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Journal of Geodynamics 38 (2004) 85–92
JOURNAL OF GEODYNAMICS
http://www.elsevier.com/locate/jog

A revision of the parameters of the NNR-NUVEL-1A plate velocity model

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Received 5 September 2003; received in revised form 8 March 2004; accepted 15 March 2004

Abstract

The NNR-NUVEL-1A model, as an international standard model of the International Terrestrial Reference Frame (ITRF), should be an accurate and rigorous enough no net rotation (NNR) model. In this paper, the new inertial *Q*-tensor of global plates is accurately calculated with a new method, and further the total angular momentum of global plates is obtained. The result shows that the reference frame constrained by NNR-NUVEL-1A is rotating at 0.012°/Ma with respect to the Earth lithosphere, and the NNR constraint of NNR-NUVEL-1A is not realized completely. A new revised model NNR-NUVEL-1B is presented.

1. Introduction

The terrestrial reference frame is an important reference benchmark for researching global and regional motions of the Earth. At present, it is maintained by International Earth Rotation Service (IERS) incorporating space geodetic techniques and following four principles (Jin and Zhu, 2003). One principle that should be satisfied by the terrestrial reference frame is no net rotation (NNR) with regard to the Earth's lithosphere, which can be mathematically expressed as the Tisserand Condition (namely the total angular momentum of global plates is zero). In 1996, the NNR-NUVEL-1A model established by DeMets et al. (1990, 1994) from geological and geomagnetic data in the last 3 Ma was regarded as an international standard model of the ITRF. From ITRF91 to ITRF94, NNR conditions were imposed to achieve the NNR constraint with respect to the Earth's lithosphere by using NNR-NUVEL-1A. The NNR constraint of

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doi:10.1016/j.jog.2004.03.004

ITRF2000 published recently by IERS was also satisfied by the NNR-NUVEL-1A model. Consequently the NNR-NUVEL-1A should be an accurate and rigorous enough NNR model.

The NNR-NUVEL-1A model, however, was deduced from NUVEL-1A based on NNR constraint of the Earth's lithosphere by Argus and Gordon (1991). The inferred inertial Q-tensor is subject to the following problems: (1) the sites along global plate boundaries within a subset of the NUVEL-1A model are sparse, and (2) the components of the inertial Q-tensor have been obtained through a simple and imprecise integral, so the precision of inertial Q-tensor is very low. Therefore, the NNR constraint of the NNR-NUVEL-1A model is not realized based on the inertial Q-tensor rigorously and completely. In addition, many authors have found that there is a net rotation in the ITRF96 (Zhang et al., 1999), ITRF97 (Zhu et al., 2000) and ITRF2000 (Jin and Zhu, 2002a,b) frames based on the inertial Q-tensor calculated by Argus and Gordon (1991), and have respectively established the corresponding NNR models, such as NNR-ITRF96VEL, NNR-ITRF97VEL and NNR-ITRF2000VEL. Strictly speaking, these conclusions and results are not accurate and rigorous. In this paper, we recalculate the new inertial Q-tensor of global plates with a new method, discuss the rigorous NNR condition of the reference frame constrained by NNR-NUVEL-1A, and further give a revision of the parameters of the NNR-NUVEL-1A plate velocity model.

2. New inertial Q-tensor of global plates

NNR, which is also called the Tisserand Condition, is one of the criteria of Conventional Terrestrial Reference Frame (CTRS). The strict Tisserand Condition is used to define an ideal terrestrial reference frame, i.e. the angular momentum summation of the whole Earth is zero with respect to this reference frame. It can be expressed as follows

$$\vec{L} = \int_D \vec{r} \times \vec{V}\,\mathrm{d}m = 0, \tag{1}$$

where $\vec{L}$ is the total angular momentum summation of the Earth's lithosphere, $\vec{V}$ and $\vec{r}$ are the site velocity vector and position vector respectively, D is the whole lithosphere surface of the Earth and dm is a unit mass of the Earth.

