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APPENDIX TO

3-D DATA-SPACE JOINT INVERSION OF GRAVITY AND MAGNETIC DATA USING A CORRELATION-ANALYSIS CONSTRAINT

Sheng Liu[1,2,3], Shuanggen Jin[3,4,5], Songbai Xuan[3], Xiang Liu[1]

[1] Xinjiang Astronomical Observatory, Chinese Academy of Sciences, Urumqi 830011, China
[2] University of Chinese Academy of Sciences, Beijing 100049, China
[3] Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, China
[4] School of Surveying and Land Information Engineering, Henan Polytechnic University, Jiaozuo 454000, China
[5] Zhuhai Fundan Innovation Institute, Creative Valley, Hengqin New District, Zhuhai 518057, China

APPENDIX A

Algorithm 1
Joint inversion based on the GN method of gravity and magnetic data

a) Initialization:

Prepare gravity and magnetic data $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$, reference density model $\boldsymbol{m}_1^{\mathrm{ref}}$, reference magnetization model $\boldsymbol{m}_2^{\mathrm{ref}}$, the range of density values $\left(\mathrm{m}_1^-, \mathrm{m}_1^+\right)$and the range of magnetization values $\left(\mathrm{m}_2^-, \mathrm{m}_2^+\right)$. Set the weighting parameters μ_1 and μ_2 of the correlation-analysis constraints. Calculate the initial regularization parameters λ_1^0 and λ_2^0 according to the L-curve method, and adopt λ_1^0 and λ_2^0 to perform separate inversions for gravity and magnetic data. The recovered density model and the recovered magnetization model obtained by the separate inversions are used as the initial density model $\boldsymbol{m}_1^0$ and initial magnetization model $\boldsymbol{m}_2^0$ for the joint inversion. Let k=0. Set the maximum number of iterations $\mathrm{k}_{\max}$ and the iteration threshold σ.

b) Joint inversion iteration:

While $(k<k_{max})$ and $\left[(\phi_d^1 \geq \sigma) \text{ or } (\phi_d^2 \geq \sigma)\right]$

k=k+1;

Compute $\boldsymbol{H}_{1,k}$, $\boldsymbol{H}_{2,k}$ according to equation (7) and compute $\boldsymbol{g}_1^k$, $\boldsymbol{g}_2^k$ according to equation (9);

Use CG method to calculate $\Delta \boldsymbol{m}_1$ and then update $\boldsymbol{m}_1^k = \boldsymbol{m}_1^{k-1} + \Delta \boldsymbol{m}_1$.

Use CG method to calculate $\Delta \boldsymbol{m}_2$ and then update $\boldsymbol{m}_2^k = \boldsymbol{m}_2^{k-1} + \Delta \boldsymbol{m}_2$.

Impose constraint on physical property model to force $m_1^- \leq \boldsymbol{m}_1^k \leq m_1^+$ and $m_2^- \leq \boldsymbol{m}_2^k \leq m_2^+$.

Compute $\phi_d^1 = \|\boldsymbol{d}_1 - \boldsymbol{G}_1 \boldsymbol{m}_1\|_2^2$ and $\phi_d^2 = \|\boldsymbol{d}_2 - \boldsymbol{G}_2 \boldsymbol{m}_2\|_2^2$.

End While

APPENDIX B

Algorithm 2
The MS-ICG Joint inversion of gravity and magnetic data

a) Initialization:

Prepare gravity and magnetic data $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$, reference density model $\boldsymbol{m}_1^{ref}$, reference magnetization model $\boldsymbol{m}_2^{ref}$, the range of density values (m_1^-, m_1^+) and the range of magnetization values (m_2^-, m_2^+). Set the weighting parameters μ_1 and μ_2 for the correlation-analysis constraints, the initial regularization parameters λ_1^0 and λ_2^0, and the initial density model $\boldsymbol{m}_1^0$ and the initial magnetization model $\boldsymbol{m}_2^0$. Let k=0, i=0 and calculate $\boldsymbol{w}_{1,0}$, $\boldsymbol{w}_{2,0}$. Set the maximum number of iterations of the outer loop k_{max}, the maximum number of iterations for the inner loop i_{max}, the threshold of the outer loop σ, the threshold of the inner loop eps and the value of q.

b) Joint inversion iteration:

While $(k<k_{max})$ and $\left[\left(\phi_d^1 \geq \sigma\right) \text{ or } \left(\phi_d^2 \geq \sigma\right)\right]$ (**The outer loop**)

k=k+1.

1) Iteration of gravity data.

Update $\lambda_1^k = \lambda_1^{k-1} q$ and $\boldsymbol{w}_{1,k}$.

