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# Improvement of Earth orientation parameters estimate with Chang'E-1 $\Delta$ VLBI observations

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## ABSTRACT

Earth orientation parameters (*EOP*) are essential for the interconnection of different reference systems involved in Chang'E-1 (CE-1) lunar exploration, such as the Earth fixed reference system, celestial reference system and dynamical reference system. To improve the accuracy of predicted *EOP* values and to reduce their influence on the accuracy of CE-1 orbital parameters, a relativistic mathematical model of differential VLBI ( $\Delta$ VLBI) time delay observations for the CE-1 transfer orbit is derived in this paper, which is generated by differencing CE-1 time delay observations with a simulated radio source's time delay observations. The CE-1 orbital parameters and *EOP* are simultaneously estimated with least squares adjustment using the measured time delay observations of the CE-1 transfer orbit. The results show that the accuracy of the CE-1 orbital arc length and the win-win approach is able to improve the accuracy of both the CE-1 orbital parameters and *EOP* estimates. The estimated CE-1 orbital accuracy can achieve a few hundred meters and the estimated *EOP* accuracies are better than their predicted values.

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# 1. Introduction

Deep space exploration has become hot topics in the 21st century (Wei et al., 2013). China's first lunar probe Chang'E-1 (CE-1) was successfully launched on 24 October 2007. Abundant information has been obtained by CE-1 for China's lunar exploration, such as lunar topography, gravitational field, atmosphere and so on (Ouyang et al., 2010). In the process of lunar exploration, precise orbit determination of CE-1 in the transfer orbit is very important because it directly relates to the success of subsequent scientific experiments, e.g., whether the lunar probe can enter the mission planned orbit, whether high orbital accuracy can be provided for the lunar exploration application and so on. On the other hand, Earth oriental parameters (EOP) refer to the interconnection of the Earth fixed reference system and lunar reference system for the lunar probe positioning. So the orbit and EOP determination in the lunar transfer orbit has important practical significance and application value.

In recent years, Very Long Baseline Interferometry (VLBI) has become one of the main technologies in deep space probe orbit determination because of its high precision and high angular resolution. For example, Liu et al. (2009) have shown that the orbit determination accuracy of a lunar probe by VLBI technology can achieve the level of tens of meters so that VLBI has great potential for this application. In practical engineering application, unified Sband monitoring system (USB) and VLBI have been used for CE-1's orbit determination, in which VLBI also played an important role when the probe entered the transfer orbit. Therefore, it is necessary to further study the CE-1 orbit determination by VLBI technology to promote the development of China's deep space exploration program.

A number of researches have been performed about CE-1 orbit determination by VLBI time delay and delay rate data (e.g., Huang, 2006; Li et al., 2009). However, most results show that the orbit determination accuracy of CE-1 with just 1–2 km for the transfer orbit and several hundred meters for the mission orbit cannot meet the high precision requirements. Therefore, how to improve the probe orbit determination accuracy has become the main challenge of China's Lunar Exploration Program. On the other hand, Earth orientation parameters (*EOP*) are still not well estimated or understood due to the complex geophysical mechanisms (Jin et al., 2010, 2011, 2012), which are essential for the interconnection of various reference systems. While the predicted *EOP* values are used in practice, whose influence on CE-1 orbit determination accuracy are not clear. Furthermore, the





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Fig. 1. The orbit of CE-1.

relativistic effect correction is also relevant to orbit determination because the current accuracy of the CE-1 VLBI time delay observations has achieved the same level of the relativistic effects on the time delay observations with a level of *ns* (Zheng, 1999), which should be considered.

To solve the problems above, differential VLBI ( $\Delta$ VLBI) technology is used in this paper.  $\Delta$ VLBI observations are composed of differencing time delay observations of the probe's radio signals and nearby alternating observed radio source's signals. The differential technology will eliminate the common errors of the probe's radio signals and radio source's signals during the propagation path, such as the station location errors and transmission media delays, so high-precision parameters can be estimated. Here, *EOP* and relativistic effect corrections are involved into the CE-1 orbit determination. A relativistic  $\Delta$ VLBI time delay mathematical model for orbital parameters and *EOP* estimate is derived and unknown parameters are estimated simultaneously using the measured time delay data. The accuracies of the CE-1 orbital parameters and *EOP* estimate are assessed and discussed.

