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A Tikhonov regularization method to estimate Earth's oblateness variations from global GPS observations

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ABSTRACT

Earth's oblateness is varying due to the redistribution of Earth's fluid mass and the interaction of various components in the Earth system. Nowadays, continuous Global Positioning System (GPS) observations can estimate Earth's oblateness (J_2) variations with the least squares method, but are subject to ill-conditioned equations with limited GPS observations and aliasing errors from truncated degrees. In this paper, a Tikhonov regularization method is used to estimate J_2 variations from global continuous GPS observations. Results show that the J_2 has been better estimated from GPS observations based on a Tikhonov regularization method than the usual least squares method when compared to SLR solutions. Furthermore, the amplitudes and phases of the annual and semi-annual J_2 variations are closer to the SLR results with truncated degrees from 2 to 5. Higher truncated degrees will degrade the J_2 estimate. Annual J_2 variations are best estimated from GPS observations with truncated degree 4 and semi-annual J_2 variations are best estimated with truncated degree 2.

1. Introduction

Because of gravitational and centrifugal forces, the rotating Earth is a flattened ellipsoid, whose dynamic oblateness is commonly called J_2. The redistribution of the Earth's atmosphere, ocean and hydrosphere fluid masses and the movement within the Earth's interior lead to changes of the Earth's rotational momentum and Earth's oblateness (J_2) (Cazenave and Nerem, 2002; Jin and Zhang, 2012). The investigation of J_2 can contribute to understanding of the redistribution and movement in the Earth's interior as well as the interaction and coupling of different components in the Earth system (Cox and Chao, 2002). In addition, J_2 has been correlated with lunisolar precession in astronomy and plays an important role in the change of the amplitude of nutation. Therefore it is of great significance to precisely estimate J_2 and understand its change and mechanism.

Satellite laser ranging (SLR), a well-established technique, can well estimate the degree-2 zonal gravitational coefficient, which has been widely used and studied (Yoder et al., 1983; Cheng and Tapley, 2004; Jin et al., 2011). Recently the new generation of gravity satellites provides a new opportunity to measure the global time-varying gravity field with high coverage, spatial resolution and precision. The Gravity Recovery and Climate Experiment (GRACE), a joint mission of US National Aeronautics and Space Administration (NASA) and German Aerospace Center (DLR), has made very successful measurements of the Earth's long and medium wavelengths of the gravity field (Tapley et al., 2004), which has been widely used in geophysics, oceanography, hydrology, glaciology and geodesy (e.g., Rangelova and Sideris, 2008; Jin et al., 2010, 2012; Jin and Feng, 2013). However, GRACE is not sensitive to J_2.

With the increase of global continuous GPS stations and the accumulation of observation data at high precision and spatial resolution, the longer precise 3-dimensional coordinate time series can be obtained from GPS, which can be used to estimate the variations of low degree gravity field coefficients using the least squares method (Kusche and Schrama, 2005; Wu et al., 2006). However, the estimation of low degree gravity field coefficients using the traditional least squares method is subject to ill-conditioned equations with limited observations and aliasing errors from truncated degrees. For example, the J_2 estimated from GPS observations with a truncated degree 9 has smaller annual amplitude than the SLR

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solution (Jin and Zhang, 2012). In this paper, the Earth's oblateness variations are estimated and investigated from global continuous GPS loading displacements based on a Tikhonov regularization method with different truncated degrees. Furthermore, the correlation and seasonal variations of J_2 with SLR solutions are evaluated and discussed.

2. Theory and methods

2.1. Low degree coefficient estimation

Global continuous GPS observations can measure the 3-D coordinates and according to the theory of Farrell (1972), the surface load $T(\Omega)$ results in the change of gravitational potential $V(\Omega)$ and the displacement of the solid Earth:

$$\begin{aligned} V(\Omega) &= \sum_{n=1}^{\bar{n}}\sum_{m=0}^{n}\sum_{\phi}^{\{C,S\}} V_{nm}^{\phi} Y_{nm}^{\phi}(\Omega) \\ &= \frac{3\rho_W}{a\rho_E}\sum_{n=1}^{\bar{n}}\sum_{m=0}^{n}\sum_{\phi}^{\{C,S\}} \frac{(1+k'_n)}{(2n+1)} T_{nm}^{\phi} Y_{nm}^{\phi}(\Omega) \\ H(\Omega) &= \frac{3\rho_W}{\rho_E}\sum_{n=1}^{\bar{n}}\sum_{m=0}^{n}\sum_{\phi}^{\{C,S\}} \frac{h'_n}{(2n+1)} T_{nm}^{\phi} Y_{nm}^{\phi}(\Omega) \\ L(\Omega) &= \frac{3\rho_W}{\rho_E}\sum_{n=1}^{\bar{n}}\sum_{m=0}^{n}\sum_{\phi}^{\{C,S\}} \frac{l'_n}{(2n+1)} T_{nm}^{\phi} Y_{nm}^{\phi}(\Omega) \end{aligned} \tag{1}$$

where $H(\Omega)$ and $L(\Omega)$ are the radial and horizontal displacement at a point ($\Omega = (\lambda, \theta)$), $Y_{nm}^{\phi}(\Omega)$ is the spherical harmonic function of degree n and order m, T_{nm}^{ϕ} is the spherical harmonic coefficient of the surface load density, the average radius of Earth is $a = 6371$ km, k'_n, h'_n, l'_n are the load Love numbers, the average density of water is $\rho_W = 1025\ \text{kg/m}^3$ and the average density of the Earth ρ_E is about $5514\ \text{kg/m}^3$. The change of surface density $\Delta\sigma$ can be expressed as follows (Wahr et al., 1998):

