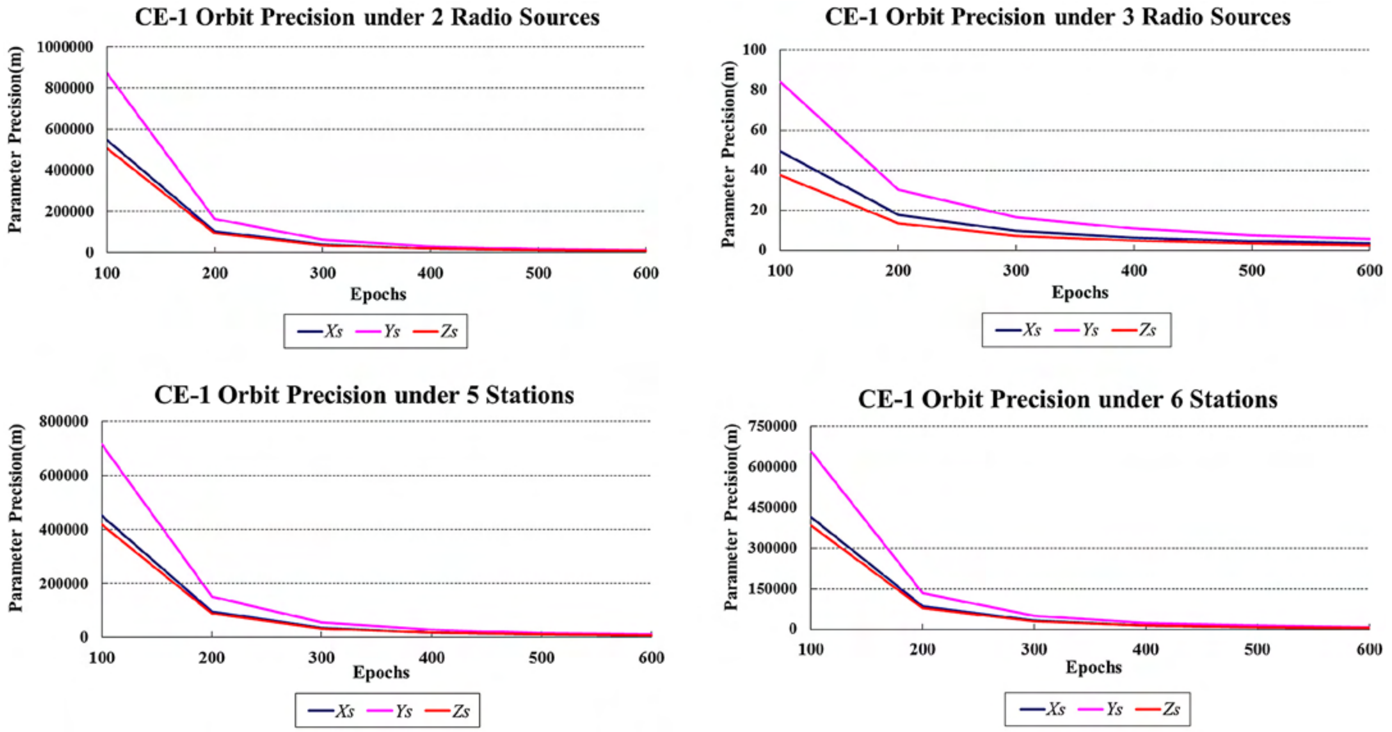

Fig. 3. Orbital parameters' precision by the simulated SVLBI observables under different observation condition.

cluded that SVLBI observables simulated under current observation condition of CE-1 are not sensitive to orbital parameters. This can be explained as CE-1 moved fast during the transfer orbit, the geometry structure of SVLBI observables is quite weak when stations just received signals from one target radio source, which will cause ill-conditioned normal equation and affect parameter estimation precision.

In order to improve CE-1 orbit determination precision, the observation condition needs to be optimized. We increase the number of ground stations and radio sources in this paper for SVLBI observables simulation to study the improvement of CE-1 orbital parameters estimation precision. The results are illustrated as follows.