#### 2. Methods and models

## 2.1. $\Delta$ VLBI for the CE-1 transfer orbit

CE-1 was launched on 24 October 2007 from Xichang Satellite Launch Center. The entire flight progress of CE-1 can be divided into the phasing orbit, transfer orbit and mission orbit (Fig. 1) (Huang, 2006). USB and VLBI have been used for CE-1 orbit determination during the flight, in which VLBI played an important role because of its high measurement accuracy when the probe entered the transfer orbit. This paper will focus on the estimation of the probe orbital parameters and *EOP* in the CE-1 transfer orbit.

Currently the CE-1 orbit was determined without estimation of *EOP*. Because CE-1 time delay observations contain the components of the probe orbital parameters and *EOP*, it is possible to estimate these parameters simultaneously. Here the USB observations are not used, which may affect the accuracy of CE-1 orbit determination from VLBI time delay observations. In order to obtain high accuracy CE-1 orbit and *EOP* parameters, the  $\Delta$ VLBI technology is used in this paper.

The principle of  $\Delta$ VLBI is illustrated in Fig. 2(a), in which *k* is the direction vector of the CE-1 signal,  $\overline{I}$  is the direction vector of a alternating observed radio source signal, and  $\overline{B}$  is the baseline vector of two ground stations. Two factors must be considered during the derivation of mathematical model:



**Fig. 2.** Principle of  $\Delta$ VLBI observations (a),  $R_{10}$  and  $R_{20}$  represent the distance of the CE-1 to ground station 1 and 2 respectively (b), which are not equal so that  $\vec{k}$  is neither parallel to  $R_{10}$  nor  $R_{20}$ ,  $\psi$  is the angle between  $\vec{I}$  and  $\vec{B}$  (c) and  $\tau$  is the time delay of the radio source signals. It indicates that  $\vec{k}$  and  $\vec{I}$  are not parallel to each other.

- 1. The propagation paths of the CE-1 signals and radio source signals are different (Fig. 2(b) and (c)). The former composes a small angle whose vertex is the probe, and the latter can be seen as parallel. As the result,  $\bar{k}$  and  $\bar{I}$  are different as shown in Fig. 1 (Sekido and Fukshima, 2005);
- 2. The relativistic effects must be considered because the accuracy of the CE-1 VLBI time delay observations has achieved the level of the relativistic effects of the time delay observations and will affect estimated parameter accuracy.

According to the analysis above, the  $\Delta$ VLBI time delay mathematical model is derived as follows:

$$L = c\Delta\tau + c\tau' = c(\tau_{CE} - \tau_{S}) + c\tau' = B \cdot (k - l) + c\tau'$$

$$= -\left(R \begin{bmatrix} X_{1} - X_{2} \\ Y_{1} - Y_{2} \\ Z_{1} - Z_{2} \end{bmatrix}\right)^{T} \cdot \left(R \begin{bmatrix} X_{1} + X_{2} \\ Y_{1} + Y_{2} \\ Z_{1} + Z_{2} \end{bmatrix} - 2 \begin{bmatrix} X_{S} \\ Y_{S} \\ Z_{S} \end{bmatrix}\right) \cdot \left|2 \begin{bmatrix} X_{S} \\ Y_{S} \\ Z_{S} \end{bmatrix}^{T} - \left(R \begin{bmatrix} X_{1} + X_{2} \\ Y_{1} + Y_{2} \\ Z_{1} + Z_{2} \end{bmatrix}\right)^{T}\right|^{-1} - a \begin{bmatrix} \cos \delta \cos \alpha \\ \cos \delta \sin \alpha \\ \sin \delta \end{bmatrix}\right\} + c\tau'$$