Assuming the Earth is a unit sphere and the density of the Earth's crust distributes uniformly, Eq. (1) can be approximately written as

$$\vec{L} = \sum_{i=1}^{k} \vec{Q}_i \vec{\Omega}_i = 0, \tag{2}$$

where k is the total number of plates, $\vec{\Omega}_i$is the Euler vector of plate i, and $\vec{Q}_i$ is the inertial tensor of plate i which can be written as

$$\vec{Q}_{ij} = \int (\delta_{ij} - x_i x_j)\,\mathrm{d}A = \int (\delta_{ij} - x_i x_j) \sin\varphi\,\mathrm{d}\varphi\,\mathrm{d}\lambda \tag{3}$$

where $\vec{Q}_{ij}$ are the components of the inertial Q-tensor, x_k ($k = 1, 2, 3$) are Cartesian coordinates along plate boundaries, δ_{ij} is the Kronecker symbol, and φ and λ are the latitude and longitude respectively. Argus and Gordon (1991) calculated the coefficients $\vec{Q}_{ij}$ through Eq. (3).Here we present a new method to calculate the coefficients $\vec{Q}_{ij}$. Firstly let $\vec{r}$ and $\vec{V}$ in Eq. (1) be expressed using Cartesian coordinates (Li, 2003), so that $\vec{r} \times \vec{V}\,\mathrm{d}m$ can be written as

$$\vec{r} \times \vec{V}\,\mathrm{d}m = [(yv_x - zv_y)i + (zv_x - xv_z)j + (xv_y - yv_x)k]\,\mathrm{d}m \tag{4}$$

where

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} R\cos\varphi\cos\lambda \\ R\cos\varphi\sin\lambda \\ R\sin\varphi \end{pmatrix}, \qquad \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} y\omega_z - z\omega_y \\ z\omega_x - x\omega_z \\ x\omega_y - y\omega_x \end{pmatrix}, \qquad \mathrm{d}m = \rho R^2\cos\varphi\,\mathrm{d}\varphi\,\mathrm{d}\lambda\,\mathrm{d}R,$$

where $\mathrm{d}\varphi$, $\mathrm{d}\lambda$ and $\mathrm{d}R$ are differences of latitude, longitude and radius, respectively, and $(\omega_x, \omega_y, \omega_z)$ are the components of Euler vectors.

Under the assumptions that the Earth is a unit sphere and the density of the crust distributes uniformly, we can easily obtain the following equations through an integral over R (Earth radius) on the two sides of Eq. (4):

$$\int_D [(\sin^2\varphi + \cos^2\varphi\sin^2\lambda)\omega_x - \cos^2\varphi\sin\lambda\cos\lambda\omega_y - \sin\varphi\cos\varphi\cos\lambda\omega_z]\cos\varphi\,\mathrm{d}\lambda\,\mathrm{d}\varphi = 0$$

$$\int_D [(-\cos^2\varphi\sin\lambda\cos\lambda)\omega_x + (\sin^2\varphi + \cos^2\varphi\cos^2\lambda)\omega_y - \sin\varphi\cos\varphi\sin\lambda\omega_z]\cos\varphi\,\mathrm{d}\lambda\,\mathrm{d}\varphi = 0$$

$$\int_D [(-\sin\varphi\cos\varphi\cos\lambda)\omega_x - \sin\varphi\cos\varphi\sin\lambda\omega_y + \cos^2\varphi\omega_z]\cos\varphi\,\mathrm{d}\lambda\,\mathrm{d}\varphi = 0$$

where D is the whole lithosphere surface. Eq. (2) can be reduced to the following expression:

$$\sum_{i=1}^{m} \begin{pmatrix} Q_{11,i} & Q_{12,i} & Q_{13,i} \\ Q_{12,i} & Q_{22,i} & Q_{23,i} \\ Q_{13,i} & Q_{23,i} & Q_{33,i} \end{pmatrix} \begin{pmatrix} \omega_{x,i} \\ \omega_{y,i} \\ \omega_{z,i} \end{pmatrix} = 0 \tag{5}$$

where $Q_{11} = [Q_{11,1}, Q_{11,2}, \ldots, Q_{11,m}]$ are

$$\begin{aligned} Q_{11,i} = \sum_{j=1}^{n_j} \Bigg[& \frac{1}{3}(\sin^3\varphi_{i,j,2} - \sin^3\varphi_{i,j,1})(\lambda_{i,j,2} - \lambda_{i,j,1}) \\ & + \frac{1}{2}\left(\sin\varphi_{i,j,2} - \sin\varphi_{i,j,1} - \frac{1}{3}(\sin^3\varphi_{i,j,2} - \sin^3\varphi_{i,j,1})\right) \\ & \times \left(\lambda_{i,j,2} - \lambda_{i,j,1} - \frac{1}{2}(\sin 2\lambda_{i,j,2} - \sin 2\lambda_{i,j,1})\right)\Bigg] \end{aligned}$$

