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Effects of physical correlations on long-distance GPS positioning and zenith tropospheric delay estimates

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Abstract

The global positioning system (GPS) has become an essential tool for the high precision navigation and positioning. The quality of GPS positioning results mainly depends on the model's formulations regarding GPS observations, including both a functional model, which describes the mathematical relationships between the GPS measurements and unknown parameters, and a stochastic model, which reflects the physical properties of the measurements. Over the past two decades, the functional models for GPS measurements have been investigated in considerable detail. However, the stochastic models of GPS observation data are simplified, assuming that all the GPS measurements have the same variance and are statistically independent. Such assumptions are unrealistic. Although a few studies of GPS stochastic models were performed, they are restricted to short baselines and short time session lengths. In this paper, the stochastic modeling for GPS long-baseline and zenith tropospheric delay (ZTD) estimates with a 24-h session is investigated using the residual-based and standard stochastic models. Results show that using the different stochastic modelling methods, the total differences can reach as much as 3–6 mm in the baseline component, especially in the height component, and 10 mm in the ZTD estimation. Any misspecification in the stochastic models will result in unreliable GPS baseline and ZTD estimations. Using the residual-based stochastic model, not only the precision of GPS baseline and ZTD estimation is obviously improved, but also the baseline and ZTD estimations are closer to the reference value.

Keywords: Stochastic modelling; GPS; Baseline; ZTD

1. Introduction

Nowadays, the global positioning system (GPS) has been widely used for various high precision positioning and navigation. Traditionally, data processing for precise GPS positioning is invariably performed using the least squares (LS) method. The quality of LS solutions depends on the model's formulation regarding GPS observations, including both a functional model, which describes the mathematical relationships between the GPS measurements and unknown parameters, and a stochastic model, which reflects the physical correlations of the measurements (Brown and Hwang, 1992; Han and Rizos, 1995; Hofmann-Wellenhof et al., 1997; Blewitt, 1998; Barnes et al., 1998; Brunner et al., 1999). Over the past two decades, the functional models for GPS measurements have been widely investigated in considerable detail. However, accurate stochastic modelling for the GPS measurements is still a difficult and challenging issue (Jin et al., 2005). In the current stochastic models for GPS positioning and applications (e.g., zenith tropospheric delay estimates), it is usually assumed that all the GPS measurements have the same variance. The time-invariant covariance matrix of the double-differenced (DD) measurements is then constructed using the error propagation law. Such assumptions are unrealistic.

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As GPS measurement errors are dominated by the systematic errors caused by the multi-path, atmospheric refraction, orbit effects, and so on, it is impossible to model all systematic errors into the functional model. Therefore, modeling some systematic errors into the stochastic model is a new challenging strategy to further realize the full potential of increasingly more accurate GPS positioning and applications. Although there were only a few studies to investigate the effects of stochastic models on the GPS relative positioning (El-Rabbany, 1994; Jin and de Jong, 1996; Bona, 2000; Satirapod et al., 2003; Tiberius and Kenselaar, 2003), they were mainly restricted to short baselines and short time session lengths, and also focused only GPS baseline estimates. Up to now, the effects of stochastic modelling on GPS long baseline and zenith tropospheric delay (ZTD) estimates have not been investigated, particularly for the long baseline IGS (International GPS service) network. In this paper, the residual-based stochastic model is implemented and investigated for the GPS long baseline and ZTD estimates (with a 24-h session). In the following sections the stochastic modelling methods and results for GPS long baseline and ZTD estimates are presented and discussed.

2. Stochastic modelling methods

The linearized GPS observation equations are written as (Hofmann-Wellenhof et al., 1997; King and Bock, 1999):

$$L = Ax + v \tag{1}$$

where L is the $n \times 1$ vector of the observed-minus-computed double difference (DD) carrier phase values (O–C), A is the $n \times q$ design matrix that describes the linearized functional model corresponding to the kth observation epoch, $k = 1 \ldots n$, x is the unknown parameters including coordinate and ambiguity, etc., and v is the $n \times 1$ vector of error terms. Using the least square method (LS), the unknown parameters and their uncertainties can be determined, namely

$$\begin{aligned} \hat{x} &= (A^T C_x^{-1} A)^{-1} A^T C_x^{-1} L \\ \bar{v} &= L - A\hat{x} \\ \hat{\sigma}^2 &= \frac{v^T C_x^{-1} v}{f} \end{aligned} \tag{2}$$