$$\Delta\sigma(\lambda,\theta) = a\rho_W \times \sum_{n=0}^{N_{\max}}\sum_{m=0}^{n}(\Delta C_{nm}^{\sigma}\cos m\lambda + \Delta S_{nm}^{\sigma}\sin m\lambda) \times \bar{P}_{nm}(\cos\theta) \tag{2}$$

where $\bar{P}_{nm}$ is the associated Legendre polynomial with full normalization, ΔC_{nm}^{σ} and ΔS_{nm}^{σ} are the changes in spherical harmonic coefficients representing the variation of surface density with time, (ΔC_{nm}^{σ}, ΔS_{nm}^{σ}) are related to the change of the geoid (ΔC_{nm}^{g}, ΔS_{nm}^{g}), the vertical displacement (ΔC_{nm}^{h}, ΔS_{nm}^{h}) and the horizontal displacement (ΔC_{nm}^{φ}, ΔS_{nm}^{φ}) as follows (Wahr et al., 1998):

$$\begin{aligned} \Delta C_{nm}^{g} &= \frac{3\rho_W}{\rho_e}\frac{(1+k'_l)}{2l+1}\Delta C_{nm}^{\sigma} \\ \Delta C_{nm}^{h} &= \frac{3\rho_W}{\rho_e}\frac{h'_l}{2l+1}\Delta C_{nm}^{\sigma} \\ \Delta C_{nm}^{\varphi} &= \frac{3\rho_W}{\rho_e}\frac{l'_l}{2l+1}\Delta C_{nm}^{\sigma} \end{aligned} \tag{3}$$

For GPS observations, the displacement vector has correlation with (ΔC_{nm}^{h}, ΔS_{nm}^{h}) and (ΔC_{nm}^{φ}, ΔS_{nm}^{φ}). Therefore, using the least squares method, the unknown parameters can be estimated from GPS 3-D coordinates, including C_{20}.

2.2. Regularization method and parameter estimation

A general linear equation can be written as:

$$Ax = b + \varepsilon, A \in R^{m\times n}, x = R^n, b \in R^m \tag{4}$$

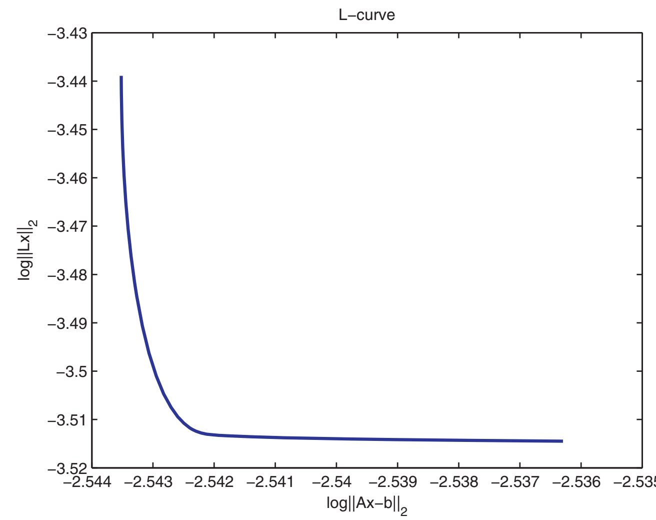

Fig. 1. An example of L-curve.

and unknown parameters can be solved through minimizing the sum of the square differences, i.e.,

$$\min_x \|Ax - b\|_2^2, A \in R^{m\times n}, m \geq n \tag{5}$$

When the coefficient matrices of the normal equation ($A^TAX = Ab$) are nonsingular, the optimal linear unbiased estimation solution is easily obtained. In practice, however, the determinant of the system normal equations is very small, and a slight perturbation of the coefficient matrix (A) will result in a big difference in the solution. This is an example of ill-conditioning. The condition number in matrix theory is commonly used to measure the stability of the coefficient matrix. The condition number of the matrix of normal equations is defined as the ratio of the maximum and minimum eigenvalues, depending on the function model and the dimension of the coefficient matrix (A). When the function model is fixed, the larger the dimension number is, the poorer the stability of equations is. In order to obtain reliable solutions from ill-conditioned equation, we can add additional constraints based on a regularization method.

2.2.1. Tikhonov regularization method

Tikhonov regularization is one of the most common regularization methods. By adding the boundary conditions, Eq. (5) can be rewritten as:

$$\min_x \{\|Ax - b\|_2^2 + \lambda\|Lx\|_2^2\} \tag{6}$$

where $||Lx||_2^2$ is the boundary conditions, $\lambda \geq 0$ is the regularization parameter, which controls the ratio between minimization of boundary conditions and the residual norm, and can be obtained by the L-curve method, and L is the regularization matrix (Hansen and O'Leary, 1993). Eq. (6) is called the Tikhonov regularization criteria, and when L is the unit matrix, Eq. (6) has the standard form of Tikhonov criteria. The selection of the regularization matrix is the key to obtaining the reasonable results (Calvetti et al., 2004).