$$Q_{12} = [Q_{12,1}, Q_{12,2}, \ldots, Q_{12,m}]$$

$$Q_{12,i} = \sum_{j=1}^{n_j}\left(-\frac{1}{2}\right)\left[\left(\sin\varphi_{i,j,2} - \sin\varphi_{i,j,1} - \frac{1}{3}(\sin^3\varphi_{i,j,2} - \sin^3\varphi_{i,j,1})\right)(\sin^2\lambda_{i,j,2} - \sin^2\lambda_{i,j,1})\right]$$

$$Q_{13} = [Q_{13,1}, Q_{13,2}, \ldots, Q_{13,m}]$$

$$Q_{13,i} = \sum_{j=1}^{n_j} \frac{1}{3}[(\cos^3 \varphi_{i,j,2} - \cos^3 \varphi_{i,j,1})(\sin \lambda_{i,j,2} - \sin \lambda_{i,j,1})]$$

$$Q_{22} = [Q_{22,1}, Q_{22,2}, \ldots, Q_{22,m}]$$

$$\begin{aligned} Q_{22,i} = \sum_{j=1}^{n_j} \Bigg[&\frac{1}{3}(\sin^3 \varphi_{i,j,2} - \sin^3 \varphi_{i,j,1})(\lambda_{i,j,2} - \lambda_{i,j,1}) \\ &+ \frac{1}{2}\left(\sin \varphi_{i,j,2} - \sin \varphi_{i,j,1} - \frac{1}{3}(\sin^3 \varphi_{i,j,2} - \sin^3 \varphi_{i,j,1})\right) \\ &\times \left(\lambda_{i,j,2} - \lambda_{i,j,1} + \frac{1}{2}(\sin 2\lambda_{i,j,2} - \sin 2\lambda_{i,j,1})\right)\Bigg] \end{aligned}$$

$$Q_{23} = [Q_{23,1}, Q_{23,2}, \ldots, Q_{23,m}]$$

$$Q_{23,i} = \sum_{j=1}^{n_j} \left[-\frac{1}{3}(\cos^3 \varphi_{i,j,2} - \cos^3 \varphi_{i,j,1})(\cos \lambda_{i,j,2} - \cos \lambda_{i,j,1})\right]$$

$$Q_{33} = [Q_{33,1}, Q_{33,2}, \ldots, Q_{33,m}]$$

$$Q_{33,i} = \sum_{j=1}^{n_j} \left[\left(\sin \varphi_{i,j,2} - \sin \varphi_{i,j,1} - \frac{1}{3}(\sin^3\varphi_{i,j,2} - \sin^3\varphi_{i,j,1})\right)(\lambda_{i,j,2} - \lambda_{i,j,1})\right]$$

$$\omega_x = [\omega_{x,1}, \omega_{x,2}, \ldots, \omega_{x,m}], \qquad \omega_y = [\omega_{y,1}, \omega_{y,2}, \ldots, \omega_{y,m}], \qquad \omega_z = [\omega_{z,1}, \omega_{z,2}, \ldots, \omega_{z,m}]$$

In the expressions above, i ($i = 1, 2, \ldots, m$) is the number of plates, n_j is the number of small regular block in the plate i, $\varphi_{i,j,1}$, $\varphi_{i,j,2}$, $\lambda_{i,j,1}$ and $\lambda_{i,j,2}$ are respectively the least value of latitude, the largest value of latitude, the least value of longitude, and the largest value of longitude in the small regular block j of plate i. Finally, the inertial Q-tensor of global plates is obtained with the plate boundary data of the current plate velocity model NUVEL-1 (DeMets et al., 1990) (Table 1, the plate boundary data can be obtained through anonymous FTP from the server ftp.iamg.org).

Schettino (1999) computed some important geometric parameters of the tectonic plates based on a triangulation algorithm for spherical polygons, such as plate areas or components of the inertial Q-tensor, and produced highly reliable results. The components of the inertial Q-tensor of 14 modern plates modified from Schettino (1999) are listed in Table 2, which almost coincide with our results. The total area of 14 plates within a subset of the NUVEL-1 model is 12.56633 which equates to 4π, entirely standing for the whole area of the Earth's surface, which shows the components of the inertial Q-tensor derived from the two methods are reliable. Additionally, the differences between components of the inertial Q-tensor of Argus and Gordon (1991) and ours are very small (Table 3).