$$(1)$$

where  $\tau_{CF}$  is the time delay observations of CE-1 signals,  $\tau_s$  is the time delay observations of radio source signals, au' is the residual time delay including the differential residual parts of the time delay observations caused by the atmosphere, station errors, clock parameters and so on. Most of these errors are removed by the difference, which can be ignored. For the expansion of  $\bar{k}$ ,  $\bar{l}$ , and  $\overline{B}$ ,  $(X_i, Y_i, Z_i)$  (i = 1, 2) are the coordinates of ground VLBI stations in ITRS,  $(X_S, Y_S, Z_S)$  are the coordinates of CE-1 in the J2000.0 ICRS, and  $(\alpha, \delta)$  are the right ascension and declination coordinates of the radio source in the J2000.0 ICRS. The  $a = c^2/c^2 - U(X_{\oplus})$  is the correction coefficient of general relativistic effect, where *c* is the light speed and  $U(X_{\oplus})$  is the gravitational potential of sun, moon and other planets (excluding Earth) in the Barycentric Celestial Reference System (BCRS), with the first order term expressed as  $U(X_{\oplus}) = \sum_{A \neq M} GM_A / r_A$ , where  $M_A$  is the lunar mass,  $r_A$  is the distance between the Earth and the Moon, and R is the rotation matrix containing the EOP, specifically xp, yp, UT1-UTC (Earth rotation parameters *ERP*),  $\Delta \psi$  and  $\Delta \varepsilon$  (nutation parameters).

#### 2.2. Adjustment model

According to the mathematical model above, the observation equation of the  $\Delta$ VLBI time delay observations can be expressed as:

$$V = Ax - l \tag{2}$$

where *A* is design matrix formed by partial derivatives of unknown parameters, *x* is correction of unknown parameters (including the CE-1 orbital parameters and *EOP*), and *l* is the difference between the observations and the model values. Accounting for the a priori accuracy of the *EOP* ( $D(X) = \sigma_{X0}^2 P_X^{-1}$ ), the estimate of *x*, denoted as  $\hat{x}$ , can be determined with least squares adjustment (Cui et al., 2005):

$$(A^{T}PA + \sigma_{0}^{2} \cdot \sigma_{X0}^{-2} \cdot P_{X})\hat{x} = A^{T}Pl$$

$$\hat{x} = (A^{T}PA + \sigma_{0}^{2} \cdot \sigma_{X0}^{-2} \cdot P_{X})^{-1}(A^{T}Pl)$$
(3)

where *P* is the weight matrix of observations and *P*<sub>X</sub> is the a priori weight matrix of the unknown parameters,  $\sigma_0^2$  is the a priori unit weight variance of observations, and  $\sigma_{X0}^2$  is the a priori unit weight variance of the unknown parameters. So the accuracy of  $\hat{x}$  can be estimated as:

$$D\{\hat{x}\} = \hat{\sigma}_0^2 (A^T P A + \sigma_0^2 \cdot \sigma_{X0}^{-2} \cdot P_X)^{-1}$$
(4)

where  $\hat{\sigma}_0^2$  is the posteriori variance of unit weight.

# 3. Observations and processing strategies

## 3.1. Observations and data

The CE-1 transfer orbit lasted from 31 October to 5 November 2007. The data used for the simultaneous estimation of the CE-1 orbital parameters and *EOP* are as follows:

- 1. Ground VLBI stations: China VLBI Net (CVN), including Shanghai (SH), Beijing (BJ), Kunming (KM) and Urumqi (UM) with ITRS coordinates.
- 2. The CE-1's time delay observations: measured time delay observations of CE-1 in the transfer orbit with a sample interval of about 5 s;
- A priori values of CE-1 orbit with J2000.0 ICRS coordinates: 1 min sample interval and interpolated every 5 s by Chebyshev polynomials (Yu et al., 2004);
- 4. The radio source's time delay observations of: the radio source is selected by the average direction of CE-1 during the orbital arc



Fig. 3. The difference of orbit determination results between two strategies.

and time delay observations are simulated under the principle of VLBI;

5. A priori values of EOP: predicted values from IERS.

#### 3.2. Orbit determination strategy

In real time applications of lunar exploration mission, predicted *EOP* values are used for the CE-1 orbit determination. Therefore, it needs to be further studied whether the prediction accuracy can meets the demand of the CE-1 orbit determination. For this purpose, the difference of orbit results between estimating orbital parameters and the simultaneously estimating orbit and *EOP* parameters is analyzed by the measured time delay observations of CE-1 on 31 October 2007. The results are shown in Fig. 3.

It can be seen that the difference between two strategies is more than 40 m in each CE-1 coordinate component, and over 100 m in the Ys component. So EOP is critical for the CE-1 orbit determination by  $\Delta$ VLBI time delay observations, which must be regarded as unknown parameters during the estimation. In addition, EOP is also important in the interconnection of Earth coordinate system and celestial reference system involved in the  $\Delta$ VLBI time delay observations. So the CE-1 orbit parameters and EOP should be estimated simultaneously.