The results show that the estimation precision of CE-1 orbital parameters is improved obviously with the optimization of observation condition. The orbit determination precision is better than one hundred meters at 600 epochs under each observation condition. Therefore, the good observation condition is significant for the estimation of CE-1 orbital parameters using SVLBI simulated observables.

4.2. *Optimal observation condition of CE-1 for SVLBI technology* [16]

According to the analysis above, a favorable observation condition is essential for parameter estimation by SVLBI time delay simulated observables. Therefore optimal observation condition of CE-1, including station numbers, observing epochs and numbers of target radio sources, must be studied to obtain optimal estimation precision of CE-1 orbital parameters and *EOPs*. The estimation precision of CE-1 orbital parameters can be significantly enhanced with the improvement of observation condition, however, the estimated precision of *EOPs* would change irregularly as showed in Fig. 3 on the next stage. This paper intends to focus on the change of *EOPs* estimation precision under different observation conditions to confirm the optimal observation condition for SVLBI time delay observables simulation.

Table 3
Observation condition of CE-1.

Parameter	Range of variability
Number of stations	4–10
Observable epochs	300–5300
Number of radio sources	1–9

Different observation conditions of CE-1 are designed in Table 3. By computing a coefficient matrix normal equation A under different observation conditions, the change of *EOPs* estimation precision can be obtained under Least Squares Adjustment Principle to analyze optimal observation condition of CE-1 for SVLBI time delay observables simulation and parameter estimation. During the calculation, the weight matrix of SVLBI time delay observables are set as unit matrix and a priori unit weight variance of parameters are determined by the precision of SVLBI observables, which is 0.05 ns according to latest research [7].

It should be mentioned that the precision of SVLBI observables, which is 0.05 ns, is just suitable for the simulated estimation in this paper. For practical application, more complex situations should be considered to ensure the authenticity and accurateness of SVLBI time delay observables, such as signal receiving ability of fast moving probe to radio source, signal-to-noise of SVLBI time delay observables and so on.

1) Impact of different station numbers on estimated parameter precision

In order to analyze the impact of different station numbers on *EOPs* estimation precision, the observable epochs are set as 1000, radio source numbers are set as 9, and station numbers can change from 4 to 10 with an interval of 1. The selection of stations is based on CVN and additional 6 stations from IVS (International VLBI Service for Geodesy and Geodynamics). The results are shown in Fig. 4 as a result of the estimation precision of *UT1-UTC* stabilizing at the magnitude order of about 0.005 ms, which is better than the precision level published by IERS. Calculation results show that the estimation precision of *EOPs* is improved gradually with

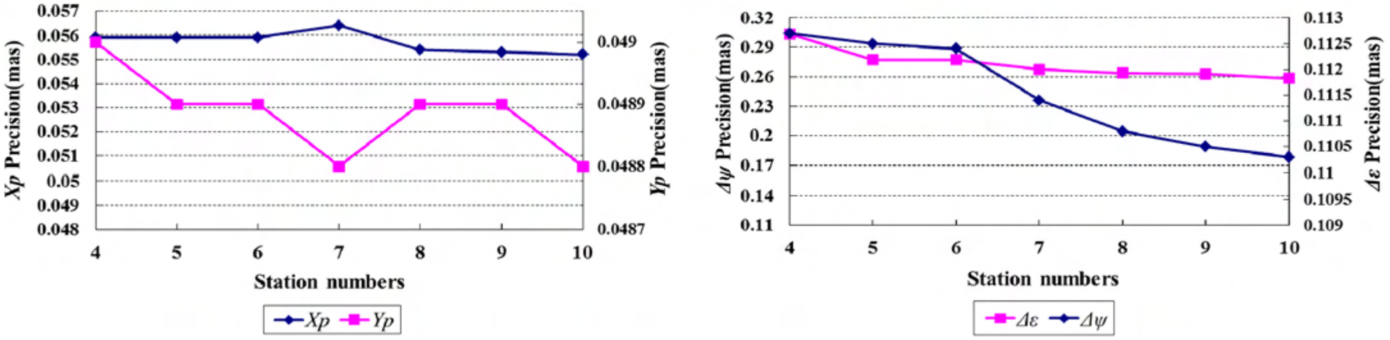

Fig. 4. *EOPs* estimation precision under different station numbers.