where $\hat{x}$ is the parameter estimate (including baseline length or coordinate, ZTD, ambiguity, etc.), C_x is the variance–covariance matrix for the double-differenced GPS measurements, called the stochastic model, $\bar{v}$ is called the LS residual vector, $\hat{\sigma}$ is the uncertainty and f is the degree of freedom. It can be seen that the estimation of the unknown parameter x and its precision indicator are dependent on the stochastic model. Any mis-specifications of the stochastic model may result in unreliable parameter estimations. The residuals obtained from the LS solution contain unmodelled biases and noise. The true residual should represent a good statistic property of GPS measurements. As it is impossible to model all systematic errors in the functional model, modelling the residuals into the stochastic model is a new challenging strategy to improve the GPS positioning solutions. Additionally, it can further test the impacts of the stochastic modelling on the GPS unknown parameter estimations. The following two easily realized stochastic modeling methods for GPS long baseline and ZTD estimates are tested and analyzed:

- A: Standard stochastic model.
- B: Residual-based stochastic model.

2.1. Standard stochastic model

In a commonly-used stochastic model, it's usually assumed that all the carrier phases or pseudo-ranges have the same variance (σ^2) and are statistically independent. Therefore, the observation Φ is treated as independent and uncorrelated, and the covariance matrix of the observations Φ can be formulated as:

$$\mathrm{Cov}(\Phi) = \sigma^2 I \tag{3}$$

where I is the unit matrix. Through the error propagation law the time-invariant covariance matrix (called the stochastic model) of the DD measurements can be determined. This is a standard stochastic model for DD measurements, which is easy to be implemented in practice. However, this simplified stochastic model may contain some mis-specifications, and thus could result in unreliable GPS results.

2.2. Residual-based stochastic model

The residual-based stochastic model is based on the classic variation covariance (VCV). The classical variance covariance matrix is defined as following:

$$\begin{aligned} C_v &= E[(v - \bar{v})(v - \bar{v})^T)] \\ &= E\left(\begin{bmatrix} v_1 - \bar{v} \\ v_2 - \bar{v} \\ \vdots \\ v_n - \bar{v} \end{bmatrix} \begin{bmatrix} v_1 - \bar{v} & v_2 - \bar{v} & \ldots & v_n - \bar{v} \end{bmatrix} \right) \\ &= \begin{bmatrix} \sigma_{v_1}^2 & \sigma_{v_1 v_2} & \cdots & \sigma_{v_1 v_n} \\ \sigma_{v_2 v_1} & \sigma_{v_2}^2 & \cdots & \sigma_{v_2 v_n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{v_n v_1} & \sigma_{v_n v_2} & \cdots & \sigma_{v_n}^2 \end{bmatrix} \end{aligned} \tag{4}$$

where v is the residual estimation from Eq. (2) and $\bar{v}$ is the mean value of residual v. With the new variance-covariance matrix of Eq. (4), the solutions of Eq. (1) can be re-obtained, including the unknown parameters and its standard deviations. For long observation period data sets, the computational load is big and consumes immense computer memory. Therefore, the entire session is divided into short segments. In a short time, as the error characteristics change slowly with time, it is appropriate to divide the entire session into short segments, in which each short segment has the same number of satellites and the measurements of the same satellite pairs have an invariant stochastic model. If the epoch is N in one short segment, the variance–covariance matrix can be obtained through averaging the residuals within the same segment

$$C_v = \frac{1}{N}\sum_{i=1}^{N} v_i v_i^T \tag{5}$$

From Eq. (5), it can see that the width of moving window (N) affects on variance–covariance matrix. Therefore, the optimal width of moving window needs to be tested and defined. The test data sets used were from the Australian IGS GPS network (Fig. 1). At first the raw GPS observations are processed with the Bernese GPS-Software 5.0. In this run program, IGS-precise orbits, and ionosphere-free linear combination are used, and then the matrices A and L in Eq. (1) created by the Bernese GPS-Software (Hugentobler et al., 2001) are drawn out. Thus the residual-based and standard stochastic models can be realized with an independent program in Matlab 6.5. The impact of stochastic modelling on the ambiguity resolution is also important but not analyzed here. Finally, the results (unknown parameters and accuracies) of the residual-based stochastic model were compared with the results of the standard stochastic model in an independent program. We tested all kinds of moving window width (N) impacts on the GPS parameter estimation from 1 to 120 epochs. It has been shown that the moving window width (N) must be larger than or equal to the satellite pairs, otherwise the matrix (C_v) is singular. Also the satellite pairs should be the same within N epoch in one short segment. Table 1 lists the test results for six pairs of satellites with different moving window width from 6 to 120 epochs. It has been shown that the standard deviation for 8-epoch window width is smallest (namely optimal), but nearly closer to other longer moving window widths, indicating that the GPS error characteristics change slowly with time in a short time.