2.2.2. Determination of regularization parameter λ

The curve is defined as:

$$L = \{(\hat{\rho}_\lambda, \hat{\eta}_\lambda) : \lambda \geq 0\} \tag{7}$$

where $\hat{\rho}_\lambda = \log\rho_\lambda$, $\hat{\eta}_\lambda = \log\eta_\lambda$, $\rho_\lambda^2 = \|Ax - b\|_2^2$, $\eta_\lambda^2 = \|Lx_\lambda\|$. Since the curve defined by Eq. (7) has the shape of the letter L, it is often called as the L-curve. Fig. 1 shows an example of an L-curve. The point on the L-curve with the maximum absolute curvature is commonly known as the "apex", and the regularization parameter of the apex is the optimal regularization parameter. The curvature $k(\lambda)$ of the L-curve is:

$$k(\lambda) = 2\frac{\left|\hat{\rho}''_{\lambda}\hat{\eta}'_{\lambda} - \hat{\rho}'_{\lambda}\hat{\eta}''_{\lambda}\right|}{\left((\hat{\rho}'_{\lambda})^2 + (\hat{\eta}')^2\right)^{3/2}} \tag{8}$$

where $\hat{\rho}'_{\lambda}$ and $\hat{\eta}'_{\lambda}$ represent the derivation of $\hat{\rho}_{\lambda}$ and $\hat{\eta}_{\lambda}$ with respect to λ, and $\hat{\rho}''_{\lambda}$ and $\hat{\eta}''_{\lambda}$ denote the second derivation of $\hat{\rho}_{\lambda}$ and $\hat{\eta}_{\lambda}$ with respect to λ. In actual calculation, we get the value according to the matrix singular value decomposition, and then obtain the interval ranges corresponding to the regularization parameter of the apex. The interval is divided by equal intervals on the logarithm, and the curvature of each point is then calculated (Calvetti et al., 2004).

3. J_2 estimation from GPS-based on the regularization method

3.1. Observation equations

Since the Earth is not a rigid body, the exchange and redistribution of the atmosphere, ocean, and continental water mass will cause variations of the solid Earth, gravity field, and the shape of the solid Earth. Therefore, the Earth's gravitational loading can be established from three-dimensional site displacements derived from GPS observations. If the Earth is assumed to be elastic spherically symmetric and radially stratified and the load is a very thin layer of Earth's surface, the surface load deformation can be computed through spherical harmonic coefficients. The seven Helmert parameters (three translation parameters, three rotation parameters and a scale factor) are introduced to absorb the possible impact of network transformation, and then GPS station displacements can be expressed as (Kusche and Schrama, 2005):

$$\begin{aligned}
\Delta h &= a\sum_{n,m}(\Delta C^{h}_{nm}\cos m\lambda + \Delta S^{h}_{nm}\sin m\lambda)\bar{P}_{nm}(\cos\theta) + \vec{e}_h\cdot\Delta\vec{x} - a\Delta s\\
\Delta e &= \frac{a}{\sin\theta}\sum_{n,m} m(-\Delta C^{\psi}_{nm}\sin m\lambda + \Delta S^{\psi}_{nm}\cos m\lambda)\bar{P}_{nm}(\cos\theta) + \vec{e}_e\cdot\Delta\vec{x} + \vec{e}_n\cdot\vec{\varepsilon}\\
\Delta n &= -a\sum_{n,m}(\Delta C^{\psi}_{nm}\cos m\lambda + \Delta S^{\psi}_{nm}\sin m\lambda)\frac{\partial\bar{P}_{nm}(\cos\theta)}{\partial\theta} + \vec{e}_n\cdot\Delta\vec{x} - \vec{e}_e\cdot\vec{\varepsilon}
\end{aligned} \tag{9}$$

where Δh, Δe and Δn are GPS displacement changes in the height, east and north directions, respectively, $(\Delta C^{h}_{nm}, \Delta S^{h}_{nm})$ are the change in spherical harmonic coefficients due to height displacement, $(\Delta C^{\varphi}_{nm}, \Delta S^{\varphi}_{nm})$ are the change in spherical harmonic coefficients due to horizontal displacement, $\bar{P}_{nm}$ is the normalization of Legendre polynomials, $\vec{e}_e$, $\vec{e}_n$ and $\vec{e}_h$ are unit vector in the east, north and radial, respectively. The degree-1 Love numbers are used with $l'_1 = 0.134$ and $h'_1 = -0.269$ in the center of whole Earth mass (Blewitt and Clarke, 2003). With the truncated degree N, Eq. (9) can be written as:

$$A_1x_1 + A_2x_2 = b + \varepsilon \tag{10}$$

where A_1 and A_2 are the coefficient matrixes, b is the observed value, x_1 is the surface load coefficient and x_2 is the Helmert transformation parameters, which are expressed as:

$$\begin{aligned}
x_1 &= [C^{\sigma}_{10}, C^{\sigma}_{11}, S^{\sigma}_{11}, \cdots, C^{\sigma}_{NN}, S^{\sigma}_{NN}]\\
x_2 &= [t_x, t_y, t_z, \omega_x, \omega_y, \omega_z, s]
\end{aligned} \tag{11}$$

Using a seven parameters transformation seems reasonable in theory, but leads to a high condition number. In the following, Tikhonov regularization is used to address this problem.