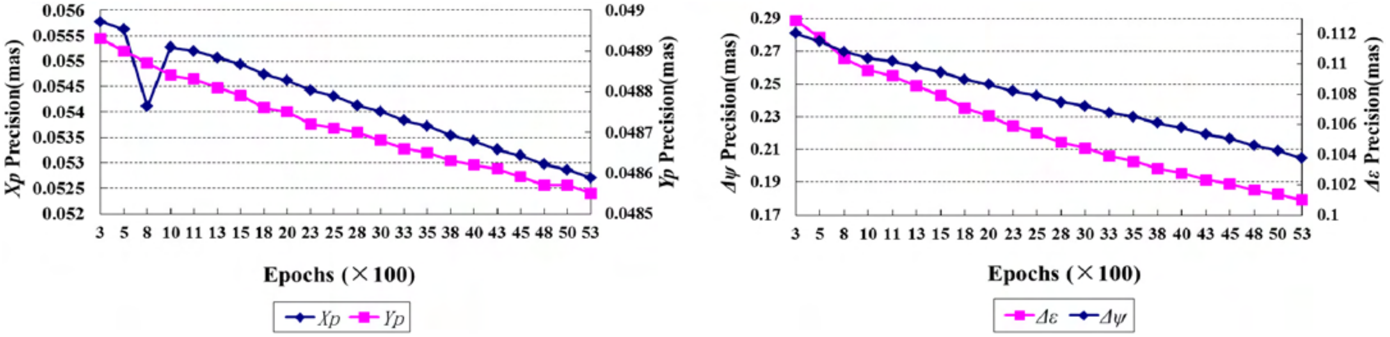

Fig. 5. *EOPs* estimation precision under different observable epochs.

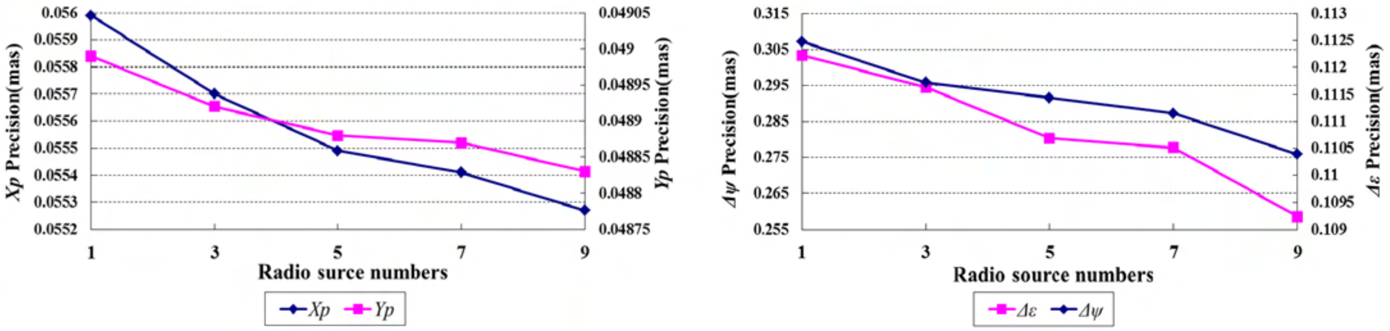

Fig. 6. *EOPs* estimation precision under different radio source numbers.

station numbers. For example, the estimation precision of polar motion parameters shows irregular fluctuations when station numbers are larger than 6, while the estimation precision of $\Delta\psi$ shows fast improving trend under the same condition. Therefore station numbers should be more than 6 to obtain stable *EOP* estimation precision.