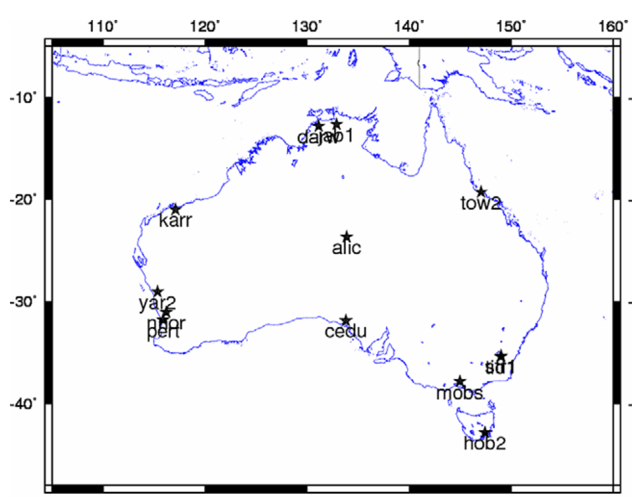

Fig. 1. IGS sites distribution in Australia.

Table 1
Effect of the moving window width on normalized RMS of GPS solutions.

Width (epochs)	6	7	8	9	10
Sigma (mm)	8.01	6.57	5.89	7.01	6.89
Width (epochs)	20	30	40	60	120
Sigma (mm)	7.17	7.47	7.87	7.98	7.61

3. Results

The GPS measurements of IGS stations in Australia (Alic, Cedu, Tow2 and Mobs) on day 151 2004 are processed using the Bernese 5.0, and the matrices A, L and v in Eq. (1) are obtained. Stochastic modeling methods A and B are then realized in an independent Matlab program. In the following, effect of stochastic models on baseline length and ZTD estimates are investigated.

3.1. Effect of moving window widths

As the width of moving window (N) directly affects the variance-covariance matrix, the different moving window widths of 8, 10, 12, 16, 20, 25, 30, 40 epochs in the baseline estimates are further tested and compared in the independent Matlab program, respectively. Fig. 2 shows a comparison of standard deviations of baseline estimations with different widths of moving window. It has been seen that the standard deviations of horizontal baseline component are twice as small as the height for each moving window width, and the standard deviation with an 8-epoch moving window width is obviously better than other cases, again indicating that 8-epoch window width is better.

3.2. Effect of stochastic models on baseline length

The stochastic modelling effects on baseline length estimates are investigated with standard stochastic model A and residual-based stochastic model B at an optimal 8-epoch window width. Fig. 3 shows a comparison of Alic–Cedu baseline solutions. Using method B, the

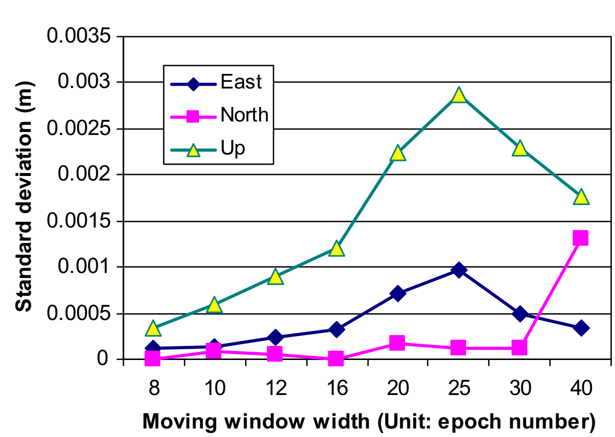

Fig. 2. Standard deviations of Alic–Cedu baseline estimation with different moving widow widths.