3.2. Regularization matrix

The Earth's surface density anomalies can be expanded by spherical harmonic coefficients as shown in Eq. (2) with $f^{C}_{nm} = \bar{P}^{m}_{n}(\cos\theta)\cos(m\lambda)$ and $f^{S}_{nm} = \bar{P}^{m}_{n}(\cos\theta)\sin(m\lambda)$. Since there are few or no GPS stations in the ocean areas, we calculate the integral of the ocean mass changes as.

$$a^2\rho_w^2\iint_{ocean}(\Delta\sigma)^2 d\sigma = a^2\rho_w^2 x_1^T R x_1 \tag{12}$$

where

$$R = \begin{bmatrix} (f^{C}_{10}, f^{C}_{10}) & (f^{C}_{10}, f^{C}_{11}) & \cdots & (f^{C}_{10}, f^{S}_{NN})\\ (f^{C}_{11}, f^{C}_{10}) & (f^{C}_{11}, f^{C}_{11}) & \cdots & (f^{C}_{11}, f^{S}_{NN})\\ \vdots & \vdots & \ddots & \vdots\\ (f^{S}_{NN}, f^{C}_{10}) & (f^{S}_{NN}, f^{C}_{11}) & \cdots & (f^{S}_{NN}, f^{S}_{NN}) \end{bmatrix} = M_1^T M_1 \tag{13}$$

$$(f, g) = \iint_{ocean} f g^* d\sigma \tag{14}$$

where $f, g = f^{C}_{nm}$ or f^{S}_{nm}, $n = 1, \ldots, N; m = 0, \ldots, n$, and g^* denotes complex conjugate. If we take M_2 as a 7th-order unit matrix, then the regularization matrix is:

$$L = \begin{bmatrix} M_1 & 0\\ 0 & M_2 \end{bmatrix} \tag{15}$$

In order to calculate the spherical integral of the product of spherical function, the plural form of the spherical functions is defined as follows:

$$\bar{Y}_{nm} = \bar{P}^{|m|}_{n}(\cos\theta)e^{im\lambda} \tag{16}$$

The inner product of the four spherical functions on the ocean is as follows:

$$\begin{Bmatrix} A_{nm,rs}\\ B_{nm,rs}\\ C_{nm,rs}\\ D_{nm,rs} \end{Bmatrix} s = s \begin{Bmatrix} (f^{C}_{nm}, f^{C}_{rs})\\ (f^{S}_{nm}, f^{S}_{rs})\\ (f^{C}_{nm}, f^{S}_{rs})\\ (f^{S}_{nm}, f^{C}_{rs}) \end{Bmatrix} = \iint_{oceans} \bar{P}^{m}_{n}(t)\bar{P}^{s}_{r}(t) \begin{Bmatrix} \cos m\lambda\cos s\lambda\\ \sin m\lambda\sin s\lambda\\ \cos m\lambda\sin s\lambda\\ \sin m\lambda\cos s\lambda \end{Bmatrix} d\cos\theta d\lambda \tag{17}$$

We can obtain

$$\begin{Bmatrix} A_{nm,rs}\\ B_{nm,rs}\\ C_{nm,rs}\\ D_{nm,rs} \end{Bmatrix} = \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4\\ 1/4 & -1/4 & -1/4 & 1/4\\ -1/4i & 1/4i & -1/4i & 1/4i\\ 1/4i & 1/4i & -1/4i & -1/4i \end{bmatrix} \begin{bmatrix} (\bar{Y}_{nm}, \bar{Y}_{r,s})\\ (\bar{Y}_{n,m}, \bar{Y}_{r,-s})\\ (\bar{Y}_{n,-m}, \bar{Y}_{r,s})\\ (\bar{Y}_{n,-m}, \bar{Y}_{r,-s}) \end{bmatrix} \tag{18}$$

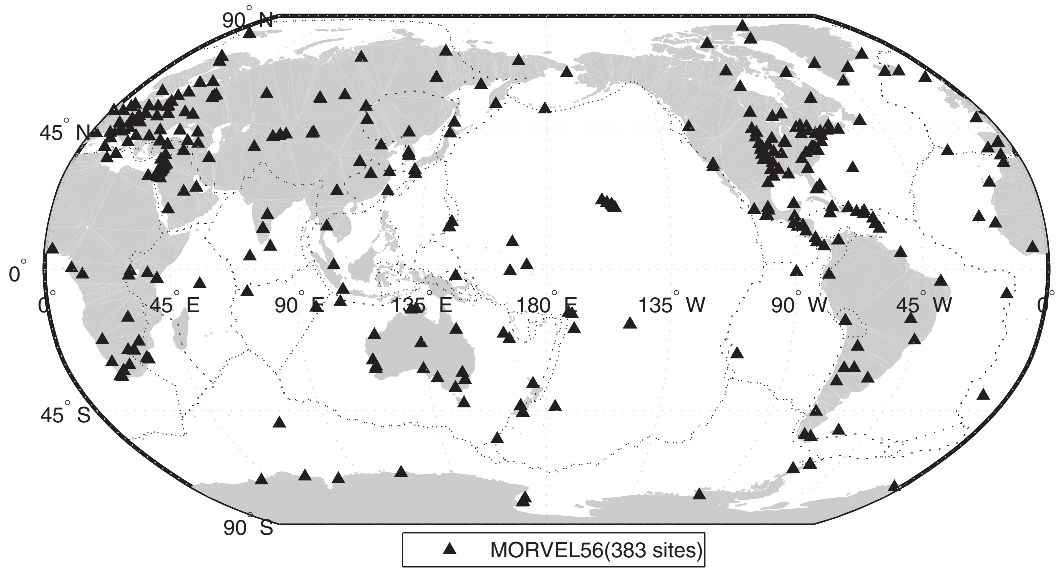

Fig. 2. Distribution of selected GPS sites in this study.