2) Impact of different observable epochs on parameter calculation precision

In order to analyze the impact of different observable epochs on *EOPs* estimation precision, the station numbers are fixed as 6, radio source numbers are fixed as 9, and observable epochs can change from 300 to 5300. The results are shown in Fig. 5.

It can be seen from Fig. 5 that the estimation precision of *EOPs* is improved with the increasing of observable epochs. The estimation precision of each parameter shows a fluctuation when observable epochs are shorter than 1100, while after that the stable decline trend exists, although yielding no stable resolutions. Because of the length of Continuous observation arc of CE-1, observables over 5300 epochs will not be calculated here. Therefore, observable epochs must be more than 1100 to obtain stable and reasonable estimation values of *EOPs*.

3) Impact of different radio sources on parameter calculation precision

In order to analyze the impact of different radio sources on *EOPs* estimation precision, the station numbers are set as 6, observable epochs are set as 1100, and radio sources are set to change from 1 to 9 with an interval of 2. Radio sources are selected from ICRS and are uniformly distributed in the sky. The results are shown in Fig. 6.

The results show that the estimation precision of EOPs is improved gradually with the increasing of radio source numbers. When the radio source numbers are set between 5 and 7, *EOPs* estimation precision exhibits a comparatively stable trend. Considering a uniform distribution, the radio source numbers are suggested to be set as 7. On the other hand, it can be concluded that increasing radio source numbers are more conducive to improve *EOPs* estimation precision by comparing Fig. 4 and Fig. 6. The geometry structure of SVLBI observables are believed to be improved when radio source numbers increase.

In summary, to apply SVLBI in the orbit determination strategy of estimating orbital parameters and *EOP* simultaneously and to obtain high estimation precision, the following observation condi-

Table 4
Parameter estimation precision from simulated SVLBI observables of CE-1 transfer orbit.

	σ_{x_p} (mas)	σ_{y_p} (mas)	$\sigma_{UT1\text{-}UTC}$ (ms)	$\sigma_{\Delta\psi}$ (mas)	$\sigma_{\Delta\varepsilon}$ (mas)	σ_{X_S} (m)	σ_{Y_S} (m)	σ_{Z_S} (m)
31 Oct.	0.046	0.041	0.005	0.093	0.249	0.35	0.51	0.24
01 Nov.	0.047	0.041	0.005	0.092	0.223	0.79	1.16	0.58
02 Nov.	0.047	0.041	0.005	0.093	0.220	1.31	1.55	0.74
03 Nov.	0.039	0.034	0.004	0.076	0.171	1.43	1.39	0.87
04 Nov.	0.043	0.038	0.005	0.085	0.197	1.25	1.18	0.80
05 Nov.	0.050	0.044	0.005	0.101	0.263	0.06	0.07	0.08

Table 5
External coincidence of *EOPs* by simulated SVLBI observables of CE-1 transfer orbit.

	x_p (mas)		y_p (mas)		*UT1-UTC* (ms)		$\Delta\psi$ (mas)		$\Delta\delta$ (mas)	
	Predicted	Estimated	Predicted	Estimated	Predicted	Estimated	Predicted	Estimated	Predicted	Estimated
31 Oct.	0.1	0.084	0.08	0.081	−0.7717	−0.192	0.091	0.060	−0.083	−0.115
01 Nov.	−0.077	−0.141	−0.44	−0.329	−0.2211	0.351	0.772	0.451	0.115	−0.399
02 Nov.	0.195	0.1	−0.693	−0.516	−0.1916	0.380	0.934	0.505	0.153	−0.584
03 Nov.	1.027	0.904	−0.708	−0.518	−0.1004	0.468	0.804	0.217	0.121	−0.616
04 Nov.	1.773	1.655	−0.44	−0.282	−0.0419	0.531	0.536	−0.002	0.022	−0.538
05 Nov.	2.334	2.301	−0.418	−0.372	0.0646	0.652	0.331	0.149	−0.067	−0.186

tion is suggested in this paper: 6 ground stations, 1100 observable epochs, and 7 radio sources for each sub-arc.