Table 1
The components of the inertial Q-tensor of 14 modern plates

Plate	Q_{11}	Q_{22}	Q_{33}	Q_{12}	Q_{13}	Q_{23}
Africa	0.66051	1.53907	1.64090	−0.25689	0.05547	0.10900
Antarctica	1.33020	1.17667	0.36426	−0.04962	0.05285	0.07723
Arabia	0.07914	0.06921	0.10397	−0.04901	−0.03053	−0.03226
Australia	0.83575	0.69715	0.94326	0.21084	−0.22493	0.29134
Caribbean	0.09684	0.01388	0.09958	0.02353	−0.00577	0.02099
Cocos	0.07266	0.00348	0.07190	−0.00617	0.00130	0.01088
Eurasia	1.43062	1.09600	0.90479	0.10812	−0.15475	−0.41478
India	0.25547	0.03997	0.25223	−0.05459	−0.01371	−0.05804
Nazca	0.38932	0.06997	0.34533	−0.01394	−0.00376	−0.11589
North America	1.26934	0.99651	0.58104	0.07511	0.01888	0.38472
South America	0.67813	0.64465	0.85539	0.35215	0.20517	−0.19160
J.de Fuca	0.00614	0.00519	0.00369	−0.00174	0.00224	0.00302
Philippine	0.07658	0.07808	0.12405	0.06082	0.02901	−0.02785
Pacific	1.19518	2.00993	2.09106	−0.39917	0.07051	−0.05852

Unit: Steradian.

Table 2
The components of the inertial Q-tensor modified from Schettino (1999)

Plate	Area	Q_{11}	Q_{22}	Q_{33}	Q_{12}	Q_{13}	Q_{23}
Africa	1.92039	0.66069	1.54150	1.63858	−0.25148	0.05431	0.10942
Antarctica	1.43634	1.33036	1.17699	0.36532	−0.05097	0.05227	0.07885
Arabia	0.12610	0.07901	0.06922	0.10397	−0.05033	−0.03114	−0.03340
Australia	1.20547	0.83653	0.63133	0.94307	0.21035	−0.22510	0.29022
Caribbean	0.10521	0.09692	0.01395	0.09955	0.02354	−0.00582	0.02111
Cocos	0.07401	0.07264	0.00351	0.07186	−0.00620	0.00130	0.01087
Eurasia	1.71569	1.43059	1.09595	0.90481	0.10818	−0.15478	−0.41443
India	0.27345	0.25452	0.04004	0.25234	−0.05468	−0.01363	−0.05808
Nazca	0.40221	0.39023	0.06993	0.34426	−0.01399	−0.00360	−0.11521
North America	1.42221	1.26952	0.99509	0.57981	0.07523	0.01896	0.38485
South America	1.08960	0.67790	0.64589	0.85540	0.35121	0.20546	−0.19116
J.de Fuca	0.00752	0.00617	0.00518	0.00369	−0.00178	0.00227	0.00298
Philippine	0.13934	0.07761	0.07713	0.12394	0.06065	0.02837	−0.02792
Pacific	2.64879	1.19484	2.01177	2.09096	−0.39975	0.07112	−0.05810
Earth	12.56633	8.37752	8.37749	8.37756	−0.00001	−0.00001	0.00001

Unit: Steradian.

3. New NNR model NNR-NUVEL-1B

The values of total angular momentum of the NNR-NUVEL-1A model based on several inertial Q-tensors are shown in Table 4. The total angular momentum of global plates based on our new inertial Q-tensors is about 0.012°/Ma, which shows that the reference frame constrained by the NNR-NUVEL-1A model is rotating at 0.012°/Ma with respect to the Earth's lithosphere, and the NNR constraint of the NNR-NUVEL-1A model is not realized rigorously.

Table 3
The differences between the inertial Q-tensor of Argus and Gordon and ours

Plate	Q_{11}	Q_{22}	Q_{33}	Q_{12}	Q_{13}	Q_{23}
Africa	0.0038	0.0025	0.0094	−0.0068	0.0002	0.0002
Antarctica	−0.0336	−0.0335	−0.0129	−0.0030	−0.0107	0.0057
Arabia	0.0014	0.0002	0.0003	0.0011	−0.0000	0.0003
Australia	0.0008	0.0647	−0.0005	−0.0085	0.0003	0.0011
Caribbean	0.0124	0.0014	0.0133	0.0028	−0.0004	0.0012
Cocos	−0.0020	−0.0002	−0.0019	0.0002	−0.0001	−0.0003
Eurasia	0.0348	0.0211	0.0297	0.0120	0.0073	−0.0062
India	−0.0221	−0.0024	−0.0230	0.0036	0.0003	0.0015
Nazca	−0.0048	−0.0010	−0.0018	−0.0005	−0.0002	0.0005
North America	−0.0024	−0.0031	0.0061	0.0021	−0.0052	0.0096
South America	0.0134	0.0192	−0.0105	−0.0025	0.0117	−0.0143
J.de Fuca	0.0004	0.0004	0.0002	−0.0000	0.0001	0.0002
Philippine	−0.0033	0.0001	−0.0013	−0.0005	−0.0003	0.0014
Pacific	−0.0006	−0.0072	−0.0032	−0.0006	−0.0014	−0.0029

Unit: Steradian.

