Modelling systematic residuals in absolute ZTD estimation from GPS

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***Abstract*—Tropsheric delay is one of important error sources in GPS positioning and InSAR applications. Accurate measurements of ZTD (Zenith Tropospheric Delay) can be used for correcting the atmospheric delay on GPS and InSAR as well as atmospheric science research and applications (e.g. numeric weather prediction). Traditionally, the GPS ZTD estimations were obtained based on the least squares (LS) principle, where the functional and stochastic models of GPS measurements need to be defined precisely. The functional models for GPS measurements have been investigated in considerable detail in the past two decades. However, most scientific GPS processing software packages, e.g. GAMIT, BERNESE or GIPSY, the stochastic models of GPS observation data are simplified, assuming that all the GPS measurements have the same variance, and that they are statistically independent in time and space. Such assumptions are unrealistic and will result in unreliable ZTD estimations as GPS observations from different satellites cannot have the same accuracy due to varying noise levels. In addition, it is impossible to model all systematic errors in the functional model, and therefore, modeling some systematic errors into the stochastic model is a current challenging topic to further realize the full potential of increasingly more accurate GPS positioning applications. This paper aims to improve the GPS ZTD estimations by modeling GPS systematic residuals into the stochastic model. The results show that the GPS ZTD estimation can be obviously improved through stochastic models, which is certainly critical for the accuracy improvement of GPS and InSAR and reliable atmospheric research and applications**

Keywords- Stochastic Modelling; GPS; ZTD, Systematic errors

I. INTRODUCTION

The global positioning system (GPS) has been widely used for positioning and navigation due to low cost, high precision, all-weather and high temporal resolution properties. However, the GPS signal propagating through the neutral atmosphere is delayed by variation of the refraction index due to temperature, pressure and water content, which results in lengthening of the ray-path, usually referred to as the "tropospheric delay". This delay is an important error source for GPS positioning, which contributes a bias in height of several centimeters even when simultaneously recorded meteorological data are used in tropospheric models [1, 2]. Nowadays, GPS has widely been used to determine the zenith tropospheric delay (ZTD) [1-4] through mapping functions [5]. The ZTD can be transformed into the precipitable water vapor (PWV) [2, 6, 7, 8]. Therefore GPS-derived ZTD provides a new tool in near real-time weather forecasting, especially improving the precision of Numerical Weather Prediction (NWP) models [9, 10]. In addition, the GPS-derived ZTD provides a new high-resolution data for research in atmospheric sciences and tropospheric delay corrections of other microwave techniques (e.g. InSAR). Therefore precise GPS-derived ZTD is valuable and beneficial.

The GPS ZTD estimation is traditionally obtained using the least squares (LS) principle. In order to employ the LS method for ZTD estimation, both the functional and stochastic models of GPS measurements need to be properly defined. The functional model, also called the mathematical model, describes the mathematical relationships between the GPS measurements and unknown parameters, e.g. baseline components and ZTD. The stochastic model describes the statistical properties of the measurements, physical property, which is mainly defined by an appropriate covariance matrix indicating the uncertainty of, and their error sources correlations between, the measurements [11, 12]. Over the past two decades, the functional models for GPS measurements have been investigated in considerable detail. However, the stochastic modeling for the GPS measurements is simplified or omitted [13].

In the current stochastic models for use in estimating the ZTD with GPS, it is usually assumed that all the GPS measurements have the same variance and are statistically independent, such as most popular scientific program GAMIT [14], BERNESE [15] and GIPSY. The time-invariant covariance matrix of the double-differenced (DD) measurements is then constructed using the error propagation law. As GPS measurement errors are dominated by the systematic errors caused by the multipath, ionosphere, and orbit effects, which are quite different for each satellite, the measurements obtained from different satellites cannot have the same accuracy due to varying noise levels [13]. Such assumptions are unrealistic and will result in unreliable ZTD estimations. Jin and Park [16] initially tested the impact of satellite elevation angle-based and baseline independent weighting stochastic models on ZTD estimates, which has shown that misspecification in the stochastic models will result in unreliable ZTD estimations, up to 2 cm deviation. Therefore, the suitable stochastic model plays an important role in ZTD estimates.

As it is impossible to model all systematic errors in the functional model, modeling some systematic errors into the stochastic model is a current challenging topic to further realize the full potential of increasingly more accurate GPS positioning applications, e.g. GPS-derived ZTD for metrological application and high-precision tropospheric delay

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correction. This paper aims to improve the GPS ZTD estimations for a long-baseline IGS GPS network in Australia by modeling GPS systematic residuals into the stochastic model**.** The section 2 describes the stochastic modeling methods, the section 3 will address results and discussions, and the final gives the conclusion.