4.3. Parameter estimation under optimal observation condition

In this section, SVLBI time delay observables are simulated under optimal observation condition discussed in Section 4.2 and unknown parameters are re-estimated. During the calculation, observable epochs of each sub-arc are fixed in 1100, while ground station numbers are set as 6 including CVN stations. As a result, 7 radio sources uniformly distributing in the sky are selected. Meanwhile, the predicted values and precision of *EOPs* are selected separately from IERS for different days. The former sub-arc's *EOPs* estimation results are seen as the a priori information of the next sub-arc's observables until the whole simulated SVLBI time delay observables of this day are calculated. The results are listed in Table 4.

The results show that the optimal estimation precision of daily CE-1 orbital parameters can all achieve a level of 2 m, which is much better than the engineering orbit determination precision and the estimation precision obtained from ground ΔVLBI measurements listed in Table 1. So SVLBI can obviously improve CE-1 orbit determination precision. It should be mentioned that Kulkarni (1992) [7] also obtained an orbit determination accuracy of centimeter level by simulated time delay observables using VSOP designed orbital elements. So the results of CE-1 orbit determination obtained in this paper are creditable. On the other hand, *EOPs* estimation precision is also very steady and can achieve a better accuracy level when compared to IERS solutions. These results reflect the technological advantages of SVLBI, which provides a brand-new method for probe orbit determination of China's Lunar Exploration Program [17].

External coincidence of each day's estimated *EOP* values during the transfer orbit are listed in Table 5. This coincidence is obtained by comparing the predicted values and the estimated values of *EOPs* with the IERS final solutions, respectively. It can be seen clearly that the estimation precision of some *EOP* components is improved when compared with their predicted values except $\Delta\varepsilon$ and *UT1-UTC*. So the predicted *EOP* precision can be improved by the orbit determination strategy of estimating orbital parameters and *EOP* simultaneously using SVLBI simulated time delay observables, which can also reflect the availability of CE-1's estimated orbit precision from another side.

Using the ground ΔVLBI time delay measurements and simulated SVLBI time delay observables, $\Delta\varepsilon$ and *UT1-UTC* estimation precision cannot be improved when compared with their predicted values. So it is suggested to take $\Delta\varepsilon$ and *UT1-UTC* as known values during the calculation to reduce the effects of their predicted precision on the estimation precision of the CE-1 orbital parameters and *EOPs*. We estimate unknown parameters using the simulated SVLBI time delay observables to analyze the estimation precision when fixing the two parameters values. The results of SVLBI simulated observables on 31 October 2007 are shown here as an example. External coincidence of x_p, y_p and $\Delta\psi$ is 0.057 mas, 0.021 mas and 0.014 mas, respectively, which are improved when compared with the corresponding values listed in Table 5. So, higher precision can be obtained by fixing $\Delta\varepsilon$ and *UT1-UTC* as known values.

5. Conclusion

SVLBI has potential in the field of deep space probe orbit determination and geodynamics. In this paper, a mathematical model for probe orbital parameters and *EOPs* estimation from relativistic SVLBI simulated observables for CE-1 transfer orbit is derived and the estimability of unknown parameters is discussed. By comparing with the results of ground ΔVLBI time delay measurements, the SVLBI has advantage to improve probe orbit determination and *EOPs* estimation. The favorable observation condition of CE-1 is crucial for the estimation of parameters from simulated SVLBI time delay observables by the analysis of parameter estimation precision under CE-1 actual observation condition. Furthermore, the optimal observation condition is determined by analyzing the precision of *EOPs* estimation. Finally, unknown parameters are estimated and effects are evaluated.

The results show that the estimation precision of CE-1 orbital parameters can achieve a level of several meters by simulated SVLBI observables under optimal observation condition, which is much better than the orbit determination precision from ground ΔVLBI time delay measurements.