If the entire surface of the Earth is divided into $N\times 2N$ equiangular blocks based on latitude and longitude, the following equation can be obtained

$$(\bar{Y}_{nm},\bar{Y}_{rs})=\sum_{k=0}^{N-1}\sum_{l=0}^{2N-1}w_{kl}\left(\int_{\theta_k}^{\theta_{k+1}}\bar{P}_n^{|m|}(\cos\theta)\bar{P}_r^{|s|}(\cos\theta)\sin\theta d\theta\right)\times\left(\int_{\lambda_k}^{\lambda_{k+1}}e^{i(m-s)\lambda}d\lambda\right) \quad (19)$$

where

$$w_{kl}=\begin{cases}0 & land\\ 1 & ocean\end{cases} \quad (20)$$

Each element in the first right parenthesis of Eq. (19) is the integral of the product of two normalized associated Legendre functions, which can be realized by recursion (Mainvile, 1986) or direct calculation (Pail et al., 2001). The product of the other three spherical functions can be similarly implemented (e.g., Hwang, 1991).

3.3. Tikhonov regularization method inversion

The linear equation for low-degree gravitational coefficients estimation from GPS data is expressed as:

$$\begin{cases}b=Ax+\Delta, E(\Delta)=0\\ \sigma_\Delta^2=\sigma_0^2 Q\end{cases} \quad (21)$$

where Δ is the residual, Q is the error matrix of observation b, and σ_0^2 is the variance of unit weight. The regularization matrix is Eq. (15), so the objective function of weighted Tikhonov regularization is expressed as follows (Bouman, 1998):

$$J=\|Ax-b\|_W^2+\lambda\|Lx\|_2^2 \quad (22)$$

where W is the covariance of observation error and a real symmetric positive definite matrix, so that W can be decomposed into the form $W=P'P$. With $\bar{A}=PAL^{-1}$, $\bar{x}=Lx$, then Eq. (22) can be written in a standard form:

$$J=\|\bar{A}\bar{x}-b\|_2^2+\lambda\|\bar{x}\|_2^2 \quad (23)$$

Once the optimal regularization parameter λ has been determined, we can further obtain

$$x_\lambda=(A^TWA+\lambda L^TL)^{-1}A^TWb \quad (24)$$

The posterior variance of unit weight is

$$\hat{\sigma}_0^2=\frac{\|Ax-b\|_W^2+\|Lx\|_2^2}{m} \quad (25)$$

The Variance and Co-Variance matrix of the estimated parameter is

$$Q_{x_\lambda x_\lambda}=(A^TWA+\lambda L^TL)^{-1}A^TWA(A^TWA+\lambda L^TL)^{-T} \quad (26)$$

3.4. J_2 estimation from GPS

The International GNSS (Global Navigation Satellite Systems) Service (IGS) coordinates a worldwide network of permanent tracking stations with about 400 GPS stations and provides high quality data and products for use in GNSS related Earth science research, navigation application and educational outreach. In this work, we use 10-year GPS coordinate solutions of 423 continuous GPS stations (2003–2012) from the Scripps Orbit and Permanent Array Center (SOPAC) (ftp://garner.ucsd.edu/pub/timeseries). In order to obtain reliable results, some deformed GPS stations are removed with deformation residuals between the observed minus estimated velocity of larger than three times of one sigma based on recent plate motion model of MORVEL56 (DeMets et al., 2010). In addition, the GPS sites with anomalous activities, such as earthquake and antenna change, are removed. Finally about 300 GPS sites are selected in this study (Fig. 2). Based on the above equations, the low degree gravity coefficients are estimated from GPS coordinate time series (January 2003–December 2012), including C_{20}. Finally J_2 can be estimated as $J_2=-\sqrt{5}C_{20}$.

4. Results and evaluation

4.1. J_2 time series

A monthly J_2 time-series is estimated from GPS observations using the Tikhonov regularization of Eq. (15) from truncated degrees 2–9. In order to check the results of the proposed methodology, the J_2 variations are also estimated from same observations based on the standard least squares method (Kusche and Schrama,

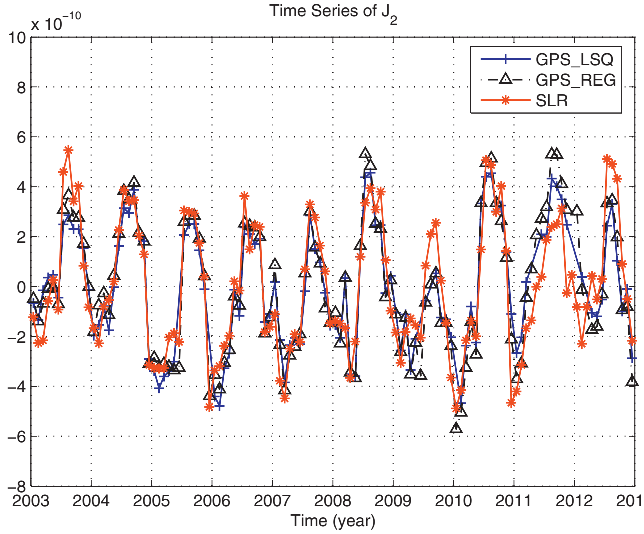

Fig. 3. The monthly J_2 time-series from SLR and GPS observations based on the least squares (GPS_LSQ) and the Tikhonov regularization method (GPS_REG) with truncated degree 4.