Calculate $\boldsymbol{H}_{1,k}$ according to equation (7) and $\boldsymbol{g}_1^k$ according to equation (9).

Let $\boldsymbol{x}_1^0 = 0$ and $\boldsymbol{r}_{1,0} = \boldsymbol{d}_{1,0} = \boldsymbol{g}_1^k$.

Calculate $\boldsymbol{t}_1^0 = \left(\boldsymbol{d}_{1,0}^T \boldsymbol{r}_{1,0}\right) / \left(\boldsymbol{d}_{1,0}^T \boldsymbol{H}_{1,k} \boldsymbol{d}_{1,0}\right)$. Let i=0;

While $i \leq i_{max}$ and $\text{sqrt}(\boldsymbol{r}_{1,i}^T \boldsymbol{r}_{1,i}) \geq \text{eps}$(**The inner loop**)

i=i+1.

Update $\boldsymbol{x}_1^i = \boldsymbol{x}_1^{i-1} + \boldsymbol{t}_1^{i-1} \boldsymbol{d}_1^{i-1}$.

Calculate $\boldsymbol{d}_1^i = \boldsymbol{r}_{1,i} + \beta_1^i \boldsymbol{d}_1^{i-1}$ and $\beta_1^i = \boldsymbol{r}_{1,i}^T \boldsymbol{r}_{1,i} / \boldsymbol{r}_{1,i-1}^T \boldsymbol{r}_{1,i-1}$.

Calculate $\boldsymbol{t}_1^i = \left(\boldsymbol{d}_{1,i}^T \boldsymbol{r}_{1,i}\right) / \left(\boldsymbol{d}_{1,i}^T \boldsymbol{H}_{1,k} \boldsymbol{d}_{1,i}\right)$.

End While

Update $\boldsymbol{m}_1^k = \boldsymbol{m}_1^{k-1} + \boldsymbol{x}_1^i$.

Impose constraint on density model to force $m_1^- \leq \boldsymbol{m}_1^k \leq m_1^+$.

2) Iteration for magnetic data.

Update $\lambda_2^k = \lambda_2^{k-1} q$ and $\boldsymbol{w}_{2,k}$.

Calculate $\boldsymbol{H}_{2,k}$ and $\boldsymbol{g}_2^k$.

Let $\boldsymbol{x}_2^0 = 0$ and $\boldsymbol{r}_{2,0} = \boldsymbol{d}_{2,0} = \boldsymbol{g}_2^k$.

Calculate $\boldsymbol{t}_2^0 = \left(\boldsymbol{d}_{2,0}^T \boldsymbol{r}_{2,0}\right) / \left(\boldsymbol{d}_{2,0}^T \boldsymbol{H}_{2,k} \boldsymbol{d}_{2,0}\right)$. Let i=0;

While $i \le i_{max}$ and $\mathrm{sqrt}(\boldsymbol{r}_{2,i}^{T}\boldsymbol{r}_{2,i}) \ge \mathrm{eps}$(**The inner loop**)

i=i+1.

update parameter $\boldsymbol{x}_2^{i} = \boldsymbol{x}_2^{i-1} + \boldsymbol{t}_2^{i-1}\boldsymbol{d}_2^{i-1}$.

Calculate $\boldsymbol{d}_2^{i} = \boldsymbol{r}_{2,i} + \beta_2^{i}\boldsymbol{d}_2^{i-1}$, $\beta_2^{i} = \boldsymbol{r}_{2,i}^{T}\boldsymbol{r}_{2,i} / \boldsymbol{r}_{2,i-1}^{T}\boldsymbol{r}_{2,i-1}$.

Calculate $\boldsymbol{t}_2^{i} = \left(\boldsymbol{d}_{2,i}^{T}\boldsymbol{r}_{2,i}\right) / \left(\boldsymbol{d}_{2,i}^{T}\boldsymbol{H}_{2,k}\boldsymbol{d}_{2,i}\right)$.

End While

Update $\boldsymbol{m}_2^{k} = \boldsymbol{m}_2^{k-1} + \boldsymbol{x}_2^{i}$.

Impose constraint on magnetization model to force $m_2^{-} \le \boldsymbol{m}_2^{k} \le m_2^{+}$.

Compute $\phi_d^{1} = \left\|\boldsymbol{d}_1 - \boldsymbol{G}_1\boldsymbol{m}_1\right\|_2^2$ and $\phi_d^{2} = \left\|\boldsymbol{d}_2 - \boldsymbol{G}_2\boldsymbol{m}_2\right\|_2^2$.

End While