Table 4
Comparisons of the total angular momentum ($|L|$, °/Ma)

	Schettino (1999)	This study	Argus and Gordon (1991)
NNR-NUVEL-1A	0.0115	0.0124	0.0047

Table 5
Comparisons of Euler parameters between NNR-NUVEL-1B and NNR-NUVEL-1A[a]

Plate	NNR-NUVEL-1A			NNR-NUVEL-1B		
	Ω (°/Ma)	λ (°)	φ (°)	Ω (°/Ma)	λ (°)	φ (°)
Africa	0.291	−74.0	50.6	0.291	−73.591	50.640
Antarctic	0.238	−115.8	63.0	0.238	−115.261	63.222
Arabia	0.543	−4.5	45.0	0.544	−4.437	44.935
Australia	0.646	33.2	33.9	0.647	33.154	33.866
Caribbean	0.214	−93.0	25.0	0.214	−92.627	25.135
Cocos	1.510	−115.8	24.5	1.509	−115.758	24.526
Eurasia	0.234	−112.3	50.6	0.234	−111.889	50.806
India	0.545	0.3	45.5	0.546	0.348	45.432
Nazca	0.743	−100.1	47.8	0.743	−99.962	47.850
North America	0.207	−85.9	−2.4	0.207	−85.540	−2.299
South America	0.116	−124.4	−25.3	0.115	−123.920	−25.341
Philippine	0.900	−35.4	−38.0	0.900	−35.319	−37.948
Pacific	0.641	107.3	−63.0	0.641	107.041	−62.990

[a] Ω is the rotation rate, and λ and φ are the longitude and latitude of rotation pole, respectively.

In order to rigorously obtain the NNR model constrained by the NNR-NUVEL-1A model, the Euler's vector of a new revised model NNR-NUVEL-1B, $\vec{\Omega}'_j$ can be obtained from the following formula (Zhang et al., 1999):

$$\vec{\Omega}'_j = \vec{\Omega}_j - \left(\sum_i Q_i\right)^{-1} \sum_i Q_i \vec{\Omega}_i = \vec{\Omega}_j - \left(\frac{3}{8}\pi\right)\vec{L} \tag{6}$$

where $\vec{\Omega}_j$ is the Euler vector of the NNR-NUVEL-1A model, and $\vec{L}$ is the total lithosphere angular momentum. The Euler parameters of the NNR-NUVEL-1B model are shown in Table 5.

4. Discussion

The differences between the NNR-NUVEL-1B and NNR-NUVEL-1A models are very small, and the largest difference of Euler rotation rate is 0.0012°/Ma that is almost equal to plate motion of 0.1 mm/a. For space geodetic studies with an accuracy of about 1 mm/a, it can be entirely ignored. However, the reference frame constrained by NNR-NUVEL-1A is rotating at 0.012°/Ma with respect to the Earth's lithosphere, which is almost equal to the long-term variation of the Earth's rotation rate and polar motion at 0.0436 mas/a (milli-arc-sec/a). The newest measured result of long-term polar motion from space geodesy has the amplitude 4.123 ± 0.002 mas/a and the direction 73.9 ± 0.03 W° (Gross and Vondrak, 1999). Thus it can be seen that the magnitude of NNR is one order of magnitude larger than the present measured long-term polar motion. Accordingly, the net rotation of reference frame constrained by NNR-NUVEL-1A, about 0.012°/Ma, cannot be ignored for the measurement of long-term polar motion. The new revised model NNR-NUVEL-1B rigorously satisfies the NNR condition and can be regarded as a better international standard model of the ITRF.

In addition, the inertial Q-tensors calculated by Argus and Gordon (1991) are imprecise. Our method to calculate the coefficients Q_{ij} yields results, which are almost consistent with the ones by Schettino (1999) with the same plate boundary data of the model NUVEL-1. The new inertial Q-tensors of this paper are useful for studying the net rotation of ITRF and realizing NNR models in the future.

Acknowledgements

This work was supported by the National Important Basic Research Project (Grant No. G1998040703) and Open Research Fund Program of the Key Laboratory of Geospace Environment and Geodesy, Ministry of Education, China (030405).

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