2005). For example, Fig. 3 shows the monthly J_2 time-series from the least squares (LSQ) and the Tikhonov regularization method (REG) with truncated degree 4, which agree well with each other usually.

The monthly J_2 time-series are further analyzed using their power spectral density (PSD) as:

$$\hat{P}(e^{i\omega}) = \frac{1}{2\pi N}\left|\sum_{k=0}^{N-1} x_k e^{-i\omega k}\right|^2 \tag{27}$$

where ω is the angular frequency, and x_k is the time series at time k. Fig. 4 shows the power spectral density of the monthly J_2 time-series from GPS observations using the least squares (LSQ) and the Tikhonov regularization method (REG) with truncated degree 4. Significant annual and semi-annual signals are found in all solutions.

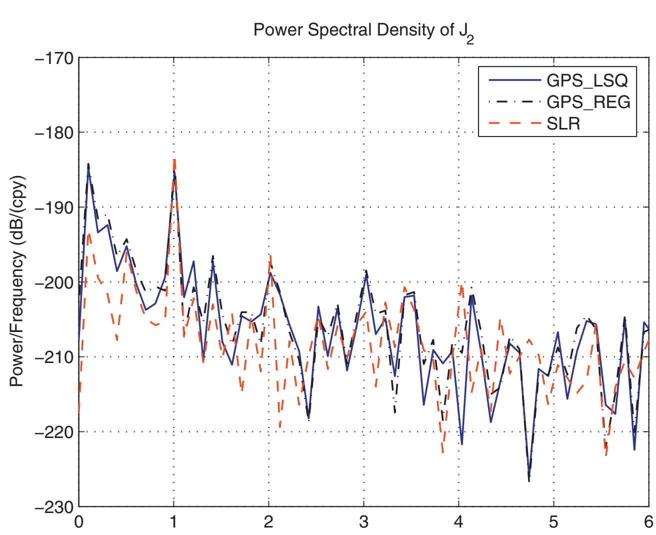

Fig. 4. Power spectral density of J_2 time series from GPS and SLR.

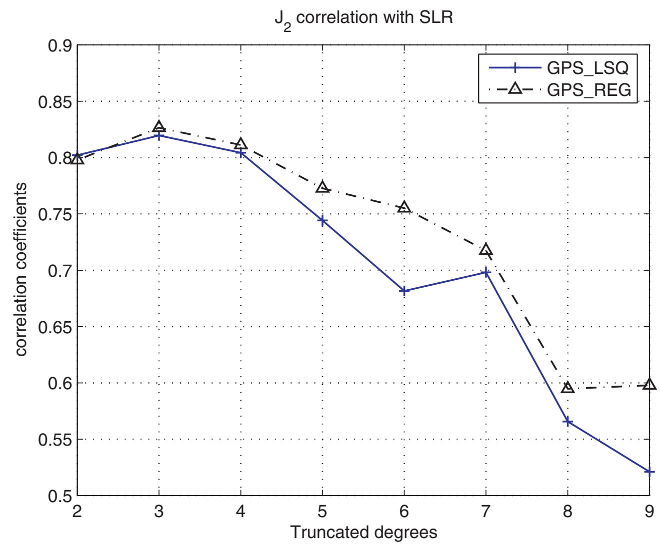

Fig. 5. Correlation coefficients of monthly J_2 time-series from GPS with different truncated degrees with SLR solution.

4.2. Correlations with SLR

We further calculate the correlation coefficients of the monthly J_2 time-series from GPS observations using the least squares (GPS_LSQ) and the Tikhonov regularization method (GPS_REG) with SLR-determined results. Here J_2 from SLR was estimated from five geodetic satellites: LAGEOS-1 and 2, Starlette, Stella and Ajisai (Cheng and Tapley, 2004), provided by the Center for Space Research, University of Texas at Austin (courtesy of Minkang Cheng). The background gravity models used in the SLR analysis are consistent with the GRACE Release-05 processing. Fig. 5 shows correlation coefficients of the monthly J_2 time-series from GPS by different truncated degrees with the SLR solution. The correlation coefficients from GPS observations based on the Tikhonov regularization method (GPS_REG) are higher than those based on the least squares method from all truncated degrees, indicating that J_2 results can be well estimated using our Tikhonov regularization method. In addition, the correlation coefficients from GPS results with truncated degree of higher than 6 are lower, which may be due to effects of unstable estimate at high degrees due to the lack of GPS observations. For example, results are not good with truncated degree 9. However, the J_2 estimates are still closer to the SLR solution when based on the Tikhonov regularization method.