# 3.3. Calculation strategy

Using the orbit determination strategy analyzed above, the unknown parameters cannot be estimated with a least squares adjustment in a single epoch algorithm because there are 11 unknown parameters (6 orbital parameters and 5 *EOP*) with just 6 observations (6 ground baselines). Therefore, an overall adjustment of multiple epochs is used in this paper. Using this method, CE-1 can be treated as one orbit for a selected orbital arc so that the orbital parameters and *EOP* can be estimated. The specific method is as follows:

- 1. The time delay observations must be selected before the orbit determination to ensure that there are 6 time delay observations per epoch.
- 2. The gross errors of time delay observations must be removed because they will affect the estimated parameter accuracy. The standard of data rejection is that the time delay residuals of each baseline at an epoch must be located in 3 times of accuracy of time delay observations  $(\pm 3\sigma)$ , and otherwise the observations will be removed. Furthermore, the amount of removed data cannot exceed 10% of the total observations.
- 3. The entire CE-1 transfer orbit is calculated in one day and each day's orbit is divided into several sub-arcs. The sub-arc's length is increased from 100 to 900 epochs with an interval of 100 or 50 epochs related to data quality.
- 4. The orbital elements of each sub-arc's initial epoch are calculated by the interpolation of the CE-1's coordinates and velocities.

Then the coordinates at subsequent epochs can be obtained by Kepler equation for the calculation of the  $\Delta$ VLBI time delay model values.

- 5. The optimal length of the sub-arc is determined by analyzing the internal and external agreement of each unknown parameter.
- 6. When the optimal length of one sub-arc is determined, the adjustment values and accuracy of the *EOP* are seen as a priori information for the next sub-arc to determine its optimal orbital arc length until all observations of this day are calculated.
- 7. The adjustment *EOP* values and accuracy of the previous day are not involved into the calculation of the next day, which means that predicted *EOP* values are taken from IERS each day.
- 8. The weight matrix of observations *P* is set as *I* with the same precision and the a priori weight matrix as the *EOP*. *P*<sub>X</sub> is set by the predicted values from the IERS with  $P_X = \sigma_{X0}^2 / \sigma_X^2$ , where  $\sigma_X$  is the a priori accuracy of the *EOP*.

#### 4. Results and discussion

In the following, the parameters of orbit and EOP are estimated by the time delay observations of CE-1 on 31 October 2007, and then the estimation results of the entire transfer orbit are analyzed and discussed.

# 4.1. Parameter estimation by time delay observations of CE-1 on 31 October 2007

## 4.1.1. Time delay residuals analysis

In the parameter estimation process, the time delay residuals will affect the value of posteriori variance of unit weight  $\hat{\sigma}_0^2$  and the estimated parameter accuracy when the number of time delay observations is constant. So the time delay residuals of the base-lines need to be analyzed. The time delay residuals of 6 baselines using the time delay observations of CE-1 on 31 October 2007 with a different number of epochs are shown in Fig. 4. The time delay



Fig. 4. Time delay residuals of each Baseline. "SH-BJ" represents the baseline from Shanghai to Beijing, "SH-KM" is the baseline from Shanghai to Kunming, "SH-UM" shows the baseline from Shanghai to Urumqi, "BJ-KM" represents the baseline from Beijing to Kunming, "BJ-UM" is the baseline from Beijing to Urumqi, and "KM-UM" is the baseline from Kunming to Urumqi.



Fig. 5. CE-1 coordinates accuracy of initial epoch.



Fig. 6. EOPs' accuracy.

residuals of each baseline are distributed between -1.5 m and 1.5 m  $(\pm 3\sigma)$ , indicating the advantage of  $\Delta$ VLBI technology. On the other hand, with the increase of the sub-arc length, the time delay residuals become larger, which will generate a bigger  $\hat{\sigma}_0^2$  to decrease the estimated accuracy of the unknown parameters according to Eq. (4). Therefore, the orbital arc length is important for the accuracy of the unknown parameters and must be selected carefully.