II. Data processing method and strategy

For a pair of GPS observation stations and satellites (seeing Fig.1) and for one given observation day, let n be the number of observation epochs and x be a $q\times 1$ vector containing the q, unknown parameters. The ionosphere-free linear combination (LC) equation of double-differenced (DD) phase and code observations can be expressed as [11]:

$$\begin{aligned} LC_{kl}^{ij} &= \frac{1}{f_1^2 - f_2^2}(f_1^2 L1_{kl}^{ij} - f_2^2 L2_{kl}^{ij}) \\ &= \rho_{kl}^{ij} + ZTD_k(t)(m(z_k^i) - m(z_k^j)) - ZTD_l(t)(m(z_l^i) - m(z_l^j)) \quad (1) \\ &\quad + \frac{c}{f_1^2 - f_2^2}(f_1^2 N1_{kl}^{ij} - f_2^2 N2_{kl}^{ij}) + \varepsilon_{kl}^{ij} \end{aligned}$$

where ZTD is the zenith tropospheric delay, m is the mapping function, f_i is the frequency (i =1,2), Li and Ni are the double-differenced phase and ambiguity of the frequency f_i, respectively, ρ_{kl}^{ij} is the double-differenced pseudorange, c is the velocity of light in vacuum, and ε_{kl}^{ij} is noise. The linearized observation equations are written as:

$$L = Ax + v \qquad (2)$$

In (2), L is an $n\times 1$ vector of the observed-minus-computed DD carrier phase values (O-C), A is the $n\times q$ design matrix that describes the linearized functional model corresponding to the k-th observation epoch, k=1… n., x is the unknown parameters including ZTD, coordinate and ambiguity, etc., and v is the $n\times 1$ vector of error terms.

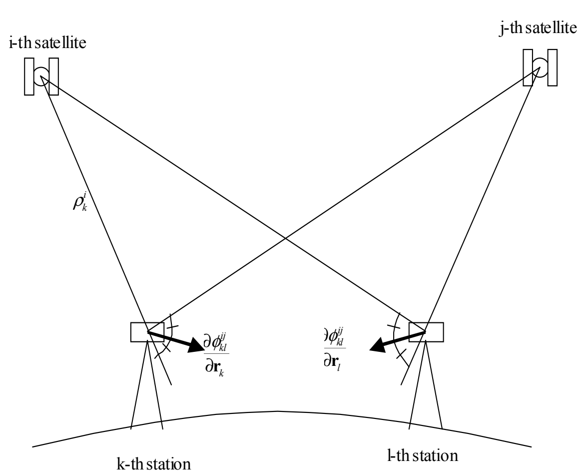

Fig. 1 GPS DD observation geometry

Using the least square method (LS), the unknown parameters and their uncertainties can be obtained, namely

$$\begin{aligned} \hat{x} &= (A^T C_x^{-1} A)^{-1} A^T C_x^{-1} L \\ \hat{v} &= L - A\hat{x} \\ \hat{\sigma}^2 &= \frac{v^T C_x^{-1} v}{f} \end{aligned} \qquad (3)$$

where $\hat{x}$ is the parameter estimate (including ZTD, coordinate , ambiguity, etc.), C_x is the variance-covariance matrix for the double-differenced GPS measurements, called the stochastic model, $\hat{v}$ is called the LS residual vector, $\hat{\sigma}$ is the uncertainty and f is the degree of freedom. It is easy to see that the estimation of the unknown parameter x and its precision indicator are dependent on the stochastic model. Any mis-specifications of the stochastic model maybe result in unreliable parameter estimations. In the following study, the stochastic models to be considered here are the ones that can be easily implemented or used within scientific GPS data processing software package of GAMIT or BERNESE.

Here, the following two stochastic modeling methods for GPS ZTD estimates are discussed and tested:

A: Standard GPS processing method with a simplified stochastic model and
B: Residual-based stochastic model

A Standard stochastic model

In a commonly used stochastic model, it is usually assumed that all the carrier phases or pseudo-ranges have the same variance (σ^2) and are statistically independent. Therefore, the observations Φ are treated as independent and uncorrelated, and the covariance matrix of the one-way observations Φ can be formulated as:

$$\mathrm{Cov}(\Phi) = \sigma^2 I \qquad (4)$$

where I is the unit matrix. Through the error propagation law the time-invariant covariance matrix (called the stochastic model) of the DD measurements can be obtained:

$$C_x = \sigma^2 \begin{bmatrix} 4 & 2 & \dots & 2 \\ 2 & 4 & \dots & 2 \\ \vdots & \vdots & \ddots & 2 \\ 2 & 2 & \dots & 4 \end{bmatrix} \qquad (5)$$

This is a standard stochastic model for DD measurements, which is easy to implement in practice. However, this simplified stochastic model may contain some mis-specifications, and thus could result in unreliable ZTD estimates.