4.3. Seasonal J_2 variations

Using the method of least squares fit to a bias, trend and seasonal terms, the amplitude and phase of the annual and semi-annual J_2 variations are estimated as

$$y(t) = a + b(t - t_0) + \sum_{k=1}^{2}\left[c_k \cos\left(\frac{2\pi(t - t_0)}{p_k} - \varphi_k\right)\right] + \varepsilon_t \tag{28}$$

where a is the constant term, b is the linear term, c_k and φ_k are amplitudes and phases of the period p_k (k = 1 and 0.5 year), t_0 is January 2003, and ε is the residual. Fig. 6 shows the annual amplitudes and phases of J_2 variations from GPS observations based on the least squares (GPS_LSQ) and a Tikhonov regularization method (GPS_REG) at different truncated degrees, SLR and GRACE RL05, respectively. The results from GPS observations based a Tikhonov regularization method (GPS_REG) are closer to the SLR solution than the least squares method (GPS_LSQ) at the annual scale. In addition, for GPS_LSQ and GPS_REG, the truncated degrees from 2 to 5 are

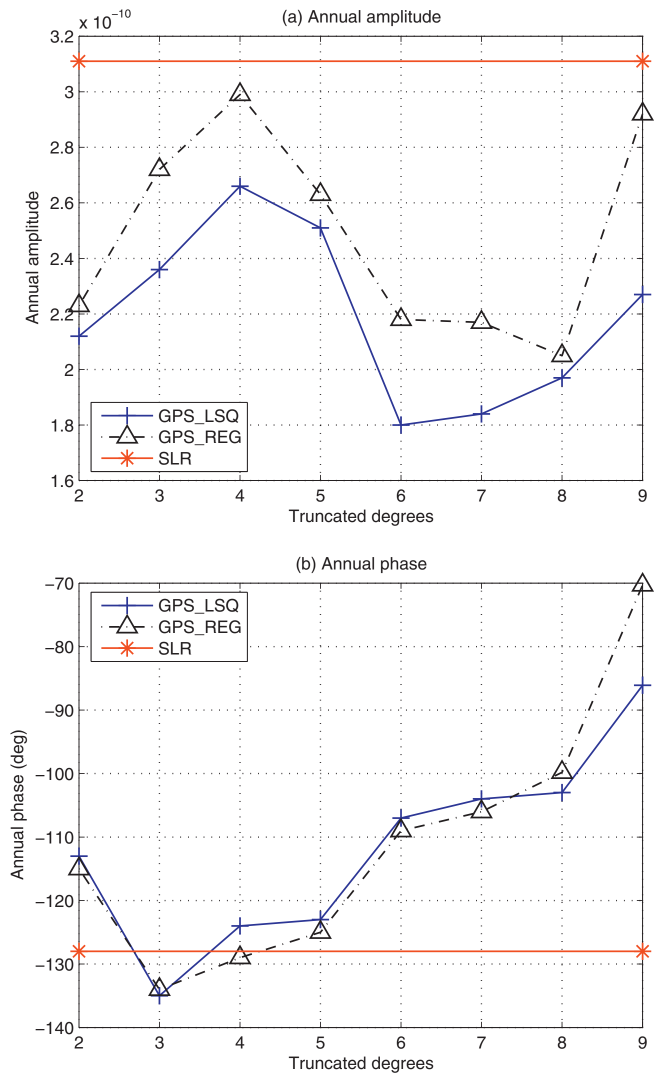

Fig. 6. Annual variations of J_2 from SLR and GPS observations based on the least squares (GPS_LSQ) and the Tikhonov regularization method (GPS_REG) with different truncated degrees. (a) is annual amplitude and (b) is annual phase.

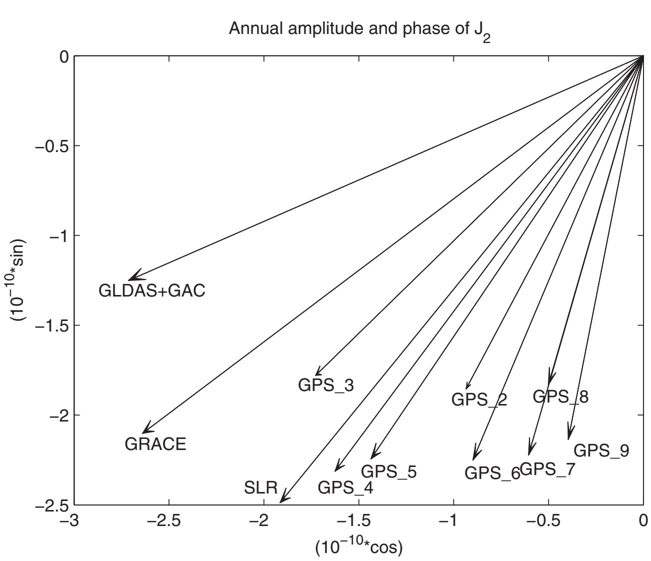

Fig. 7. Phasor plots of annual J_2 variations with different truncated degrees.