#### 4.1.2. Optimal orbital arc length

The estimated accuracy of the unknown parameters for the first sub-arc of CE-1 on 31 October 2007 with different orbital arc lengths is shown from Figs. 5 to 7. Fig. 5 shows the estimated CE-1 orbital parameters accuracy for the initial epoch. The accuracy improves from tens of kilometers to several hundred meters with the increase of the sub-arc length. So it can be concluded that the estimated CE-1 coordinates' accuracy can be improved by increasing the number of observations. The accuracy stops changing significantly after 600 epochs. Figs. 6 and 7 show that the estimated EOP accuracy decreases with the increase of the subarc length, which still maintains the same order of magnitude. The

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Externally coincident accuracy	of EOP.





Fig. 8. Externally coincident accuracy of EOP.

accuracy stops changing significantly after 600 epochs too. So the optimal orbital arc length of the first sub-arc of CE-1 on 31 October 2007 will be larger than 600 epochs according to the estimated parameter accuracy.

Then the externally coincident accuracy of EOP is calculated by differencing the IERS final EOP values with the adjusted values of this paper, which is used to determine the optimal orbital arc length with the optimal estimated accuracy of the parameters. The externally coincident accuracy of EOP improves with the increase of the sub-arc length and then decreases (Fig. 8). It reached an optimal value when the sub-arc length is about 600 epochs, and then some parameters' estimated accuracy begins to diverge, just like UT1-UTC. Unfortunately, the externally coincident accuracy of CE-1 coordinates is not calculated and discussed in this paper because no accurate real orbit determination results of CE-1 are available. Taking into account of the estimated accuracy and the external agreement of the unknown parameters, the optimal length of the first sub-arc of CE-1 on 31 October 2007 is 600 epochs for the parameter estimation by the  $\Delta$ VLBI time delay observations.

Taking the adjusted values and accuracy of the EOP as a priori information for the next sub-arc, the entire orbit's observations of CE-1 on 31 October 2007 can be calculated. According to the adjustment results, the entire orbit of CE-1 on 31 October 2007 can be divided into 3 sub-arcs with optimal lengths of 600, 800 and 184 epochs, respectively. The externally coincident accuracy of EOP of each sub-arc is shown in Table 1.

	A priori accuracy	Adjusted accuracy (1st sub-arc)	Adjusted accuracy (2nd sub-arc)	Adjusted accuracy (3rd sub-arc)
$x_p(mas)$	0.1	0.083	0.076	0.076
$y_p(mas)$	0.08	0.079	0.077	0.077
ÚT1–UTC (ms)	-0.77	-0.78	-0.77	-0.77
$\Delta \psi$ (mas)	0.091	-0.025	-0.031	-0.03
$\Delta \varepsilon$ (mas)	-0.083	-0.032	-0.151	-0.15

	$\sigma_{xp}$ (mas)	$\sigma_{y_p}$ (mas)	$\sigma_{UT1-UTC}(ms)$	$\sigma_{\Delta\psi}({ m mas})$	$\sigma_{\Delta arepsilon}({\sf mas})$	$\sigma_{Xs}(m)$	$\sigma_{Y_{S}}(m)$	$\sigma_{Zs}(m)$
31 October	0.133	0.116	0.012	0.269	0.736	371.790	631.710	330.22
1 November	0.070	0.070	0.024	0.005	0.002	459.320	717.640	430.66
2 November	0.035	0.035	0.012	0.027	0.009	778.470	966.630	571.95
3 November	0.011	0.011	0.004	0.011	0.003	1506.070	1208.950	712.99
4 November	0.013	0.013	0.004	0.024	0.004	128.130	132.960	155.33
5 November	0.035	0.035	0.002	0.027	0.008	80.400	75.670	105.14

Table 2Estimated EOP accuracy in the CE-1 transfer orbit.

## Table 3

Externally coincident accuracy of EOP in the CE-1 transfer orbit.