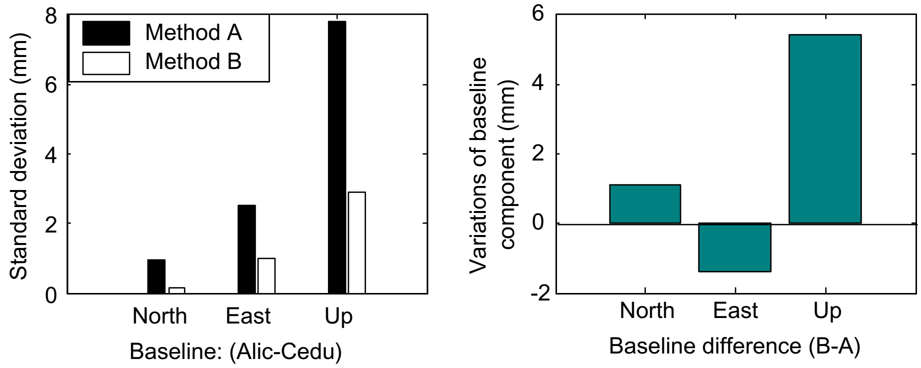

Fig. 3. Comparison of standard deviations and baseline differences (Alic–Cedu).

standard deviation of baseline component is obviously smaller than the method A (in the left panel of Fig. 3), and the corresponding difference of baseline component is about 5.8 mm in the vertical component and 1–3 mm in the horizontal component (in the right panel of Fig. 3). Fig. 4 shows the standard deviations and the baseline differences between methods B and A for the baseline Alic–Tow2, whose baseline length is 1445.5 km. It also shows that the standard deviation using method B is obviously better than method A, and the difference of baseline components between methods B and A is about 5 mm. In addition, the baselines of Alic–Mobs and Tow2–Mobs are further tested and compared. It has been shown that using different stochastic models, the difference in baseline estimates can reach 3–6 mm, mainly in GPS height component, while the precision estimations are largely improved when the residual-based stochastic model B is taken into account. Furthermore, the results using stochastic model B are closer to the reference value from the ITRF2000 (International Terrestrial Reference Frame 2000) (http://itrf.ensg.ign.fr/ITRF solutions/2000/ITRF2000).

GPS data with different time session lengths were further investigated and compared using stochastic modeling methods A and B (Fig. 5). Results show that the precision for GPS long baseline with short time observations (e.g., 4 or 6 h) is worse, at about 1-2 cm in horizontal component and 3–4 cm in vertical component, and moreover larger differences of baseline components between methods B and A are found (about 1–2 cm). When the time of GPS observations is longer than 12 h, reliable results can be obtained when compared to the reference value (Fig. 6). The difference of baseline components between methods B and A is up to 3 mm, mainly in the height component, gradually decreasing with the increase of GPS observation time length (12, 18 and 24 h). In addition, the results using stochastic model B with 24-h session are closer to the reference value.

3.3. Effect of stochastic models on ZTD estimate

The ZTD is also estimated using stochastic models A and B. Figs. 7 and 8 show the standard deviations and absolute ZTD estimations at the Alic station on day 151 2004 using stochastic models A and B. The ZTD difference can reach 10 mm between Methods A and B, but using stochastic model B the standard deviation of ZTD estimations is much improved and also the ZTD is closer to the reference value from the IGS solution. Figs. 9 and 10 show the comparisons of standard deviations and absolute ZTD estimations on day 151 2004 at the Cedu station using stochastic models A and B. It again shows that the standard deviations of ZTD estimations are much improved with Method B and also the ZTD is closer to the reference value from the IGS solution. Therefore, any misspecifications of stochastic model could result in unreliable ZTD estimates, up to 1 cm deviation. Using the residual-based stochastic

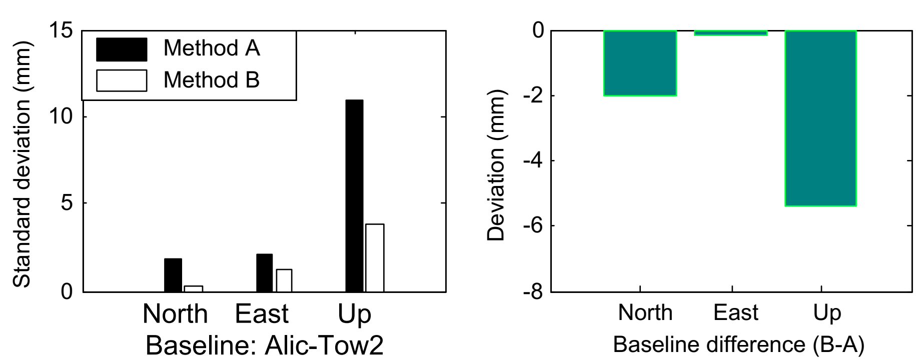

Fig. 4. Comparison of standard deviations and baseline differences (Alic–Tow2).