The estimation precision of *EOPs* also can achieve or be better than IERS accuracy level, and some components' predicted precision can be improved when compared with the predicted values by IERS and when fixing $\Delta\varepsilon$ and *UT1-UTC* as known values. The estimation precision of remaining *EOPs* components can achieve a level of 0.01 mas. Although real SVLBI has lots of challenges, such as huge antenna area, huge rocket launch demand, observation targeting ability, data transmission ability and so on, this paper presents a new advantage for SVLBI applications.

Conflict of interest statement

The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgements

This research was funded by National Natural Science Foundation of China (Grant Nos. 41374012, 41490633, 41304021, 11173050 and 11373059), National Keystone Basic Research Program (MOST 973) (Grant No. 2012CB72000), Shanghai Science and Technology Commission Project (Grant No. 12DZ2273300) and the Young Researcher Grant of National Astronomical Observatories, Chinese Academy of Sciences.

References

[1] M.T. Zuber, D.E. Smith, M.M. Watkins, et al., Gravity field of the Moon from the Gravity Recovery and Interior Laboratory (GRAIL) mission, Science 339 (6120) (2012) 668–671, http://dx.doi.org/10.1126/science.1231507.

[2] Y. Huang, Orbit determination of the first Chinese lunar exploration spacecraft CE-1, PhD thesis, Shanghai Astronomical Observatory, 2006.

[3] W. Wang, Accuracy analysis of orbit determination for Chinese lunar exploration mission, PhD thesis, Shanghai Astronomical Observatory, 2005.

[4] M. Chen, G. Tang, J. Cao, et al., Precision orbit determination of CE-1 lunar satellite, Geomat. Inf. Sci. Wuhan Univ. 36 (2) (2011) 212–217.

[5] E. Wei, W. Yan, S.G. Jin, et al., Improvement of Earth orientation parameters estimate with Chang'E-1 ΔVLBI observations, J. Geodyn. 72 (2013) 46–52.

[6] W. Yan, E. Wei, J. Liu, Determination of CE-1 orbit and EOPs with ΔVLBI observation in Earth–Moon transfer orbit, Geomat. Inf. Sci. Wuhan Univ. 37 (8) (2012) 960–962.

[7] M.N. Kulkarni, A feasibility study of space VLBI for geodesy & geodynamics, Report No. 420, Dept. of Geodetic Science & Surveying, the Ohio State University, Columbus, Ohio, 1992.

[8] J. Ádám, Estimability of geodetic parameters from space VLBI observables, Report No. 406, Dept. of Geodetic Science and Surveying, the Ohio State University, Columbus, Ohio, July 1990.

[9] Y. Zheng, Z. Qian, Using space VLBI to establish reference systems and their connection, J. PLA Inst. Surv. Mapp. 3 (1992) 19–22.

[10] Y. Zheng, VLBI Geodesy, Chinese PLA Press, Beijing, 1999.

[11] K. Lin, Very Long Baseline Interferometry, China Astronautic Publishing House, Beijing, 1985.

[12] E. Wei, Research on the designment of Chinese space VLBI system and computation simulation, PhD thesis, Wuhan University, 2006.

[13] W. Huang, Modern Adjustment Theory and Its Application, Chinese PLA Press, Beijing, 1990.

[14] X. Cui, Z. Yu, B. Tao, et al., General Surveying Adjustment, Wuhan University Press, Wuhan, 2005.

[15] P. Yu, X. Sun, S. Zhao, Chebyshev polynomial fitting model for GPS orbit calculation, Meteor. Sci. Technol. 32 (3) (2004) 198–201.

[16] W. Yan, E. Wei, J. Liu, On the sensitivity of EOPs to the priori precision of Chang'E-1's simulated parameters, in: Proceeding of CPGPS2010, Shanghai, China, 2010, pp. 321–326.

[17] S.G. Jin, S. Arivazhagan, H. Araki, New results and questions of lunar exploration from SELENE, Chang'E-1, Chandrayaan-1 and LRO/LCROSS, Adv. Space Res. 52 (2) (2013) 285–305, http://dx.doi.org/10.1016/j.asr.2012.11.022.