APPENDIX A

Algorithm 1
Joint inversion based on the GN method of gravity and magnetic data

a) Initialization:

Prepare gravity and magnetic data \( d_1 \) and \( d_2 \), reference density model \( m_1^{\text{ref}} \), reference magnetization model \( m_2^{\text{ref}} \), the range of density values \( (m_1^-, m_1^+) \) and the range of magnetization values \( (m_2^-, m_2^+) \). Set the weighting parameters \( \mu_1 \) and \( \mu_2 \) of the correlation-analysis constraints. Calculate the initial regularization parameters \( \lambda_1^0 \) and \( \lambda_2^0 \) according to the L-curve method, and adopt \( \lambda_1^0 \) and \( \lambda_2^0 \) to perform separate inversions for gravity and magnetic data. The recovered density model and the recovered magnetization model obtained by the separate inversions are used as the initial density model \( m_1^0 \) and initial magnetization model \( m_2^0 \) for the joint inversion. Let \( k=0 \). Set the maximum number of iterations \( k_{\text{max}} \) and the iteration threshold \( \sigma \).
Appendix B

Algorithm 2 The MS-ICG Joint inversion of gravity and magnetic data

a) Initialization:

Prepare gravity and magnetic data $d_1$ and $d_2$, reference density model $m_1^{ref}$, reference magnetization model $m_2^{ref}$, the range of density values $(m_1^-, m_1^+)$ and the range of magnetization values $(m_2^-, m_2^+)$. Set the weighting parameters $\mu_1$ and $\mu_2$ for the correlation-analysis constraints, the initial regularization parameters $\lambda_1^0$ and $\lambda_2^0$, and the initial density model $m_1^0$ and the initial magnetization model $m_2^0$. Let $k=0, i=0$ and calculate $w_{1,0}, w_{2,0}$. Set the maximum number of iterations of the outer loop $k_{max}$, the maximum number of iterations for the inner loop $i_{max}$, the threshold of the outer loop $\sigma$, the threshold of the inner loop $\epsilon_{ps}$ and the value of $q$.

\[b) \text{Joint inversion iteration:}\]

While $(k < k_{max})$ and $\left[ (\phi_d^1 \geq \sigma) \right.$ or $\left. (\phi_d^2 \geq \sigma) \right]$

$k = k + 1$;

Compute $H_{1,k}, H_{2,k}$ according to equation (7) and compute $g_1^k, g_2^k$ according to equation (9);

Use CG method to calculate $\Delta m_1$ and then update $m_1^k = m_1^{k-1} + \Delta m_1$.

Use CG method to calculate $\Delta m_2$ and then update $m_2^k = m_2^{k-1} + \Delta m_2$.

Impose constraint on physical property model to force $m_1^- \leq m_1^k \leq m_1^+$ and $m_2^- \leq m_2^k \leq m_2^+$.

Compute $\phi_d^1 = \|d_1 - G_1 m_1^k\|_2^2$ and $\phi_d^2 = \|d_2 - G_2 m_2^k\|_2^2$.

End While
b) Joint inversion iteration:

While \( k < k_{\text{max}} \) and \( \left[ \left( \phi_d^1 \geq \sigma \right) \text{ or } \left( \phi_d^2 \geq \sigma \right) \right] \) (The outer loop)

\[ k = k + 1. \]

1) Iteration of gravity data.

Update \( \lambda^k_i = \lambda^{k-1}_i q \) and \( w_{1,k} \).

Calculate \( H_{1,k} \) according to equation (7) and \( g^k_i \) according to equation (9).

Let \( x^0_i = 0 \) and \( r_{i,0} = d_{i,0} = g^k_i \).

Calculate \( t^0_i = \left( d^T_{i,0} r_{i,0} \right) / \left( d^T_{i,0} H_{1,k} d_{i,0} \right) \). Let \( i = 0; \)

While \( i \leq i_{\text{max}} \) and \( \sqrt{\text{tr}(r_{i,i}^T r_{i,i})} \geq \text{eps} \) (The inner loop)

\[ i = i + 1. \]

Update \( x_i^j = x_i^{i-1} + t_i^{i-1} d_i^{i-1} \).

Calculate \( d^i_i = r_{i,i} + \beta_i d_i^{i-1} \) and \( \beta_i = r_{i,i}^T r_{i,i} / r_{i,i}^T d_i^{i-1} \).

Calculate \( t_i^i = \left( d^T_{i,i} r_{i,i} \right) / \left( d^T_{i,i} H_{1,k} d_{i,i} \right) \).

End While

Update \( m_i^k = m_i^{k-1} + x_i^i \).

Impose constraint on density model to force \( m_i^- \leq m_i^k \leq m_i^+ \).

2) Iteration for magnetic data.

Update \( \lambda^k_2 = \lambda^{k-1}_2 q \) and \( w_{2,k} \).

Calculate \( H_{2,k} \) and \( g^k_2 \).

Let \( x^0_2 = 0 \) and \( r_{2,0} = d_{2,0} = g^k_2 \).

Calculate \( t^0_2 = \left( d^T_{2,0} r_{2,0} \right) / \left( d^T_{2,0} H_{2,k} d_{2,0} \right) \). Let \( i = 0; \)
While \( i \leq i_{\text{max}} \) and \( \sqrt{\langle r_{2,i}^T r_{2,i} \rangle} \geq \varepsilon \) (The inner loop)

\[
i = i + 1.
\]

update parameter \( x_2^i = x_2^{i-1} + t_2^{i-1} d_2^{i-1} \).

Calculate \( d_2^i = r_{2,i} + \beta_2^i d_2^{i-1} \), \( \beta_2^i = r_{2,i}^T r_{2,i} / r_{2,i-1}^T r_{2,i-1} \).

Calculate \( t_2^i = \left( d_2^T r_{2,i} \right) / \left( d_{2,i}^T H_{i,i} d_{2,i} \right) \).

End While

Update \( m_2^k = m_2^{k-1} + x_2^i \).

Impose constraint on magnetization model to force \( m_2^k \leq m_2^k \leq m_2^k \).

Compute \( \phi_d^i = \| d_2 - G_1 m_1 \|_{L^2}^2 \) and \( \phi_d^i = \| d_2 - G_2 m_2 \|_{L^2}^2 \).

End While