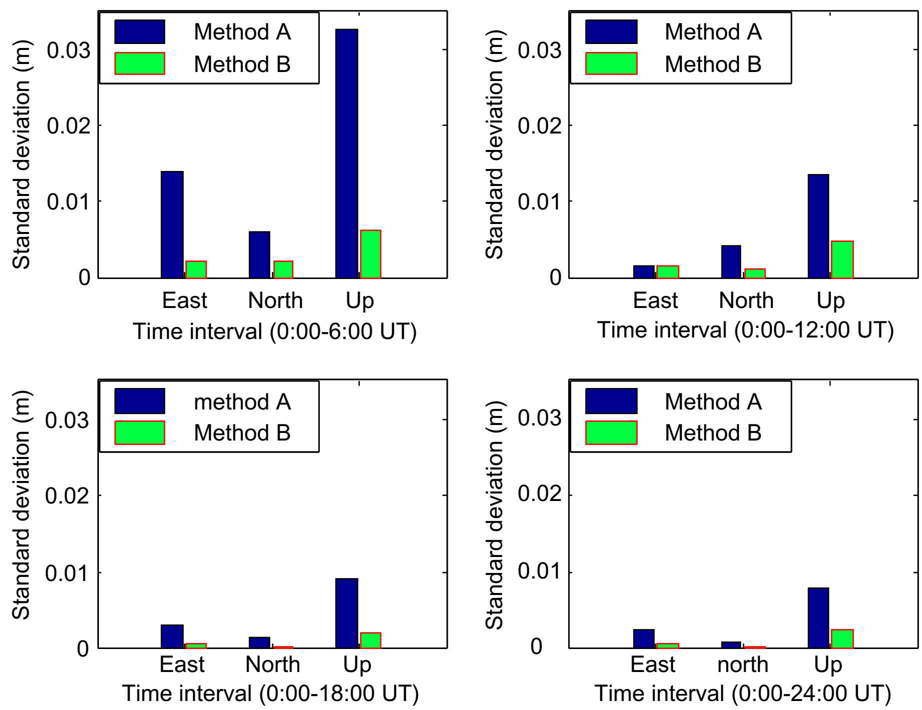

Fig. 5. Standard deviations of Alic–Cedu baseline estimation with different time intervals.

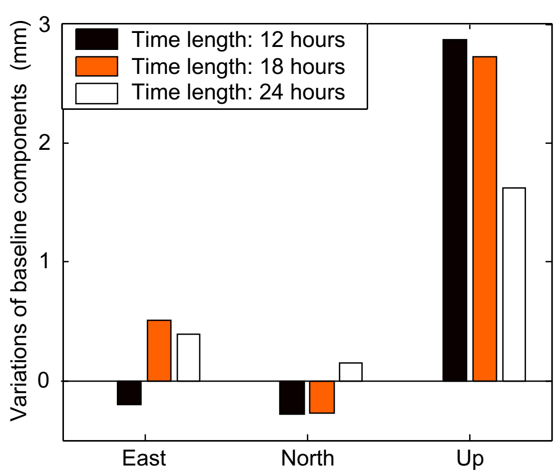

Fig. 6. Differences of Alic–Cedu baseline components between residual-based stochastic model and standard stochastic model.

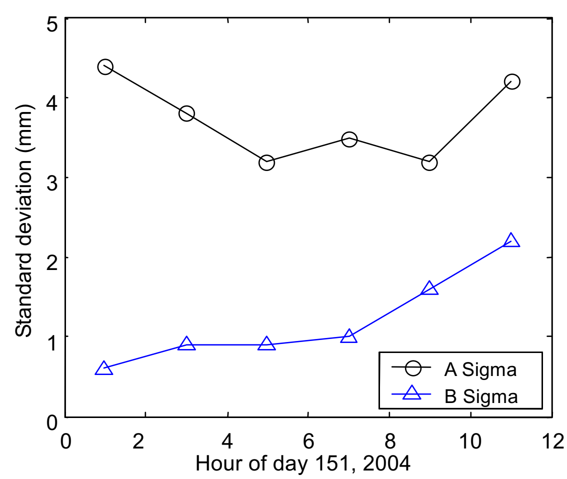

Fig. 7. Standard deviations of ZTD estimations at Alic using stochastic models A and B.

model (method B), the better performance of ZTD estimate is obtained.