	$x_p$ (mas)		$y_p(mas)$		UT1-UTC (ms)		$\Delta\psi$ (mas)		$\Delta \varepsilon$ (mas)	
	Predicted	Estimated	Predicted	Estimated	Predicted	Estimated	Predicted	Estimated	Predicted	Estimated
31 October	0.100	0.076	0.080	0.077	-0.7717	-0.770	0.091	-0.031	-0.083	-0.151
1 November	-0.077	0.012	-0.44	-0.411	-0.2211	-0.025	0.772	0.772	0.115	0.115
2 November	0.195	0.194	-0.693	-0.692	-0.1916	0.966	0.934	0.741	0.153	0.152
3 November	1.027	0.993	-0.708	-0.645	-0.1004	-0.043	0.804	0.524	0.121	0.121
4 November	1.773	1.766	-0.440	-0.416	-0.0419	0.709	0.536	0.202	0.022	0.022
5 November	2.334	2.335	-0.418	-0.417	0.0646	1.239	0.331	0.324	-0.067	-0.066

#### 4.2. Estimation of the entire transfer orbit

For the observations of the following days in the CE-1 transfer orbit, the data with gross errors are removed following the standards in Section 3.3. Fig. 9 gives a comparison of time delay residuals



**Fig. 9.** Time delay residuals of each baseline without data rejection (a) and with data rejection (b).

of each baseline before and after data rejection. It can be seen that residuals are reduced after data rejection. The observations used in Fig. 9 are from the fourth sub-arc on 2 November 2007 with 900 epochs.

According to the data processing methods mentioned above, the optimal estimated accuracy of the CE-1 orbital parameters and *EOP* in the transfer orbit using an optimal orbital arc are shown in Table 2. It can be seen that the estimated accuracy of the daily *EOP* in the CE-1 transfer orbit are better than the IERS accuracy level, which is at the level of 0.010 mas for diagonal components and 0.001 ms for *UT1–UTC*. The results of observations on 31 October 2007 are slightly worse than the other days because of short orbital arc with about 2 h. So the estimated accuracy is reasonable.

For the CE-1 orbital parameters, the optimal estimated accuracy using the optimal orbital arc can be achieved at a level of several hundred meters for each coordinate component at the beginning and end phase of the transfer orbit. For example, the accuracies of 5 November 2007 are within 100 m. It means that the orbit determination accuracy can be achieved at level of about 100 m for radial direction or 82.7 mas in angular equivalent. It shows the advantage of the  $\Delta$ VLBI technology. Therefore, the CE-1 orbit determination accuracy in the transfer orbit can be improved using the  $\Delta$ VLBI time delay observations with the simultaneous estimation of the probe's orbital parameters and EOP. However, the estimated accuracy of the medium-term phase of the transfer orbit is worse with about 1.5 km. The reason is that the probe's flight is smooth and the orbit determination accuracy at the level of kilometers can meet the demand of the engineering application so only one orbit maneuver is made during the CE-1 transfer orbit. The method used in this paper is a geometrical orbit determination so that the results reflect the accuracy of  $\Delta$ VLBI time delay observations and geometric conditions of observations, but cannot make use of orbital constraints or forecasting.

The external agreement of the daily estimated *EOP* values in the transfer orbit are listed in Table 3. The accuracy of *EOP* estimate is improved from 2 to 5 November 2007 using the method of simultaneously estimating the CE-1 orbital parameters and *EOP* when compared with their predicted values, except some day's  $\Delta \varepsilon$  and *UT1–UTC*. The results show that *UT1–UTC* is sensitive to error of  $\Delta$ VLBI time delay observations. In addition, this paper provides a useful ways to improve *EOP* estimates using  $\Delta$ VLBI observations.

#### 5. Conclusion

In this paper, a relativistic mathematical model of  $\Delta$ VLBI time delay observations for the CE-1 transfer orbit is derived with the advantage of significantly eliminating common propagated errors during the signals' propagation path. The CE-1 orbital parameters and EOP are simultaneously estimated using the measured time delay data of the CE-1 transfer orbit and higher accuracies are obtained. The estimated CE-1 orbital accuracy can achieve a few hundred meters, with a best case of 75 m (Y direction) and the estimated EOP accuracies are improved when compared with their predicted values except for some day's  $\Delta \varepsilon$  and UT1–UTC. So it suggests that  $\Delta \varepsilon$  and UT1–UTC as known values is necessary to reduce the affects of the predicted EOP values on the estimated accuracy of the CE-1 orbit and EOP parameters. In addition, this paper provides a new ways to estimate EOP with respect to the traditional observation methods. A win-win approach can improve the accuracy of CE-1 orbit and EOP estimates.

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