B Residual-based stochastic model

Although there are a number of stochastic methods, such as satellite elevation angle-based, Baseline length dependent weighting, signal-noise-ratio, the MINQUE methods, etc., but the true residual should represent a good statistic property of GPS measurements. However, it is impossible to model all

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systematic errors in the functional model, and therefore modeling some systematic errors into the stochastic model is a challenge strategy to improve the GPS positioning solutions. Additionally, it can further test whether or not the stochastic model can be neglected. The residual-based stochastic model based on the classic variation co-variance is used to test. The classical definition of the variance-covariance matrix is as following:

$$C_v = E[(v-\bar{v})(v-\bar{v})^T)] = E(\begin{bmatrix} v_1-\bar{v} \\ v_2-\bar{v} \\ \vdots \\ v_n-\bar{v} \end{bmatrix}[v_1-\bar{v} \quad v_2-\bar{v} \quad \ldots \quad v_n-\bar{v}]) = \begin{bmatrix} \sigma_{v_1}^2 & \sigma_{v_1v_2} & \cdots & \sigma_{v_1v_n} \\ \sigma_{v_2v_1} & \sigma_{v_2}^2 & \cdots & \sigma_{v_2v_n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{v_nv_1} & \sigma_{v_nv_2} & \cdots & \sigma_{v_n}^2 \end{bmatrix} \tag{6}$$

where v is the residual estimation of (3) and $\bar{v}$ is the mean value of residual v. With the new variance-covariance matrix of (6), the solutions of (2) can be re-obtained, including the unknown parameters and its standard deviations. For long observation period data sets, the computational load is big and consumes immense computer memory. Therefore, the entire session is divided into short segments. In a short time, (6) error characteristics change slowly with time, it is appr to divide the entire session into short segments, in which each short segment has the same number of satellites and the measurements of the same satellite pairs have an invariant stochastic model. If the epoch is N in one short segment, the variance-covariance matrix can be obtained through averaging the residuals within the same segment.

$$C_v = \frac{1}{N}\sum_{i=1}^{N} v_i v_i^T \tag{7}$$

From (7), the width of moving window (N) affects directly on variance-covariance matrix and indirectly on GPS parameter estimations. Therefore, the optimal width of moving window needs to be decided or defined.

III. Results and discussion

The test data sets used were from the Australian IGS GPS network. At first the raw GPS observations were processed with the Bernese GPS-Software 4.2. In this run program, IGS-precise orbits, ionosphere-free linear combination, etc. were used. And the matrices A and L in Eq.(2) that were created in the Bernese GPS-Software can be drawn out. Thus the residual-based stochastic and standard stochastic models can be realized with an independent program in Matlab 6.5. The impact on the ambiguity resolution is also important but not analyzed here. Finally, the results (unknown parameters and accuracies) of the residual-based stochastic model were compared with the results of the standard stochastic model in an independent program. From (7), the width of moving window (N) affects directly on variance-covariance matrix, and the optimal width of moving window needs to be identified. We tested all kinds of moving window width (N) impacts on the ZTD estimation, from 1 to 120 epochs. It has shown the moving window width (N) must be larger than or equal to the satellite pairs, otherwise the matrix (C_v) is singular. Also the satellite pairs should be the same within N epoch in one short segment. This means that few GPS satellite observations without the same satellite pairs will be excluded.

Table.1 is a test result for 6 pairs of satellites with different moving window width from 6 to 120 epochs, which shows the standard deviation for 8-epoch window is smallest (namely optimal), but closer to other longer moving window width's, verifying that the GPS error characteristics change slowly with time in a short time. In addition, it has shown the moving window width (N) doesn't heavily affect on stochastic model and GPS solutions. However, for different conditions or different GPS network, such as satellite pair, observation time and observation station, the optimal epoch number is possible variable and need to be investigated further.

TABLE 1 Effect of the moving window width on GPS ZTD estimation

Width (epochs)	6	7	8	9	10
Sigma (mm)	8.01	6.57	5.89	7.01	6.89
Width (epochs)	20	30	40	60	120
Sigma (mm)	7.17	7.47	7.87	7.98	7.61

The GPS measurements of Australian IGS stations on day 151 2004 are processed using the BERNESE 4.2, and the matrices A, L and v in (2) are obtained. In practice, if the epoch number of the same satellite pairs is more than 24 epochs in some time session, 8 epochs are regarded as the width of moving window; if the epoch number of the same satellite pairs is less than 16 epochs in some time session, the total number of epochs is regarded as the width of moving window; and besides these, the approximate mean number of epochs is regarded as the width of moving window. The ZTD with 2-hour interval are estimated with an independent program in Matlab 6.5 and standard Bernese GPS-Software, respectively. Fig.2 are the comparisons of standard deviations (left) and absolute ZTD estimations (right) at the ALIC station on day 151 2004 using stochastic models A and B. The standard deviations of ZTD estimations are much reduced with Method B, and corresponding ZTD difference can reach 1 cm between Methods A and B. Compared to the solution of standard Bernese GPS-Software, the one of method B is closer.