4. Conclusions and discussion

Stochastic models are tested for long baseline and ZTD estimations of the IGS network in Australia with 24-h observations. It has been noted that, with the different stochastic models, the total differences can reach as much as 10 mm in the ZTD and 3–6 mm in the estimated baseline components, especially in the height component, which is not ignored for the current sub-millimeter precision GPS positioning and applications. Any mis-specification in the stochastic models will result in unreliable GPS baseline and ZTD estimations. Using the stochastic model B, residual-based stochastic model, not only the GPS baseline and ZTD estimations are closer to the reference value, but also their precisions are obviously improved. Therefore, the residual-based stochastic model has a best performance and is proposed to use in scientific software packages, e.g., GAMIT, Berenese and GIPSY. In addition, the optimal moving window width is tested using all kinds of moving widths from 1 to 120 epochs. It has been shown that the moving window width (N) must be larger than or equal to the satellite pairs, otherwise the matrix (C_x) is singular. Also the satellite pairs should be the same within N epoch

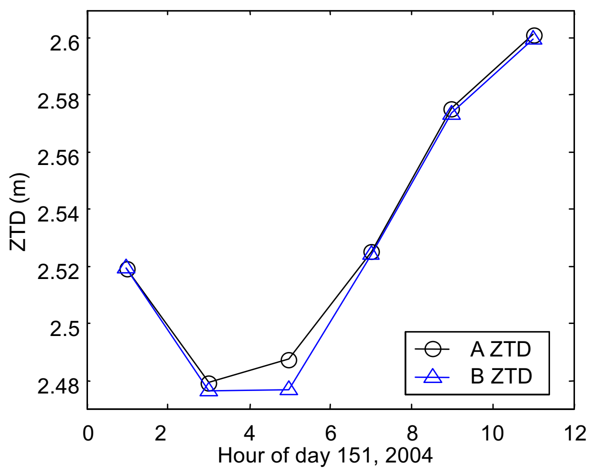

Fig. 8. ZTD estimation at Alic using stochastic models A and B.

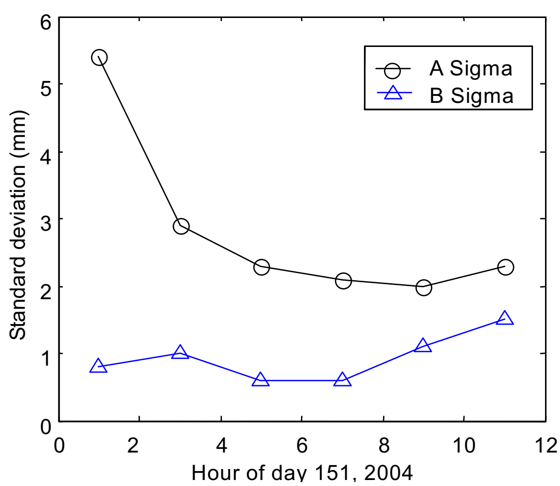

Fig. 9. Standard deviations of ZTD estimations at Cedu using stochastic models A and B.

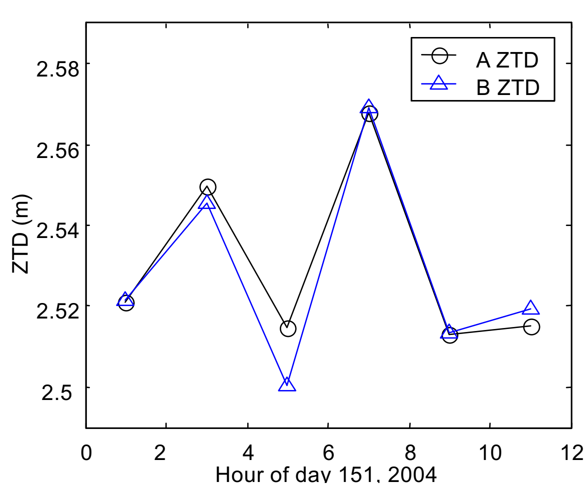

Fig. 10. ZTD estimations at Cedu using stochastic models A and B.

in one short segment. The optimal moving window width is 8 epochs in tested cases, but does not largely affect on stochastic model and GPS positioning solutions in a short time.

As GPS observation errors are dominated by the systematic errors, such as the multi-path, atmospheric delays, receiver noise and orbit errors, it is quite different for each satellite and GPS receiver. Although some errors can be mitigated or minimized by some models and appropriate processing techniques, parts of error sources are still not well eliminated, particularly for a low GPS satellite elevate angle. Such errors are difficult to be taken into account in the functional models. This paper demonstrated that GPS results could be improved by modelling some unmodelled errors into the stochastic model. This initial study has shown that the stochastic model methods play an important role in the GPS baseline and ZTD estimations. Suitable stochastic modeling strategies for GPS positioning applications should be further investigated in the future.

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