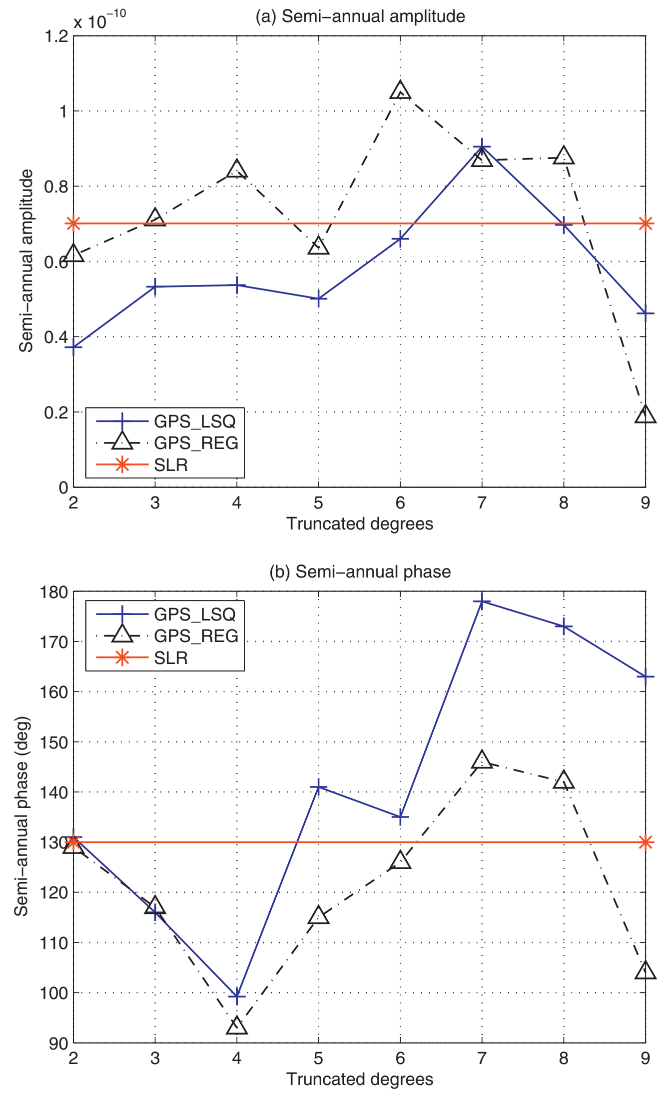

Fig. 8. Semi-annual J_2 variations from SLR and GPS observations based on the least squares (GPS_LSQ) and the Tikhonov regularization method (GPS_REG) with different truncated degrees. (a) is semi-annual amplitude and (b) is semi-annual phase.

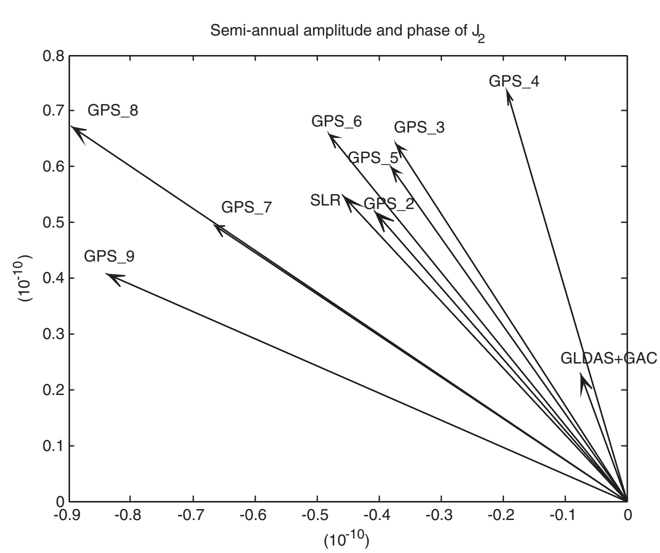

Fig. 9. Phasor plots of semi-annual J_2 variations with different truncated degrees.

better to estimate the J_2 variations than other truncated degrees for annual amplitudes and phases. Furthermore, annual J_2 variations are best estimated from GPS observations with truncated degree 4 (Fig. 7), while models based on GLDAS + GRACE GAC have larger difference and therefore we do not discuss them further here. The GLDAS model is the land surface simulation system from the National Aeronautics and Space Administration (NASA) Goddard Space Flight Center (GSFC) and the National Oceanic and Atmospheric Administration (NOAA) National Centers for Environmental Prediction (NCEP), and is the integration of the ground and space-based high-resolution observations (Rodell et al., 2004). GRACE GAC is a GRACE Release-05 processing products (representing the ocean and atmosphere mass variations). Similarly the semi-annual amplitude and the semi-annual phase with truncated degrees from 2 to 5 are also better to estimate the J_2 variations (Fig. 8) and the truncated degree 2 is the best for the semi-annual variations (Fig. 9).

5. Conclusion

The Earth's oblateness variations are estimated from GPS observations using a Tikhonov regularization method, which is much better than the standard least squares method when compared to SLR with better correlations at all truncated degrees. The correlation coefficients from GPS results with truncated degree 6 or larger are lower and are affected by unstable estimates of higher truncated degree coefficients due to the lack of GPS observations. For annual variations, the results from GPS observations based on a Tikhonov regularization method are closer to the SLR solution than those from the least squares method. In addition, with truncated degrees from 2 to 5, the amplitudes and phases of annual and semi-annual J_2 variations are much closer to the SLR results. Higher truncated degrees appear to degrade the J_2estimate. Furthermore, annual J_2 variations estimated from GPS observations with truncated degree 4 are much better than those with other truncated degrees. Semi-annual J_2 variations estimated with truncated degree 2 are much better than those obtained with other truncated degrees. Since most GPS stations are located on land and 70% of the Earth's surfaces consist of oceans with fewer GPS stations, and hence there are less observations from oceanic areas. In the future, J_2 variations will be further estimated from the combination of GPS and ocean bottom pressure observations data.

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