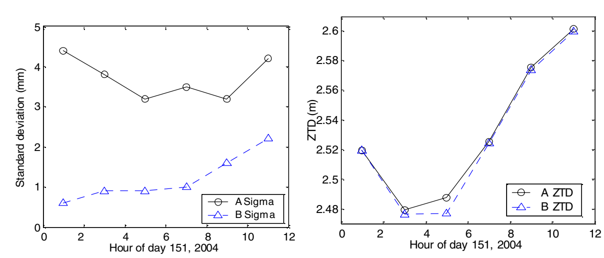

Fig.2 Standard deviations (left) and ZTD estimations (right) on the ALIC station using stochastic models A and B

Fig. 3 show the comparisons of standard deviations (left) and absolute ZTD estimations (right) on day 151 2004 at the CEDU station using stochastic models A and B. It has shown the same conclusion that the standard deviations of ZTD estimations are much reduced with Method B, and corresponding ZTD difference can reach 1 cm between Methods A and B. Compared to the solution of standard

BERNESE GPS-Software, the one of method B is also closer. Therefore, any misspecifications of stochastic model could result in unreliable ZTD estimates, up to 1cm deviation. Using the residual-based stochastic model, namely method B, the standard deviations of ZTD estimations are smaller, and corresponding ZTD estimations are closer to the reference value, standard BERNESE GPS-Software solution.

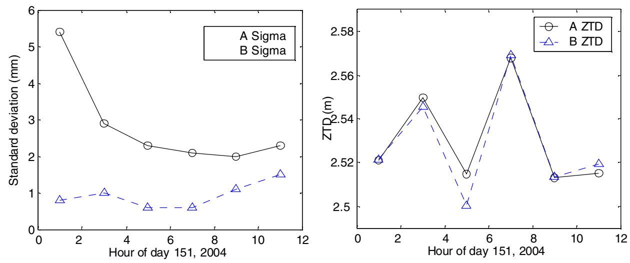

Fig.3 Standard deviations (left) and ZTD estimations (right) on the CEDU station using stochastic models A and B

As GPS measurement errors are dominated by the systematic errors caused by the multi-path, atmosphere, receiver noise and orbit effects, etc., which are quite different for each satellite, the measurements obtained from different satellites cannot have the same accuracy. Therefore, the ZTD was usually estimated under the assumptions that all the GPS measurements have the same variance and are statistically independent, and such assumptions are unrealistic. Although some errors can be mitigated or minimized by some correction models and appropriate processing techniques, the most error sources, especially in a low GPS satellite elevate angle, are not well eliminated, e.g. the multi-path delay error. Such error increases when the satellite elevation cut-off angle decreases and is also difficult to be taken into account in the functional models. In the theory, it can further improve accurate GPS ZTD estimations by modeling some systematic errors into the stochastic model. The above tested results show that using the residual-based stochastic model, not only the ZTD is closest to the reference value, but also the precision is obviously improved.

IV. Conclusions

The GPS ZTD was usually estimated using LS technique under the assumptions that all the GPS measurements have the same variance and are statically independent. In this paper, stochastic models are tested for GPS ZTD estimation with a long baseline IGS network in Australia. It has been noted that, with the different tested stochastic modeling methods, the total changes in the estimated ZTD values can reach as much as 10 mm, which is not omitted for the uncertainty of GPS ZTD with the order of several millimeters. Any mis-specification in the stochastic models will result in unreliable ZTD estimations. Using good stochastic model, residual-based stochastic model, not only the precision of ZTD estimation is obviously improved, but also the ZTD estimation is closer to reference value. In addition, the optimal moving window width is identified by testing all kinds of moving width from 1 to 120 epochs. It has shown the moving window width (N) must be larger than or equal to the satellite pairs, otherwise the matrix (C_x) is singular. Also the satellite pairs should be the same within N epoch in one short segment. The optimal moving window width is 8 epochs, but doesn't heavily affect on stochastic model and GPS solutions in a short time.

Therefore, the residual-based stochastic model has a best performance and is proposed to use in current popular scientific software package, GAMIT, BERNESE and GIPSY. This improvement of ZTD estimation is certainly critical for reliable GPS meteorology applications and other tropospheric delay corrections. This initial study has demonstrated that the stochastic model methods play an important role in the ZTD estimation process. Suitable stochastic modeling strategies for GPS ZTD parameter estimation should be further investigated.

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